

ANALYSIS OF THE QUALITY OF SERVICES
FOR CHECKOUT OPERATION IN ICA
SUPERMARKET USING QUEUING THEORY

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ABSTRACT

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This paper describes a queuing model for a single server process. This model is developed for a checkout sales operation in ICA supermarket, Borlange. The model contains one server which is checkout sales counter area. This server consists of a series of Queues. In any service system, a queue forms whenever current demand exceeds the existing capacity to serve. This occurs when the checkout operation unit is too busy to serve the arriving costumers, immediately. After the office hours, in which arrivals of customers occur at varying times hence the service times also vary accordingly, the chances of number of customers may increase.

This paper contains the analysis of Quality service of supermarket account service unit. One of the expected gains from studying queuing systems is to make the services more efficient in terms of resources utilization, hence eliminating the queues so customers will not have to wait when servers are too busy. In other words, trying to detect the variability in service and time, and eliminate that variability to improve the quality of the services where required, is the aim of our research paper.

This study requires an empirical data which may include the variables like, Arrival Time in the checkout operating unit, Departure Time, etc. A questionnaire is developed to collect the data for such variables and the reaction of the ICA Supermarket from the customers separately.

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OBJECTIVES OF STUDY

The purpose of this study is to evaluate the quality of the service using the observed data and to develop a mathematical model to analyze the performance of checking out sales service unit of ICA Supermarket. The main idea in the application of a mathematical model is to measure the service rate provided to the customers while checking out, and to measure the gaps between the arrivals of each customer per hour. The other important factor analyzed is about the perspective of the customers regarding the quality of service using questionnaire.

INTRODUCTION

Customer's perception of the quality of the service is an important part of a business for any organization. This paper is about the analysis of quality service of supermarket account service unit by using queuing theory to describe a model. The research object of the model is ICA Supermarket Borlange.

ICA AB is a Swedish corporate group in the retail business, which started in 1938 and operated 1,668 stores as of 2003. The stores have different profiles, depending on location, range of products and size:

1. ICA Nara ("ICA Near-by", convenience-type stores for daily retail needs);
2. ICA Supermarket (Mid-size supermarkets, located near where customers dwell or work carrying a wide range of products);
3. ICA Kvantum (Superstores for large, planned, purchases. Large spaces allocated for traffic and parking. Typically located outside of the cities);
4. MAXI ICA Stormarknad (these are Hypermarkets with a full range of groceries as well as fashions, home wares, entertainment and electrical. Smaller stores do not offer the fashion and electrical ranges while the largest stores also have a DIY and Garden department).

Each store is owned and operated separately, but operations are coordinated within the group. All feature ICA brand products.

There are two ICA Supermarkets in Borlange, the bigger one was chosen to be the research object and to collect data from.

The main purpose of this paper is to evaluate the quality of the service for the sales checkout operation in ICA Supermarket. Therefore, a mathematical model is developed to analyze the performance of the checking out service unit. Two important results need to be known from the data collected in the supermarket by the mathematical model: one is the 'service rate' provided to the customers during the checking out process, and the other is the gaps between the arrival times of each

customer per hour. In order to get an overall perspective of the customer's quality of service, the questionnaires which indicate the result in percentages, are also used to get the evaluation from the customers directly.

There are seven counters in ICA Borlange at one place, which means consisting of one server (single service facility) by the Queuing Theory. A queue forms whenever current demand exceeds the existing capacity to serve when each counter is so busy that arriving customers cannot receive immediate service facility. So one-server process queuing model works in this situation.

The data used in the Queuing Theory is collected for various arrival times of each customer in two days by the questionnaire form. The observations for number of customers in a queue, their arrival-time and departure-time were taken without distracting the employees. The whole procedure of the service unit each day was observed and recorded using a time-watch during the same time period for each day. In addition, the questionnaires are conducted at the same timings for each day.

The aim of studying queuing system is trying to detect the variability in service and time, and find the ways to eliminate it to improve the quality of the services where required. The results obtained from ICA Supermarket using queuing model suggest that sales checkout operating unit is rather busy each day of a week but the service is satisfactory.

Chapter 2

BACKGROUND

Quality of Service is defined as the perception of the customers about the service they avail. The customer's perception of the quality of the service being delivered will determine whether or not they are satisfied with that service and whether or not they shall want to continue to work with that service provider. There may be the temptation to consider the adoption of a customer's perspective to be biased and possibly even one-sided. The most important point is that the customer is concerned with the business functionality, that is, customers need to be able to use the checking out service in ICA to perform as a business function.

