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MASTER THESIS IN MICRODATA ANALYSIS

**Modeling and forecasting regional GDP in Sweden
using autoregressive models**

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Abstract

Regional Gross Domestic Product (GDP) per capita is an important indicator of regional economic activity, and is often used by decision makers to plan economic policy. In this thesis, based on time series data of regional GDP per capita in Sweden from 1993 to 2009, three autoregressive models are used to model and forecast regional GDP per capita. The included models are the Autoregressive Integrated Moving Average (ARIMA) model, the Vector Autoregression (VAR) model and the First-order Autoregression (AR(1)) model. Data from five counties were chosen for the analysis, Stockholm, Västergötland, Skåne, Östergötland and Jönköping, which are the top 5 ranked counties in Sweden with regard to regional GDP per capita. Data from 1993 to 2004 are used to fit the model, and then data for the last 5 years are used to evaluate the performance of the prediction. The results show that all the three models are valid in forecasting the GDP per capita in short-term. However, generally, the performance of the AR(1) model is better than that of the ARIMA model. And the predictive performance of the VAR model was shown to be the worst.

KEY WORDS: Autoregressive model, GDP per capita, ARIMA model, VAR model, AR(1) model

*This thesis is dedicated to my parents
for their love and support
throughout my life.*

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1. Introduction

Gross domestic product (GDP) refers to the value of all final goods and services produced within a country or an area in a period of time (a quarter or a year), and is often considered the best standard of measuring national economic conditions (Mankiw & Taylor 2007). According to the data of GDP all over the world (the latest update is 24th March 2013), the United States was ranked at the top of the list, while Sweden was ranked 21st. The GDP of Sweden in 2012 was 524 billion US dollars, and the growth rate of the GDP was 1.5%.¹

Regional gross domestic product (GDPR) refers to the final results of all resident units' production activities in a certain region in a period of time (Pav á & Cabrer 2007). GDPR estimated from the production side is the aggregate of value added in a region. The sum of all regions' GDPR is equal to the GDP of the country.

Gross Domestic Product per capita (GDP per capita) is often used as a measure of economic development, and is one of the most important measures in macroeconomics. GDP per capita is a useful tool to study the macroeconomic situation of a country or a region. We use a real GDP in a national accounting period (usually a year) divided by the resident population (registered population) to get GDP per capita. GDP per capita is often combined with measures of the purchasing power parity (PPP) to measure people's living standard more objectively (Larsson & Harrell 2007). The GDP per capita in Sweden in 2012 was 56,956 US dollars, which was ranked 8th in the world.² Although the GDP of Sweden was ranked 21th in the world, the GDP per capita of Sweden was ranked a lot higher.

¹ Source: World Economic Outlook Database, April 2012. Official website of IMF.

² Source: World Economic Outlook Database, April 2012. Official website of IMF.

The significance of GDP per capita as a measure of economic development can be seen in three aspects. Firstly, GDP per capita reflects the level and degree of economic development in industrialized countries. For instance, the GDP per capita of Luxembourg was ranked 1st in the world in 2012. Although the international status and influence of Luxembourg is inferior to India's, which was ranked 140th in the world, in terms of education, health, social security etc., the Luxembourg social development level and the balanced development between urban and rural areas would by most be considered better than India (Yan 2011).

Secondly, if individual income levels in a country do not vary much between residents, the data collected to measure GDP per capita can also be used to measure social justice and equality. In fact, the countries which emphasize GDP-growth per capita usually also pay attention to improve the level of income per capita and social equity.

Thirdly, GDP per capita has also been shown to be related to the level of social stability in a country. At a certain stage, the growth of GDP per capita is often related to social stability. Research indicates that 1000-3000 US dollars per capita is considered as the initial stage of industrialization, 4000-6000 US dollars per capita is considered as the medium stage of industrialization. After the initial stage of industrialization, and compared with a traditional society, the social instability factor can increase. Some countries in the process of modernization will often get into the high-risk stage of decreased social stability when the GDP per capita reaches 4000-6000 US dollars. Nevertheless, once the GDP per capita reached 6000-8000 US dollars, especially exceeding 8000 US dollars, the nation basically will enter into a new social stable state (Chen 2011).

The GDP measure has also been criticized. First, GDP does not measure important

non-market economic activities. In developed countries, the degree of domestic labor market is relatively high (Yan & Zhu 2003). For example, most families raise their children to go to kindergarten; many families often go to restaurants, etc. The degree of domestic labor market use in developing countries is relatively low, as family members always do the housework by themselves. Thus, the contribution to GDP in developed countries is higher than in developing countries. Due to this, the GDP of developed countries and developing countries are not entirely comparable.

Secondly, GDP does not reflect possible negative impact of economic development on natural resources and the environment. For instance, cutting trees will increase GDP, but also result in a reduction of forest resources. Obviously, in this case, GDP reflects only the positive side of economic development, but does not reflect the environment damage.

Since the GDP and GDP per capita are such important indicators, forecasting GDP can be useful for decision makers not only in drawing up economic development plans but also to be able to counter potential recessions in advance. A lot of models could be used to do forecasting, each of which has its own characteristics, advantages and disadvantages. In this paper, three time series models are applied to forecast regional GDP per capita, the Autoregressive Integrated Moving Average (ARIMA) model, the Vector Autoregression (VAR) model and the First-order Autoregression (AR(1)) model.

The purpose of this thesis is to test and distinguish which of the three different autoregressive models performs best in forecasting regional GDP per capita. The results show that, to some degree, all the three models are valid in forecasting GDP per capita in a short-term. However, as the sample size is very small, the simpler AR(1) model performs better than the other two models.

2. Literature review

Wang and Wang (2011) forecast the GDP of China based on the time series methods developed by Box and Jenkins (1976). They set up an ARIMA model of the GDP of China from 1978 to 2006. They then choose the best ARIMA model based on statistical tests and forecast the GDP from 2007 to 2011. The result shows that the error between the actual value and the predicted value is small which indicates that the ARIMA model is a high precision and effective method to forecast the GDP time series.

Sheng (2006) analyze and forecast the GDP per capita development in the Zhejiang province in China based on the back propagation (BP) neural network and an ARIMA model. The result indicates that, from 2006 to 2010, the average GDP per capita of Zhejiang province during these five years will be 40624.53 Yuan, and the average growth rate of GDP per capita is 10.01% per year.

Wei, Bian and Yuan (2010) forecast the GDP of the Shanxi province in China based on the ARIMA model. Using GDP data from 1952 to 2007, they set up an ARIMA (1, 2, 1) time series model, and compare the actual and predicted values from 2002 to 2007. The result indicates that the error between the real GDP value and the predicted value is within 5%.

Mei, Liu and Jing (2011) constructed a multi-factor dynamic system VAR forecast model of GDP by selecting six important economic indicators, which include the social retail goods, fiscal revenue, investment in fixed assets, secondary industry output, tertiary industry output, and employment rate, based on data from the Shanghai region in China. The analysis show that the significance of model is high

and the results show that the relative forecast error is quite small, leading the authors to conclude that the VAR model has a considerable practical value.

Hui and Jia (2003) investigate the forecasting performance of the non-linear series self-exciting threshold auto regressive (SETAR) model using Canadian GDP data from 1965 to 2000. Besides the within-sample fit, a standard linear ARIMA model for the same sample has also been generated to compare with the SETAR model. Two forecasting methods, one-step ahead and multi-step ahead forecasting, are compared for each type of model. In one-step-ahead forecasting, actual data is used to predict for every forecasting period. While in the multi-step ahead forecasting, previous periods' predictions are used as part of the forecasting equation. Their results show that the two forecasting methods can both offer good forecasting results. But in real life, the multi-step-ahead forecasting tends to be more practical.