The concerns of the customers can be classified into three broad categories: Availability, Performance and Accuracy. Whereas, the Affordable cost of the service is a concern of ICA.

Availability focuses upon the question whether the services are available to the customer when the customer wishes to use them, that is, whether the salesman is available or all of the required components are available at the cash counter (checkout operation unit.)

Performance can hold various meanings for different people. The main factor of performance is time, the service should be provided sufficiently in acceptable time. For instance, the checkout operation unit of ICA provides sufficient information and service to the customers for purchasing the items.

Accuracy is concerned with the question of whether or not the service performs accurately. If the services do not accurately perform their functions, then high availability and high performance are worthless, that is, all the required functions from scanning item's price to taking money from the customer (in form of cash, credit card payment, or by a maestro card) should be completed accurately by the employee.

Affordable, the cost of the service cannot be ignored from the list of the components of the quality of services. There is always an exchange between cost of the service and the quality of the service

provided, as well as the timeliness of the service function. Cost is not directly part of quality of service being delivered; it is a limiting factor and cannot be ignored.

ICA supermarket in Borlange is emphasizing on all of these above categories. To analyze the three main concerns of customer: availability, performance and accuracy, regarding checkout sales service, the main service unit is chosen as a server of Queuing model that is the sales checkout operating unit.

Delimitations in improving Quality of Service by customer's perceptions:

The major factors that determine a customer's expectations and perceptions concerning service quality, and three gaps that can lead to dissatisfaction with service delivery

Gap between the customers' expectations and the marketers perceptions

Managers do not always have an accurate understanding of what customers want or how they will evaluate firms' service efforts. The first step in providing a good service, then, is to collect information through customer surveys, evaluations of customer complaints, or other methods, to determine which service attributes customers consider important.

Gap between management perceptions and service quality specifications

Even when management has a clear understanding of customer's want, that understanding might not get translated into effective operating standards. The policies concerning customer service may be unclear or poorly communicated to employees. Unless the employees know what the company's service policies are and believe that management is seriously committed to those standards, their performance is likely to fall short of desired levels.

Gap between perceived service and expected service

This results when management fails to close one or more of the two above-mentioned gaps. It shows the difference between customer's expectations, and his or her actual experience with the firm that leads to dissatisfaction.

METHODOLOGY

Queuing Theory

Delays and queuing problems are most common features not only in our daily-life situations such as at a bank or postal office, at a ticketing office, in public transportation or in a traffic jam but also in more technical environments, such as in manufacturing, computer networking and telecommunications. They play an essential role for business process re-engineering purposes in administrative tasks.

Whenever customers arrive at a service facility, some of them have to wait before they receive the desired service. It means that the customer has to wait for his/her turn, may be in a line. Queuing theory uses mathematics to calculate how to provide services that will minimize the waiting time of the customers.

Sometimes, insufficiencies in services also occur due to an undue wait in service may be because of new employee. Delays in service jobs beyond their due time may result in losing future business opportunities. Queuing theory is the study of waiting in all these various situations. It uses queuing models to represent the various types of queuing systems that arise in practice. The models enable finding an appropriate balance between the cost of service and the amount of waiting.

Queuing Model (Multiple Queue, Single Stage (facility) Model)

The supermarkets may consist of multiple units to perform same checkout operation of sales, which are usually set all together besides the entrance of the supermarket. Each unit contains one employee. This kind of a system is called a single-server system with single service facility and multiple checkout counters to allow multiple queues in a system. Using the same concept of model, the sales checkout operating units are all together taken as a series of queues for one server (single service facility) where the average arrival rate of customers in a queuing system and average service rate per busy server are constants regardless of the state of the system (busy or idle). For such a model the following assumptions are made:

Assumptions

a) Arrivals follow a Poisson distribution

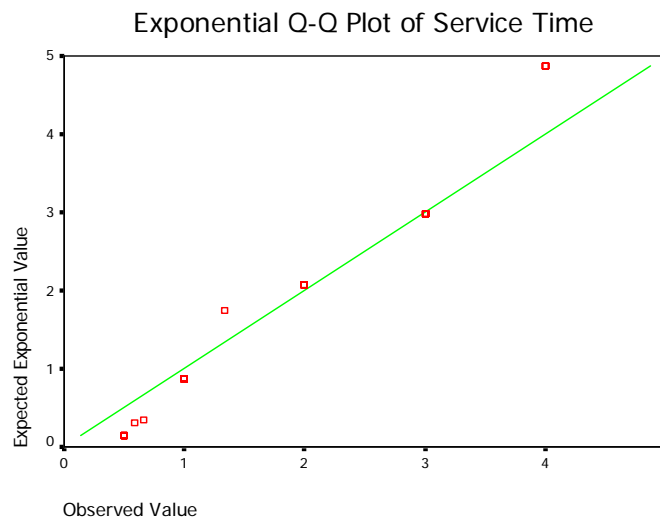
1. The number of the customers that come to the account during time period $[t, t+s)$ only depends on the length of the time period 's' but no relationship with the start time 't'
2. The number of the customers that come to the account during time period $[t, t+s)$ is independent with the coming of customers that before time t
3. If s is small enough, there will be at most one customer come to the account during time period $[t, t+s)$

So, the number of customers that come to the counter is a Poisson process and the arrivals follow a Poisson distribution.

b) Service times are exponentially distributed

The arrivals follow a Poisson distribution, so the length of the time between two arrivals should follow exponential distribution. And according to our observation, the supermarket is so busy that all the accounts work continuously. That means the length of the time between two arrivals is just the length of the service time. So the service times are exponentially distributed.

Q-Q plot is used to check whether the service time is exponentially distributed:



And there is one more thing to mention is that there are only a few points on the graph but the number of the original data is nearly 100. The reason for this condition is that, the data was not observed per seconds, therefore, it is not so accurate and some service time has repeated time.

c) Identical service facilities

- d) No customer leaves the queue without being served
- e) Infinite population
- f) FIFO (First In First Out)+
- g) The arrival rate is smaller than the combined service rate

Characteristics of Queuing Model

Time: neither continuous nor in equidistant discrete steps, but proceeding from event to event;

Object: at least three different kinds of objects namely customer (dynamic object), queue and a server (static object)

Stochastic: arrival and serving times are in-deterministic (or uncertain)

Dynamic: states of servers, queues, and customers depend on past states

Basic Queuing Process

Customers requiring service are generated over time by an input source. The required service is then performed for the customer by the service mechanism, after which the customer leaves the queuing system.

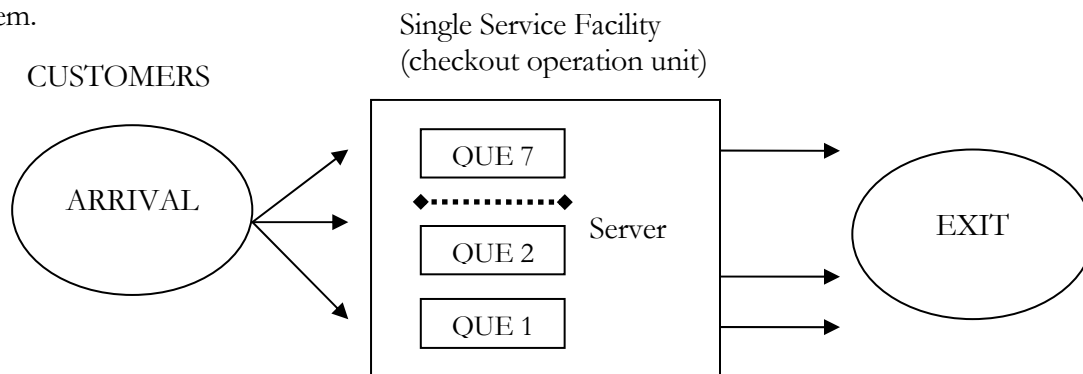


Fig. 1: Single Stage Queuing Model with Multiple Queues in Single Server of Sales Checkout Operation in ICA supermarket

In this model, three various sub-processes may be distinguished:

- Arrival Process: includes finite/infinite number of customers arriving, one/several types of customers, one/several types of customer demands, deterministic or stochastic arrival distance, and arrival intensity.
- Waiting Process: includes length of queue, server order (First In First Out/Last In First Out)

- Server Process: includes a type of a server, serving rate and serving time

Parameters in Queuing Models (Single Server, Single Phase Model)

S	number of servers (Checkout sales operation units in ICA)
λ	arrival rate (Average number of customers arriving in each queue at a server in one hour)
μ	serving rate (Average number of customers being served in a system per hour)
ρ	System intensity or load, utilization factor ($= \lambda/(s\mu)$) (the expected factor of time the server is busy, that is service capability being utilized on the average arriving customers)

The Queuing model is commonly labeled as M/M/s/K, where first **M** represents Markovian exponential distribution of inter-arrival times, second **M** represents Markovian exponential distribution of service times, **s** (a positive integer) represents the number of servers, and **K** is the specified number of customers in a queuing system. This general model contains only limited number of K customers in the system. However, if there are unlimited number of customers exist, which means $K = \infty$, then our model will be labeled as M/M/s (Hillier & Lieberman, 2001.)