Clarida and Friedman (1984) use a VAR model and forecast the United States short-term interest rates during April 1979 to February 1983. A constant-coefficient, linear VAR model is generated to estimate the pre-October 1979 probability structure of the quarterly data, which takes six important United States macroeconomic factors into consideration. The result shows that short-term interest rates in the United States have been "too high" since October 1979. Because, based on their VAR model, the prediction results of conditional and unconditional forecast are both lower than the actual United States short-term interest rates during this period.

3. Methodology

3.1 ARIMA Time Series Analysis

Autoregressive integrated moving average (ARIMA) model was first popularized by Box and Jenkins (1970). It forecasts future values of a time series as a linear combination of its own past values and a series of errors (also called random shocks or innovations). ARIMA models are always applied in some cases where time series show evidence of non-stationarity by using an initial differencing step to remove the non-stationarity (Hamilton 1994).

3.1.1 Testing the stationarity of the time series

First, we have to test the stationarity of the time series. We can use scatter plots or line plots to get an initial idea of the problem. Then, an Augmented Dickey-Fuller (ADF) unit root test is used to determine the stationarity of the data. If the data is non-stationary, we do a logarithm transformation or take the first (or higher) order difference of the data series which may lead to a stationary time series. This process will be repeated until the data exhibit no apparent deviations from stationarity. The times of differencing of the data is indicated by the parameter d in the $ARIMA(p,d,q)$ model. Theoretically, differencing the time series repeatedly will eliminate the non-stationarity of the time series. However, it does not mean that the more differencing, the better. Since differencing is a procedure of extracting information and processing the data, each time the procedure is performed it will lead to a loss of information (Harvey 1989).

After transforming the data into a stationary time series by differencing, the $ARIMA(p,d,q)$ model can be taken as $ARMA(p,q)$, which is the combination of autoregression and moving average. Generally, the $ARMA(p,q)$ model can be expressed as follows:

$$X_t - \phi_1 X_{t-1} - \dots - \phi_p X_{t-p} = Z_t + \theta_1 Z_{t-1} + \dots + \theta_q Z_{t-q} \quad (3.1)$$

where the ϕ_i are the parameters of the autoregressive part of the model, and θ_i are the parameters of the moving average part.

It is convenient to use the more concise form of (3.1)

$$\Phi(L)X_t = \Theta(L)Z_t \quad (3.2)$$

where $\Phi(\cdot)$ and $\Theta(\cdot)$ are the p th and q th-degree polynomials

$$\Phi(z) = 1 - \phi_1 z - \dots - \phi_p z^p$$

and

$$\Theta(z) = 1 + \theta_1 z + \dots + \theta_q z^q$$

And where L is the lag operator ($L^j X_t = X_{t-j}, L^j Z_t = Z_{t-j}, j = 0, \pm 1, \dots$). The time series $\{X_t\}$ is said to be an autoregressive process of order p (or $AR(p)$) if $\Theta(z) \equiv 1$, and a moving-average process of order q (or $MA(q)$) if $\Phi(z) \equiv 1$.

3.1.2 Model identification

The Autocorrelation Function (ACF) plots and the Partial Autocorrelation Function (PACF) plots can help us to determine the properties and number of lags in the models. If the ACF plot displays an exponentially declining trend and the PACF plot spikes in the first one or more lags, it suggests that the process best fits the AR models. The number of spikes in the PACF plot indicates the order of the AR terms. If the ACF plot spikes in the first one or more lags and the PACF plot displays an exponentially declining trend, it suggests that the process best fits the MA models. The number of spikes in the ACF plots indicates the order of MA terms. If both ACF and PACF plots display exponentially declining trend, it suggests that the process best fits the mixed model, i.e. the ARMA model. (Robert, 2005).

After the tentative identification of the orders p and q in the $ARMA(p,q)$ model, the model that best describes the dataset at hand can be constructed using the Akaike Information Criterion (AIC) and Schwarz Criterion (SC) (Harvey, Leybourne & Newbold 1998).

The Akaike Information Criterion (AIC) was developed by Hirotugu Akaike, under the name of "an information criterion" (Akaike, 1974). The AIC rule provides the best number of lags and parameters to be estimated in the ARMA(p, q) models. In the ARMA model, the AIC function can be defined as follows

$$AIC = -2 \log(L) + 2(p + q) \quad (3.3)$$

where L indicates the likelihood of the data with a certain model, p and q indicate the lag orders of AR term and MA term.

The AIC rule used to determine the lags can be expressed as follows:

$$AIC(p, q) = \min_{k, l} AIC(k, l) \quad (3.4)$$

where k and l indicates the different choices of lag orders.

The Schwarz Criterion (SC, also called Schwarz information criterion (SIC) or Bayesian information criterion (BIC)) was popularized by Schwarz (1978), who gave a Bayesian argument for adopting it. The SC is a criterion used for selecting the best fitted model among different ones. Partly related to AIC, the SC is based on the likelihood function. In the ARMA model, the SC function can be expressed as follows:

$$SC = -2 \log(L) + (p + q) \log T \quad (3.5)$$

where T indicates the number of observations in the stationary time series, L indicates the likelihood of the data from a certain model, p and q indicate the lag orders of AR term and MA term.

A lower value of SC means either better fit, fewer independent variables, or both. So when we compare two models, the model with the lower SC is better than the other one.

3.1.3 The estimation of parameters

After determining the number of lags in the model, we have to estimate the parameters of the

ARMA model. In this paper, the OLS method is used to estimate the parameters.

3.1.4 Model diagnostics

While we estimate the model parameters, it is also necessary to do model diagnostics, in order to check whether the fitted model is appropriate. If not, we have to know how to adjust it. So in this step, the main objective is to examine the validity of the fitted model. Firstly, we have to check whether the estimated parameters of the model are significant; secondly, we have to test whether the residuals are white noise. As to check the significance of the parameters, we use the t test. And the Ljung and Box (1978) Q test (also called portmanteau test) is applied to do the white noise test. The Q-statistic is defined as:

$$Q = T(T + 2) \sum_{k=1}^m \frac{r_k^2}{T - k} \quad (3.6)$$

where T is the sample size, r_k is the sample autocorrelation at lag k and m is the lag order that needs to be specified. Under the null hypothesis that the ARMA model is adequate, the Q-statistic is Chi-squared distributed, $\chi^2(m - p - q)$, where p and q are the lag orders of AR term and MA term. The judge criteria can be written as:

if $Q \leq \chi_{\alpha}^2(m - p - q)$, then H_0 is not rejected;

if $Q > \chi_{\alpha}^2(m - p - q)$, then H_0 is rejected,

where α is the significance level.

3.2 VAR time series analysis

The Vector Autoregression (VAR) model, proposed by Sims (1980), is one of the most successful, flexible, and easy to use models for analysis of multivariate time series. It is applied to grasp the mutual influence among multiple time series. VAR models extend the univariate autoregressive (AR) model to dynamic multivariate time series by allowing for more than one evolving variable. All variables in a VAR model are treated symmetrically in a structural sense; each variable has an equation explaining its evolution based on its own lags

and the lags of the other model variables (Walter 2003).

Let $\mathbf{Y}_t = (y_{1t}, y_{2t}, \dots, y_{nt})'$ denote an $(n \times 1)$ vector of time series variables. A VAR model with p lags can then be expressed as follows:

$$\mathbf{Y}_t = \mathbf{c} + \mathbf{\Pi}_1 \mathbf{Y}_{t-1} + \mathbf{\Pi}_2 \mathbf{Y}_{t-2} + \dots + \mathbf{\Pi}_p \mathbf{Y}_{t-p} + \boldsymbol{\varepsilon}_t, \quad t = 1, \dots, T \quad (3.7)$$

where $\mathbf{\Pi}_i$ is a $(n \times n)$ coefficient matrix, $\boldsymbol{\varepsilon}_t$ is an $(n \times 1)$ unobservable zero mean white noise vector process, and \mathbf{c} is an $(n \times 1)$ vector of constants (intercepts).

3.2.1 The choice of variables

We have to determine a list of variables which can be assumed to affect each other intertemporally. When we choose the variables, it is necessary to take three aspects into consideration: 1) The chosen variables should be related to the research problem; 2) The choice of the variables should be in accordance with the theoretical hypothesis; 3) Data used for fitting the model must be available and of good quality.

In this study, two kinds of VAR models were set up. First, we take the GDP per capita of the target county and its adjacent counties as inter-affect variables to set up the structural models. Second, we take the GDP per capita of the target county and the national average GDP per capita (except the target county) as inter-affect variables to generate the VAR model.

3.2.2 Testing the stationarity of time series

Sim, Stock and Watson (1990) suggest that non-stationary time series are still feasible in VAR modeling. But in practice, using the non-stationary time series in VAR modeling is problematic with regards to statistical inference since the standard statistical tests used for inference are based on the condition that all of the series used must be stationary. In the VAR

modeling, we continue to use the ADF unit-root test to check the stationarity of the time series.

3.2.3 Model identification

It is known that the more the lags there are, the less the degrees of freedom are. When we determine the number of lags, we choose the one with the minimum AIC and SC value. If the AIC and SC value are not minimized using the same model, we instead apply a likelihood-ratio (LR) test (Johansen 1995). The LR-statistic can be expressed as follows:

$$LR = -2(\log L_{(k)} - \log L_{(k+1)}) \sim \chi^2(n^2) \quad (3.8)$$

where k is the lag order, L is the maximized likelihood of the model and n is the number of variables.

If $LR \leq \chi_{\alpha}^2$, we do not reject the null hypothesis that all the elements in the coefficient matrix are zero. Then we can reduce the lag order until the null hypothesis is rejected.

3.2.4 The estimation of parameters and the model diagnostics

Although the structure of the VAR model looks very complex, the estimation of the parameters is not difficult. The most common methods are the Maximum Likelihood Estimator (MLE) and the Ordinary Least Square Estimator (OLS) (Yang & Yuan 1991). In this study, we use the OLS method to estimate the parameters.

Similar as ARIMA modeling, a Q test is applied to test whether the residuals of the VAR models are white noise.

4. Empirical analysis

4.1 Data description

The data include the regional GDP and regional GDP per capita of 25 counties in Sweden during 1993 to 2009. We use the regional GDP per capita to fit the model. Among the 25 counties, we choose to study the five of them with the highest regional GDP per capita. The regional GDP is calculated as the sum of value added (roughly the sum of firm profits and salaries to employees) of firms and residents in the region. The sum of all regional GDP is then equal to the total value added of Sweden, i.e. the GDP of Sweden.

We use data from 1993 to 2004 to fit the model, and then the last 5 years' data (2005-2009) are used to assess the forecasting ability of the models. In order to eliminate the influence of inflation from the analysis, we calculate the real GDP using the consumer price index (CPI) of 1990 as the baseline.

In the modeling section of this thesis, we only take the Stockholm region as an example to explain in detail since the modeling procedure and forecasting method for the other counties is similar. And the forecasts of other counties are presented in the last part.

4.2 ARIMA modeling

4.2.1 Testing the stationarity

We denote the GDP per capita of Stockholm as Y_t . The line plot of the Y_t is presented in Figure 1. Obviously, the GDP per capita in Stockholm has an increasing trend

showing that it is a non-stationary process. Furthermore, we can also use the ADF test to test the stationary of the time series. As shown in Table 4.1, the same conclusion is obtained. In order to reduce problems with heteroscedasticity, we take the logarithm transformation of GDP per capita in Stockholm, $Ln(Y_t)$. What is more, first order differencing is needed to ensure that the stationarity assumption of the ARIMA model is satisfied.

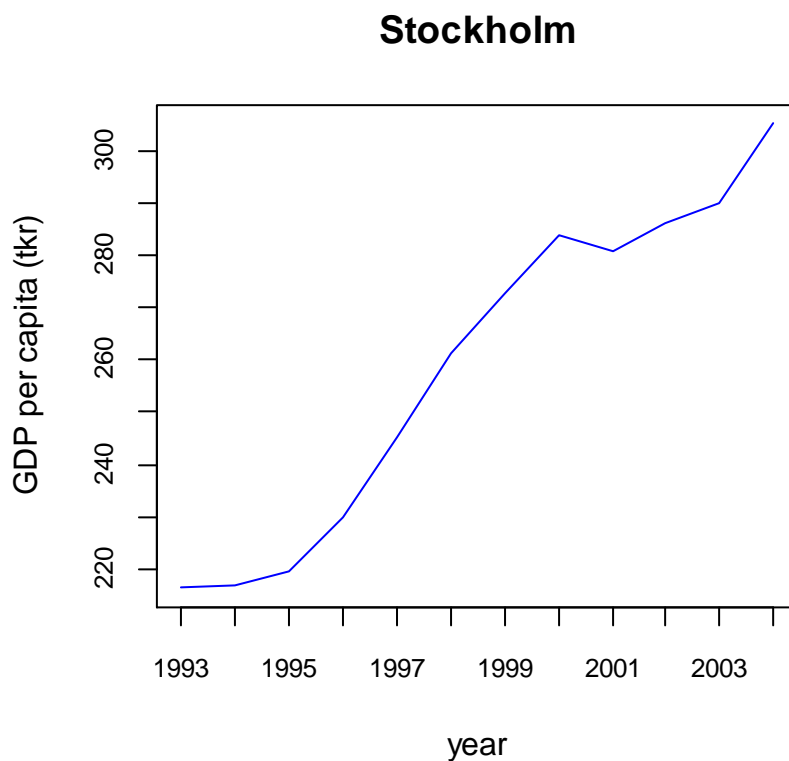


Figure 4.1 The line plot of GDP per capita in Stockholm

Table 4.1 The ADF test result of Y_1

	t-Statistic	Prob.*
ADF test statistic	-0.862534	0.9356
Test critical values:		
1% level	-4.667883	
5% level	-3.7333200	
10% level	-3.310349	

The line plot of $\Delta \ln(Y_1)$ is presented in Figure 4.2. The ADF test results presented in Table 4.2 indicates that we can reject the null hypothesis that the first difference of GDP has a unit root (non-stationary) at the 5% significance level. It means that the first order difference of the Y_1 time series turns out to be stationary.