Other Notations & their Description assuming the system is in steady-state condition

P_n	Probability of exactly n customers in the system
P_0	Probability of no (or zero) customers in the system (= Probability that the system is idle)
L	Average number of customers in the system (waiting to be served) = $\sum_{n=0}^{\infty} nP_n$
L_q	Average number of customers in the waiting line (queue) = $\sum_{n=s}^{\infty} (n-s)P_n$
W	Average waiting time a customer spends in the system including the service time (Average waiting time for each customer for waiting and being served) = $\frac{L}{\lambda}$ where $\bar{\lambda} = \sum_{n=0}^{\infty} \lambda_n P_n$
W_q	Average waiting time a customer spends waiting in line excluding the service time = $\frac{L_q}{\lambda}$

Chapter 4

ANALYSIS OF CHECKOUT SALES OPERATION SERVICE IN ICA

A sales checkout service has 7 waiting lines in a form of parallel cash counters (see fig.1 in the last chapter). Customers are served on a first-come, first-served (FIFO) basis as a salesman of checkout operation unit becomes free. The data has been collected for two different days, which is Wednesday (weekday) and Friday (weekend). It was assumed that the customers crowd in more, on average, on weekends rather than the weekdays.

Analysis for Weekends

On Friday, customers arrive at an average of 82 customers per hour (Poisson-distributed), and an average of 97 customers can be served per hour by a salesperson (Poisson distributed).

Results

The parameters and corresponding characteristics in Queuing Model M/M/1, assuming system is in steady-state condition, are (for a weekend):

- S number of servers = 1
- λ arrival rate = 82 per hour
- μ serving rate = 87 per server per hour
- Ms Queue Capacity (maximum waiting space) = 7
- M Customer population = M
- $s\mu$ (1) (87) = 87 (average service time)
- ρ = $\lambda/(s\mu) = 82 / 87 = 0.9425$

Overall system utilization = $\rho = 94.2529\%$

Average number of customers in the system (L) = $\sum_{n=0}^{\infty} nP_n = \rho/(1 - \rho) = 16.4$ (where $P_n = \rho^n(1 - \rho)$)

Average number of customers in the queue (Lq) = $\sum_{n=1}^{\infty} (n-1)P_n = \rho^2/(1 - \rho) = 15.4575$

Average time customer spends in the system (W) = $1/(\mu(1 - \rho)) = 0.2000$ hours

Average time customer spends in the queue (Wq) = $\rho/(\mu(1 - \rho)) = 0.1885$ hours

The probability that all servers are idle (P_0) = $(1 - \rho) = 5.7471 \%$

Interpretation of results

The performance of the checkout service is very good. On Friday, the waiting time ($W_q = 11.31$ min) is a bit long, whereas on average 16 customers per hour has to wait in a queue. Although the sales checkout unit has 7 parallel counters (each of them has an individual salesman to deal with the customers in a queue), it is possible that some of the checkout units are idle.

Analysis for Weekdays

On Wednesday, customers arrive at an average of 118 customers per hour (Poisson-distributed), and an average of 125 customers can be served per hour by a salesperson (Poisson distributed).

Results

The parameters and corresponding characteristics in Queuing Model M/M/1, assuming system is in steady-state condition, are (for a weekday):

- S number of servers = 1
- λ arrival rate = 118 per hour
- μ serving rate = 125 per server per hour
- M_s Queue Capacity (maximum waiting space) = 7
- M Customer population = M
- $s\mu$ (1) (125) = 125 (average service time)
- ρ = $\lambda/(s\mu) = 118 / 125 = 0.9440$

Overall system utilization = $\rho = 94.40 \%$

Average number of customers in the system (L) = $\sum_{n=0}^{\infty} nP_n = \rho/(1 - \rho) = 16.8571$ (where $P_n = \rho^n(1 - \rho)$)