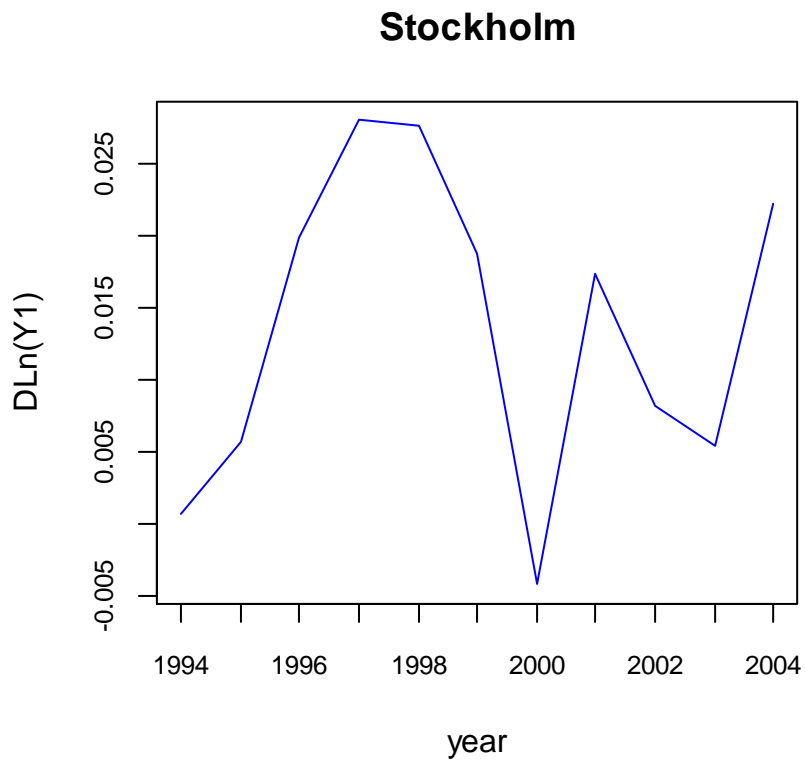


Figure 4.2 The line plot of differenced of $\ln(Y_1)$

Table 4.2 The ADF test result of first order difference of $\ln(Y_1)$

		t-Statistic	Prob.*
ADF test statistic		-5.646789	0.0042
Test critical values:	1% level	-4.992279	
	5% level	-3.875302	
	10% level	-3.388330	

4.2.2 Model identification and parameters estimation

The ACF plots and PACF plots of the differenced series can be used to tentatively identify the order of autoregressive terms and/or moving average terms (Robert 2005). Seeing the ACF and PACF plot in Figure 4.3, both ACF and PACF spike at the 3rd lag and then cut off. This suggests that the process that may fit the analysis is the MA(3), AR(3), or ARMA(3,3). And considering the AIC rule and the significance of parameters, MA(3) best fitted the process. And since the first two lag order of the autocorrelation are not significant, only the 3rd lagged MA term is included.

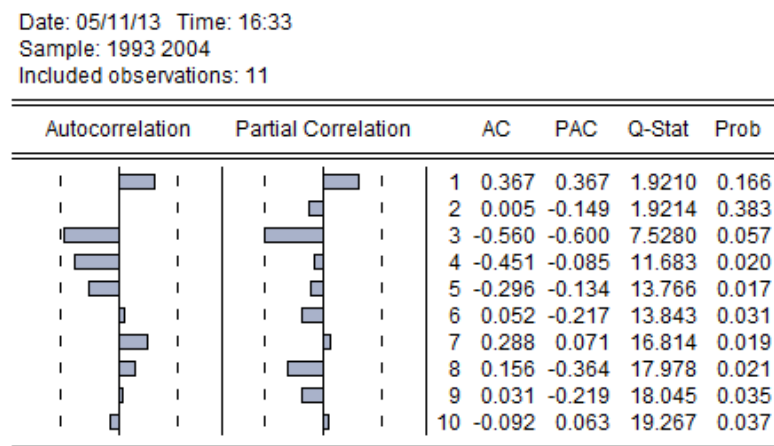


Figure 4.3 Autocorrelation and Partial Correlation plot

Table 4.3 The results of estimated ARIMA model

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.032581	0.003398	9.588604	0.0000
MA(3)	-0.992215	0.206934	-4.794829	0.0010
R-squared		0.670270		
Adjusted R-squared		0.633634		

The results of estimated ARIMA model are presented in Table 4.3. If we take a glance at the *t-Stat* values and *p-values* of each coefficient, we find that all coefficients are

significantly different from zero for a confidence level of 95%.

As a result the fitted model is:

$$\Delta \ln(Y_1)_t = 0.0326 - 0.9922v_{t-3} + v_t$$

4.2.3 Model diagnostics

The Q test is applied on the residuals of the fitted ARIMA model (Box & Pierce 1970). And the results are presented in Table 4.4, they indicate that all the values of the Q-statistic are less than that of the critical χ^2 value with the corresponding degrees of freedom and all the p -values are bigger than α (0.05), which provide the evidence that we cannot reject the white noise null hypothesis. Therefore, the fitted ARIMA model is considered to be a valid model.

Table 4.4 The results of Q test for the residuals of ARIMA model

Lag	Q-Statistic	Prob.
1	1.1058	
2	1.2184	0.0270
3	3.4668	0.177
4	7.6033	0.055
5	9.3140	0.054
6	9.7897	0.081
7	11.620	0.071
8	11.699	0.111
9	11.835	0.159
10	11.952	0.216

4.2.4 Prediction results

Then we apply the model above to predict the GDP per capita of Stockholm during 2005 to 2009. Here multi-step ahead method is applied to forecast the values, which uses the predicted value of the current time step to forecast its value in the next time step (Cheng, Tan, Gao & Scripps 2006). The forecasting results are presented in Table 4.5.

Table 4.5 The forecasting results of ARIMA model

Year	Actual value	Prediction value	Error
2005	317.5851	315.2066	-0.75%
2006	324.1260	328.2307	1.27%
2007	335.6788	334.9167	-0.23%
2008	329.1474	344.2186	4.58%
2009	332.8755	344.3206	3.44%

4.3 VAR modeling-1

In the first VAR model, we take the GDP per capita of the target county and its adjacent counties as inter-affect variables to do the structural models. In this case, Stockholm is the target county, and its adjacent counties include Uppsala and Södermanland. As a result, the VAR model will contain 3 variables, the GDP per capita of Stockholm (Y_1), Uppsala (Y_2) and Södermanland (Y_3).

4.3.1 Testing the stationarity

As shown in the Figures 4.5, 4.6 and 4.7, all the Y_i time series are non-stationary. Here we again take the logarithm transformation of GDP per capita of each county to reduce potential problems with heteroscedasticity. Then we do the differencing for

each series, denoting the differenced time series as $\Delta \ln(Y_i)$.

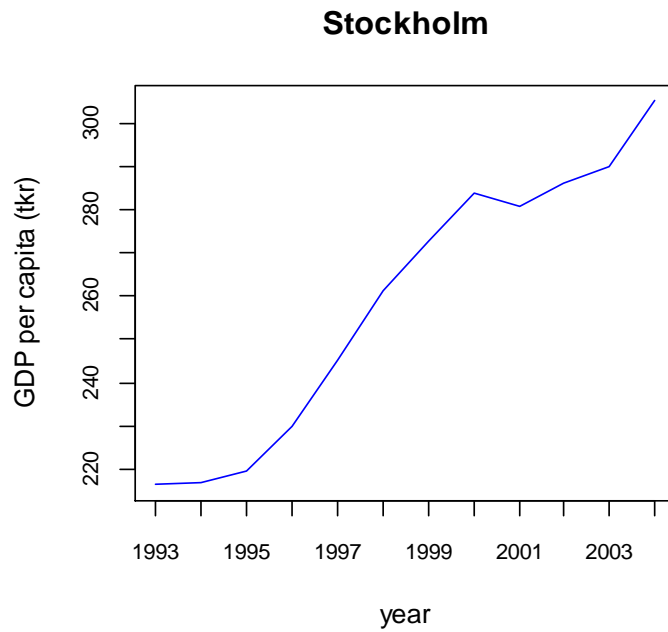


Figure 4.5 The line plot of GDP per capita in Stockholm

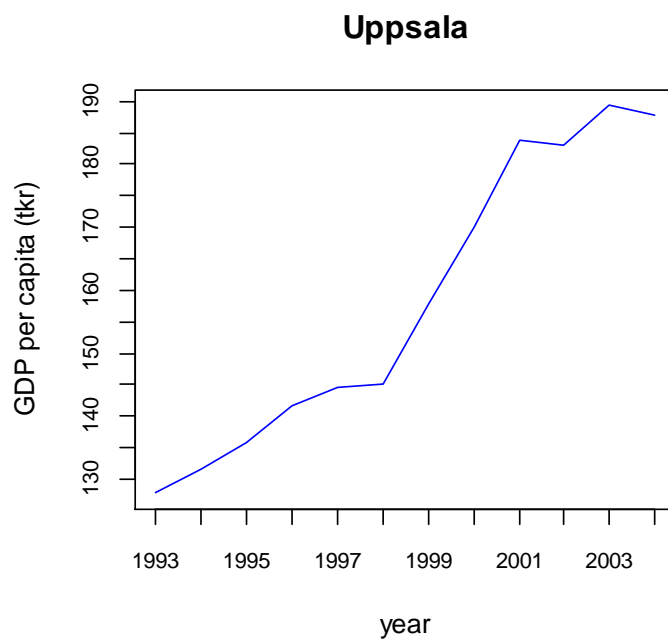


Figure 4.6 The line plot of GDP per capita in Uppsala

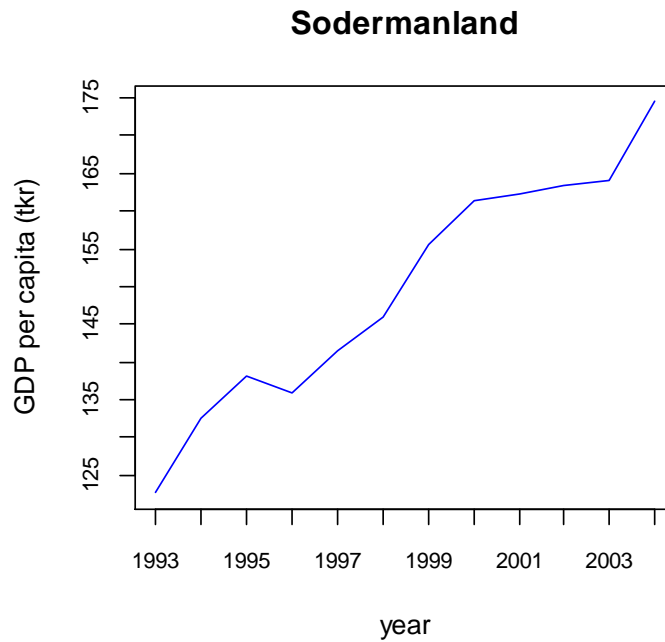


Figure 4.7 The line plot of GDP per capita in Södermanland

After the first order differencing, we apply the ADF unit-root test for each series. The results, as shown in Table 4.6 and 4.8, indicate that all the series are stationary. It should however be noticed that when we test the stationarity of $\Delta \ln(Y_2)$, this time series is not stationary under all kinds of ADF tests. However, it does not mean that $\Delta \ln(Y_2)$ is non-stationary. This is so because when the sample size is small, the ADF test is not always reliable (Zhang 2000). Thus, we use the DF-GLS test instead of ADF test, and the result of the DF-GLS test, presented in Table 4.7, indicates that the $\Delta \ln(Y_2)$ is stationary.

Table 4.6 The ADF test result of differenced $\ln(Y_1)$

	t-Statistic	Prob.*
ADF test statistic	-5.646789	0.0042
Test critical values:		
1% level	-4.992279	
5% level	-3.875302	
10% level	-3.388330	

Table 4.7 The DF-GLS test result of differenced $Ln(Y_2)$

		t-Statistic	Prob.*
DF-GLS test statistic		-2.233120	0.0401
Test critical values:	1% level	-2.886101	
	5% level	-2.995865	
	10% level	-1.599088	

Table 4.8 The ADF test result of differenced $Ln(Y_3)$

		t-Statistic	Prob.*
ADF test statistic		-3.617635	0.0382
Test critical values:	1% level	-4.803491	
	5% level	-3.403313	
	10% level	-2.841819	

4.3.2 Model identification and parameter estimation

Since we only have 12 annual observations in the sample, when constructing the VAR model, the number of lags could not be large. Given the minimum AIC and SC rules, as shown in Table 4.9, and in order to avoid the loss of information, we determine that the optimal number of lags is 2. Then there are 21 parameters to be estimated. The result of the estimated model is presented in Table 4.10.

Table 4.9 VAR model-1 lag order selection criteria

Lag	AIC	SC
1	-4.20105	-4.25173
2	-4.40488*	-4.39175*

Table 4.10 The result of estimated VAR model-1

	$\Delta Ln(Y_1)$	$\Delta Ln(Y_2)$	$\Delta Ln(Y_3)$
$\Delta Ln(Y_1)(-1)$	0.339193 (0.53509) [0.63390]	0.499960 (0.75704) [0.66042]	0.157901 (0.29556) [0.53424]
$\Delta Ln(Y_1)(-2)$	-0.213079 (0.58906) [-0.36861]	0.294562 (0.81784) [0.36017]	0.691388 (0.31930) [2.16532]
$\Delta Ln(Y_2)(-1)$	0.026123 (0.41503) [0.06294]	-0.266387 (0.58718) [-0.45367]	-0.157239 (0.22925) [-0.68589]
$\Delta Ln(Y_2)(-2)$	-0.553319 (0.29470) [-1.87758]	0.153307 (0.41694) [0.36770]	-0.481125 (0.16278) [-2.95567]
$\Delta Ln(Y_3)(-1)$	-0.211743 (0.45125) [-0.4692]	0.740328 (0.63842) [0.75118]	-0.641854 (0.24925) [-2.57510]
$\Delta Ln(Y_3)(-2)$	-0.119832 (0.35107) [-0.34133]	0.740328 (0.49669) [1.49501]	-0.144210 (0.19392) [-0.74366]
C	0.062069 (0.02646) [2.34562]	-0.019556 (0.03744) [-0.52237]	0.044531 (0.01462) [3.04663]

Therefore, the estimated VAR model-1 can be expressed as following:

$$\begin{aligned} \Delta Ln(Y_1)_t = & 0.3392 * \Delta Ln(Y_1)_{t-1} - 0.2131 * \Delta Ln(Y_1)_{t-2} + 0.0261 * \Delta Ln(Y_2)_{t-1} \\ & - 0.5533 * \Delta Ln(Y_2)_{t-2} - 0.2117 * \Delta Ln(Y_3)_{t-1} - 0.1198 \\ & * \Delta Ln(Y_3)_{t-2} + 0.0621 \end{aligned}$$

$$\begin{aligned} \Delta Ln(Y_2)_t = & 0.5000 * \Delta Ln(Y_1)_{t-1} + 0.2946 * \Delta Ln(Y_1)_{t-2} - 0.2664 * \Delta Ln(Y_2)_{t-1} \\ & + 0.1533 * \Delta Ln(Y_2)_{t-2} + 0.4796 * \Delta Ln(Y_3)_{t-1} + 0.7403 \\ & * \Delta Ln(Y_3)_{t-2} + 0.0196 \end{aligned}$$

$$\begin{aligned} \Delta Ln(Y_3)_t = & 0.1579 * \Delta Ln(Y_1)_{t-1} + 0.6914 * \Delta Ln(Y_1)_{t-2} - 0.1572 * \Delta Ln(Y_2)_{t-1} \\ & - 0.4811 * \Delta Ln(Y_2)_{t-2} - 0.6419 * \Delta Ln(Y_3)_{t-1} - 0.1442 \\ & * \Delta Ln(Y_3)_{t-2} + 0.0445 \end{aligned}$$

4.3.3 Model diagnostics

A Q test for residual autocorrelation is applied to the estimated model. The Q test results for the VAR model-1 are presented in Table 4.11. The results indicate that all the values of the Q-statistic are less than that of the critical χ^2 value with the corresponding degrees of freedom and all the p -values are bigger than α (0.05), which offer the evidence that we cannot reject the white noise null hypothesis. Therefore, it is reasonable to consider that our VAR model-1 is valid.

Table 4.11 The results of Q test for residual autocorrelation of VAR model-1

Lag	$\Delta \ln(Y_1)_t$		$\Delta \ln(Y_2)_t$		$\Delta \ln(Y_3)_t$	
	Q-Statistic	Prob.	Q-Statistic	Prob.	Q-Statistic	Prob.
1	0.0503	0.823	0.0257	0.873	0.0115	0.915
2	0.1199	0.942	0.0669	0.967	0.0709	0.965
3	0.2891	0.962	0.0816	0.994	0.1096	0.991
4	0.5996	0.963	0.1784	0.996	0.2263	0.994
5	1.7870	0.787	2.7409	0.740	0.8836	0.971
6	2.2153	0.899	2.7761	0.836	1.3929	0.966
7	4.0573	0.773	4.5899	0.710	3.1321	0.873
8	5.7826	0.672	4.6280	0.796	5.3304	0.722

4.3.4 Prediction results

Then we apply the VAR model above to predict the GDP per capita of Stockholm during 2005 to 2009 by using the multi-step ahead forecast method. The forecasting results are presented in Table 4.12.

Table 4.12 The forecasting results of VAR model-1

Year	Actual value	Prediction value	Error
2005	317.5851	318.8651	0.40%
2006	324.1260	335.8369	3.61%
2007	335.6788	334.2892	-0.41%
2008	329.1474	341.6079	3.79%
2009	332.8755	347.6571	4.44%

4.3 VAR modeling-2

In the second VAR model, we take the GDP per capita of the target county (Stockholm) and the average regional GDP per capita (except the target counties) as the inter-affect variables to set up the structural models. As a result, the VAR model will contain two variables, the GDP per capita of Stockholm and the average regional GDP per capita except Stockholm, noted as Y_1 and Y .

4.3.1 Testing the stationarity

Since the stationarity of the $\Delta \ln(Y_1)$ has been presented in the first VAR model, here we just test the stationarity of Y . As shown in Figure 4.8, Y is not stationary. Here we again take the logarithm transformation of average regional GDP per capita (except Stockholm) in order to avoid heteroscedasticity, noting this variable as $\ln(Y)$. Then we take the first order difference of the time series $\ln(Y)$ and denote it as $\Delta \ln(Y)$.

Average GDP per Capita (Except Stockholm)

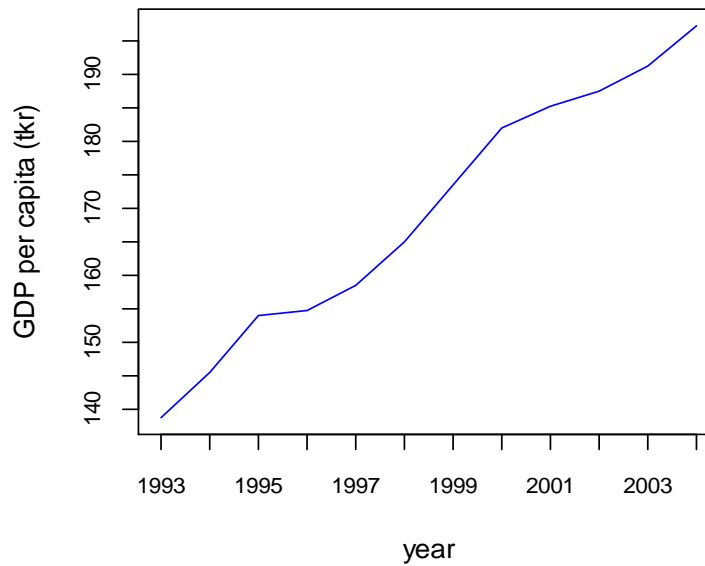


Figure 4.8 The line plot of average GDP per capita (except Stockholm)

After the first order differencing, we again do the ADF unit-root test. The result, as shown in Table 4.13, indicates that the first-differenced time series is stationary.

Table 4.13 The ADF test result of differenced $\ln(Y)$

		t-Statistic	Prob.*
ADF test statistic		-3.515918	0.0349
Test critical values:	1% level	-4.420595	
	5% level	-3.259808	
	10% level	-2.771129	

4.3.2 Model identification and parameters estimation

Given the minimum AIC and SC rule, as shown in Table 4.14, and in order to avoid the loss of information, we determine that the optimal number of lags is 3. As a result,

there are then 14 parameters to be estimated. The result of the estimated model is presented in Table 4.15.

Table 4.14 VAR model-2 lag order selection criteria

Lag	AIC	SC
1	-4.41011	-4.31933
2	-3.82217	-3.71260
3	-4.71215	-4.64264

Table 4.15 The result of estimated VAR model-2

	$\Delta Ln(Y_1)$	$\Delta Ln(Y)$
$\Delta Ln(Y_1)(-1)$	0.386627 (0.48760) [0.79292]	0.263411 (0.24788) [1.06267]
$\Delta Ln(Y_1)(-2)$	0.624794 (0.067029) [0.93212]	0.247666 (0.34075) [0.72682]
$\Delta Ln(Y_1)(-3)$	-0.524337 (0.68265) [-0.76809]	0.057537 (0.34703) [0.016580]
$\Delta Ln(Y)(-1)$	-1.020122 (1.55965) [-0.65407]	-0.270666 (0.79287) [-0.80467]
$\Delta Ln(Y)(-2)$	-0.019949 (0.60197) [-0.03314]	-0.246245 (0.30602) [-0.80467]
$\Delta Ln(Y)(-3)$	-0.247727 (0.75328) [-0.32887]	-0.352661 (0.38294) [-0.92094]
C	0.054577 (0.04404) [1.23923]	0.038507 (0.02239) [1.71989]

Therefore, the estimated VAR model-2 can be expressed as follows:

$$\begin{aligned} \Delta \ln(Y_1)_t &= 0.3866 * \Delta \ln(Y_1)_{t-1} + 0.6248 * \Delta \ln(Y_1)_{t-2} - 0.5243 * \Delta \ln(Y_1)_{t-3} \\ &\quad - 1.0201 * \Delta \ln(Y)_{t-1} - 0.0200 * \Delta \ln(Y)_{t-2} + 0.2477 * \Delta \ln(Y)_{t-3} \\ &\quad + 0.05458 \end{aligned}$$

$$\begin{aligned} \Delta \ln(Y)_t &= 0.2634 * \Delta \ln(Y_1)_{t-1} + 0.2477 * \Delta \ln(Y_1)_{t-2} + 0.0575 * \Delta \ln(Y_1)_{t-3} \\ &\quad - 0.2707 * \Delta \ln(Y)_{t-1} - 0.2462 * \Delta \ln(Y)_{t-2} - 0.3527 * \Delta \ln(Y)_{t-3} \\ &\quad + 0.0385 \end{aligned}$$

4.3.3 Model diagnostics

VAR model-2 is also tested for the serial correlation. Q test for residual autocorrelation is applied. The Q test results for the VAR model-2 are presented in the Table 4.16. The results indicate that all the values of the Q-statistic are less than that of the critical χ^2 value with the corresponding degrees of freedom and all the p -values are bigger than α (0.05), which shows that we cannot reject the white noise null hypothesis. Therefore, it is reasonable to consider that the VAR model-2 is valid.

Table 4.16 The results of Q test for residual autocorrelation of VAR model-2

Lag	$\Delta \ln(Y_1)_t$		$\Delta \ln(Y)_t$	
	Q-Statistic	Prob.	Q-Statistic	Prob.
1	4.0885	0.054	4.0736	0.053
2	4.0889	0.129	4.149	0.133
3	4.8787	0.181	4.7950	0.190
4	5.1220	0.276	5.0240	0.292
5	5.1163	0.402	5.0108	0.411
6	5.1222	0.528	5.1365	0.517
7	5.1224	0.645	5.1396	0.663

4.3.4 Prediction results

Then we again apply the VAR model above to predict the GDP per capita of Stockholm during 2005 to 2009 by using the multi-step ahead forecast method. The forecasting results are shown in Table 4.17.

Table 4.17 The forecasting results of VAR model-2

Year	Actual value	Prediction value	Error
2005	317.5851	316.5259	-0.33%
2006	324.1260	338.1571	4.33%
2007	335.6788	324.9721	-3.19%
2008	329.1474	336.9865	2.38%
2009	332.8755	351.1386	5.49%

4.4 AR(1) modeling

In practice, a simple time series model could offer good forecasts of regional GDP. So here we use an AR(1) model, that is an autoregression with one lag, to do the forecasting. In order to keep the modeling as simple as possible, we ignore testing for stationarity, white noise, etc.

4.4.1 Model specification

We use the time series data of Stockholm GDP per capita to fit the AR(1) model. The model was estimated as presented in Table 4.18.

Table 4.18 The result of estimated AR(1) model

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2125.875	31196.42	0.068145	0.9472
AR(1)	0.995690	0.072329	13.76606	0.0000
R-squared	0.954661			
Adjusted R-squared	0.949623			

As a result, the AR(1) model can be expressed as following:

$$Y_t = 2125.8750 + u_t$$

$$u_t = 0.9957u_{t-1} + v_t$$

Therefore,

$$Y_t = 9.1413 + 0.9957Y_{t-1} + v_t$$

4.4.2 Prediction results

Then we apply the AR(1) model above to predict the GDP per capita of Stockholm during 2005 to 2009. The forecasting results are presented in Table 4.19.

Table 4.19 The forecasting results of AR(1)

Year	Actual value	Prediction value	Error
2005	317.5851	312.9179	-1.47%
2006	324.1260	325.3788	0.39%
2007	335.6788	331.8961	-1.13%
2008	329.1474	343.3946	4.33%
2009	332.8755	336.8913	1.21%

4.5 Performance comparison, Stockholm

We test and distinguish the performance of the different models for the purpose of finding a better forecasting model. Figure 4.9 gives us a very clear picture of the forecasting results from different models. And the percentage errors and mean absolute percentage errors (MAPE) are used to evaluate the performance of different autoregressive models (Diebold 1995). The results are presented in Table 4.20. The measure of MAPE can be expressed as follows:

$$MAPE = \left[\frac{1}{n} \sum_{i=1}^n \text{abs}((\hat{y}_t - y_t)/y_t)_t \right] * 100\% \quad (4.1)$$

where \hat{y}_t is the predicted value, y_t is the actual value, and n indicates the number of fitted points.

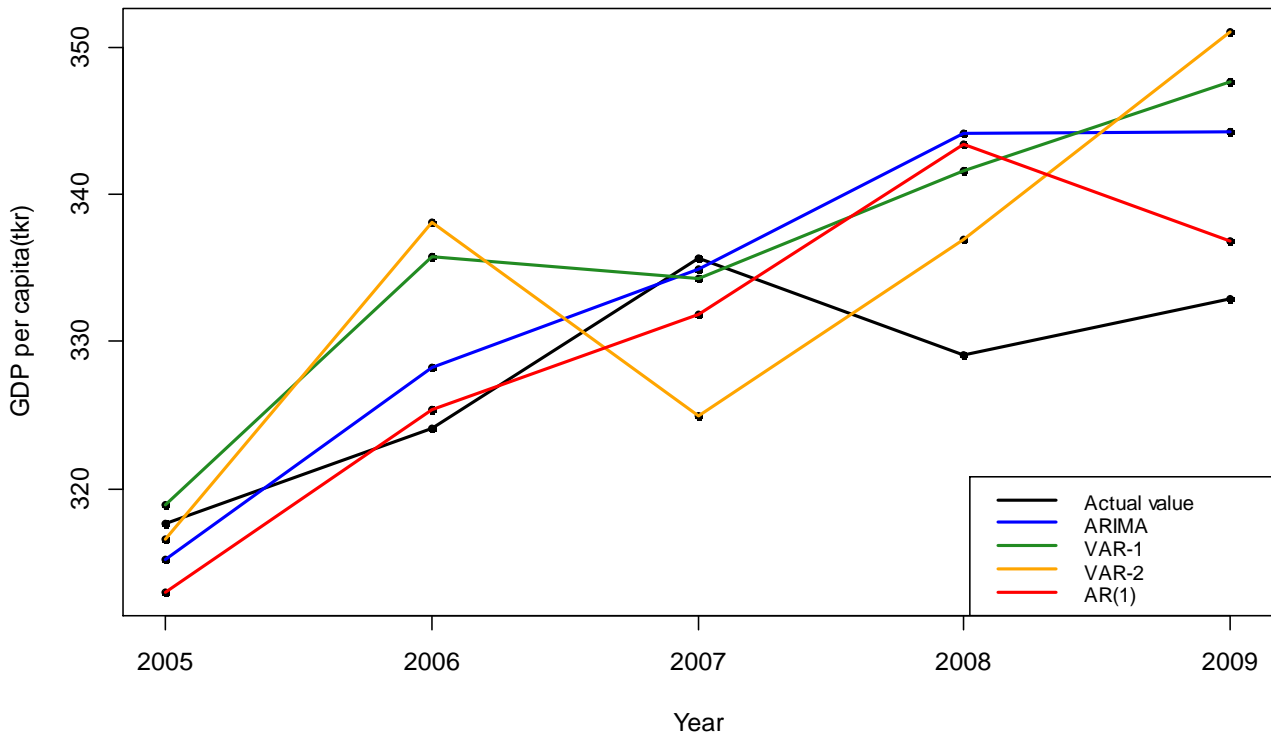


Figure 4.9 Actual value of GDP per capita and predicted values from ARIMA model, VAR model-1, VAR model-2, AR(1) model for Stockholm

We find that, for 2008 when the economic crisis took place, each model offers a prediction with a big percentage error. Therefore, when we assess the performance of

the different models, we offer two kinds of MAPE, one is with 2008, and the other without 2008. If we are just focusing on the prediction accuracy of the models, obviously, the AR(1) model is the best one in forecasting the GDP per capita in the 5 years with less than 2% mean absolute percentage error. Then the next is the ARIMA model, the VAR model-1 and the VAR model-2.

Table 4.20 The percentage error of different models for Stockholm

Stockholm	Year	ARIMA	VAR-1	VAR-2	AR(1)
	2005	-0.75%	0.40%	-0.33%	-1.47%
	2006	1.27%	3.61%	4.33%	0.39%
Percentage	2007	-0.23%	-0.41%	-3.19%	-1.13%
error	2008	4.58%	3.79%	2.38%	4.33%
	2009	3.44%	4.44%	5.49%	1.21%
	MAPE	2.05%	2.53%	3.14%	1.71%
	MAPE except 2008	1.42%	2.22%	3.34%	1.05%

4.6 Predictive performance, four other regions

In this thesis, we used the same method to do the forecasts for four other counties with top GDP per capita in Sweden. These counties include Västergötland, Skåne, Östergötland, and Jönköping. Similarly, three kinds of models, ARIMA model, VAR model and AR(1) model, were applied to predict GDP per capita for these four counties and the performance of the different models was compared as shown in the following tables.

As seen in the Table 4.21, 4.22, 4.23 and 4.24, generally, the performance of the AR(1) model is impressive. For Västergötland, Skåne and Jönköping, the forecast accuracy of the AR(1) model is better than the other three kinds of models with less

than 5% mean absolute percentage error, and the performance of the ARIMA model is better than that of the VAR models. However, for Östergötland, there is an exception in that the VAR-2 model is the best one when forecasting the GDP per capita. Similarly, when the different models are applied to forecast the GDP per capita in 2008, the results are less encouraging due to the impact of the financial crisis on the forecasts.

Table 4.21 The forecasting results of different models for V ästra G ötaland

Year	Actual value	ARIMA		VAR-1		VAR-2		AR(1)	
		Prediction	Error	Prediction	Error	Prediction	Error	Prediction	Error
2005	223.0481	225.2787	1.00%	227.7786	2.12%	229.3912	2.84%	220.8801	-0.97%
2006	233.8077	229.7377	-1.74%	232.1032	-0.73%	242.9987	3.93%	227.6731	-2.62%
2007	238.7524	244.8899	2.57%	244.2546	2.30%	256.3606	7.38%	237.9503	-0.34%
2008	232.0558	249.4900	7.51%	251.6321	8.44%	258.9629	11.60%	242.6898	4.58%
2009	222.1490	236.8741	6.63%	240.6474	8.33%	240.8355	8.41%	236.2711	6.36%
MAPE			3.89%		4.38%		6.83%		2.97%
MAPE except 2008			2.98%		3.37%		5.64%		2.57%

Table 4.22 The forecasting results of different models for Sk åne

Year	Actual value	ARIMA		VAR-1		VAR-2		AR(1)	
		Prediction	Error	Prediction	Error	Prediction	Error	Prediction	Error
2005	200.8910	200.8684	-0.01%	200.0771	-0.41%	199.8236	-0.53%	198.2622	-1.31%
2006	209.0430	206.4495	-1.24%	209.2804	0.11%	190.313	-8.96%	203.523	-2.64%
2007	223.0732	214.9976	-3.62%	212.0661	-4.93%	210.6163	-5.58%	219.0791	-1.79%
2008	209.3322	232.3611	11.00%	232.5437	11.09%	203.3057	-2.88%	224.0836	7.05%
2009	202.0795	218.5786	8.16%	219.3208	8.53%	235.6575	16.62%	211.3472	4.59%
MAPE			4.81%		5.01%		6.91%		3.47%
MAPE except 2008			3.26%		3.50%		7.92%		2.58%

Table 4.23 The forecasting results of different models for Östergötland

Year	Actual value	ARIMA		VAR-1		VAR-2		AR(1)	
		Prediction	Error	Prediction	Error	Prediction	Error	Prediction	Error
2005	192.7667	194.6072	0.95%	196.2929	1.83%	193.8943	0.58%	189.2694	-1.81%
2006	198.8458	205.4270	3.31%	199.9511	0.56%	196.7858	-1.04%	193.1309	-2.87%
2007	205.9685	211.4364	2.65%	196.9811	-4.36%	209.8506	1.88%	199.6478	-3.07%
2008	197.6261	215.3611	8.97%	225.8221	14.27%	207.5362	5.01%	206.1119	4.29%
2009	197.9272	210.5786	6.39%	209.5141	5.85%	194.4022	-1.78%	198.5409	0.31%
MAPE			4.46%		4.64%		2.06%		2.47%
MAPE except 2008			3.33%		2.24%		1.32%		2.02%

Table 4.24 The forecasting results of different models for Jönköping

Year	Actual value	ARIMA		VAR-1		VAR-2		AR(1)	
		Prediction	Error	Prediction	Error	Prediction	Error	Prediction	Error
2005	196.4596	200.8684	2.24%	214.4111	9.14%	209.1093	6.44%	203.0992	3.38%
2006	211.2282	206.4495	-2.26%	192.6479	-8.80%	213.5043	1.08%	205.1331	-2.89%
2007	225.2113	214.9976	-4.54%	227.4501	0.99%	203.0187	-9.85%	217.6095	-3.38%
2008	219.6611	232.3611	5.78%	239.5150	9.04%	225.8590	2.82%	226.3761	3.06%
2009	200.0033	223.5786	11.79%	216.1845	8.09%	226.3786	13.19%	219.3057	9.65%
MAPE			5.32%		7.21%		6.68%		4.47%
MAPE except 2008			5.21%		6.75%		7.64%		4.82%

5. Discussion and Conclusions

The purpose of this thesis is to test and distinguish which of the three time series models performs best in forecasting regional GDP per capita. Here, three kinds of models were taken into consideration, the ARIMA, VAR and AR(1) models. In the empirical analysis, 17 annual observations from 1993 to 2009 are available. The original data are separated into 12 in-sample years (1993-2004), thus using approximately 70% of the original data for fitting the model, and 5 out-of-sample years (2005-2009) for prediction. Since there are so few observations in total available, increasing the size of the out-of-sample data is difficult and would probably decrease the accuracy of the forecasts.

As the sample size is very small, with only 12 annual observations available for each region, the performance of complex models is very limited. Taking Stockholm as an example, the mean absolute percentage error of two kinds of VAR models are 2.53% and 3.14%, respectively. The performance of the ARIMA model is better than the VAR models with a 2.05% mean absolute percentage error. However, the performance of the simplest time series model, the AR(1) model, is impressive by offering a 5 year prediction with only 1.71% mean absolute percentage error. The results are similar for the three of the other four regions, Västergötland, Skåne, and Jönköping. However, for Östergötland, there is an exception in that the performance of the VAR model-2 is the best.

In practice, we often face the situation that the number of observations we can get is few. However, the decision makers still need to forecast the economic development and decide the government policy. Based on the analysis above, under such conditions, the simple AR(1) model is still quite useful. Sometimes, as shown in this empirical

study, the performance of the AR(1) model is even better than other complex ones.

Obviously, the prediction of GDP per capita is very complicated, since the value is affected by a great deal of factors, such as prices, disasters, and the economic crisis and so on. Therefore, the simple time series models are not always enough to offer an accurate prediction of GDP per capita. However, for short-term forecasting, the results of time series models could be used as preliminary predictions, which can be used for the regional government to draw up economics plans and policies.

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