Average number of customers in the queue (L_q) = $\sum_{n=1}^{\infty} (n-1)P_n = \rho^2/(1 - \rho) = 15.9131$

Average time customer spends in the system (W) = $1/(\mu(1 - \rho)) = 0.1429$ hours

Average time customer spends in the queue (W_q) = $\rho/(\mu(1 - \rho)) = 0.1349$ hours

The probability that all servers are idle (P_0) = $(1 - \rho) = 5.60 \%$

Interpretation of results

The performance of the service is very good. On Wednesday, the waiting time ($W_q = 8 \text{ min}$) is not that long; whereas on average (L_q) 16 customers per hour has to wait in a queue. Since the sales checkout unit has 7 parallel counters each of them has an individual salesman to deal with the customers in a queue, so it reduces the waiting time of the customers to be served.

If we consider each salesperson in sales checkout unit as a server then it may reduce waiting time approximately zero. That is, our model will be changed as **Multiple Queues, Multiple Servers, Single Stage (checkout facility) Model** (Sheu, C., Babbar S. (Jun 1996)).

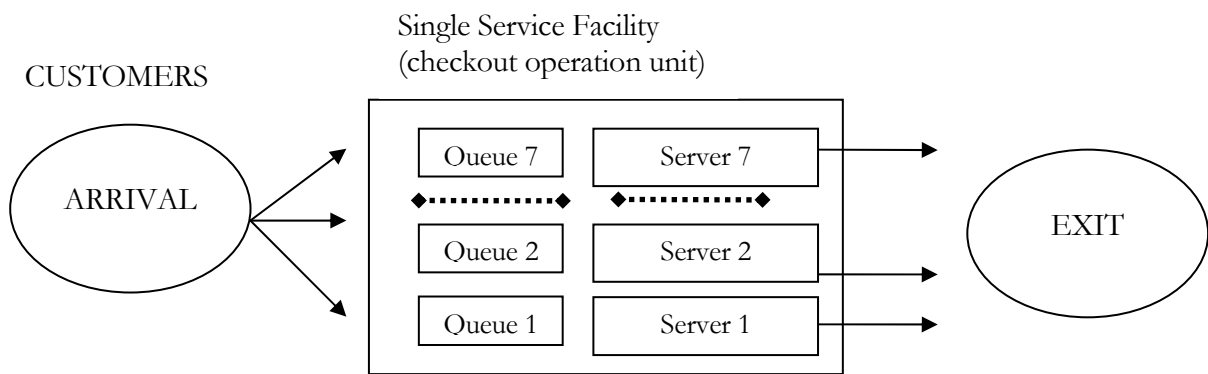


Fig. 2: Single Stage Queuing Model with Multiple Queues and Multiple Servers of Sales Checkout Operation in ICA supermarket

In that case, we can take the results for Friday's data as an example:

Results for Weekends

The parameters and corresponding characteristics in Queuing Model $M/M/7$, assuming system in a steady-state condition, are (for a weekend):

- S number of servers = 7
- λ arrival rate = 82 per hour
- μ serving rate = 87 per server per hour
- M_s Queue Capacity (maximum waiting space) = 1
- M Customer population = M
- $s\mu$ $(7)(87) = 609$ (average service time)
- ρ $= \lambda / (s\mu) = 82 / 609 = 0.1346$

Overall system utilization = $\rho = 13.4647\%$

Average number of customers in the system (L) = 0.9425

Average number of customers in the queue (L_q) = 0

Interpretation of results

The performance of the service on Friday, the average number of customers waiting in a queue is zero ($L_q = 0$) whereas on average (L) only 1 customer per hour has to wait in a queuing system. Which contradicts the queuing system; that is, no queue is formed in a server. In other words, the entire sales checkout units are functional unnecessarily whereas numbers of customers are limited.

DISCUSSION

The insight gained from studying queuing systems is that they may not be very efficient in terms of resources utilization. Queues form and customers wait even though servers may be idle much of the time. The fault is not in the model or underlying assumptions. It is a direct consequence of the variability of the arrival and service processes. If variability could be eliminated, system could be designed economically so that there would be little or no waiting, and hence no need for queuing models.

With the increasing number of customers coming to ICA for shopping, either for usual grocery or for some house wares, there is a trained employee serving at each service unit. Sales checkout service has sufficient number of employees (Queues) which is helpful during the peak hours of weekends or weekdays. Other than these hours, there is no need of various Queues in a model. Increasing more than sufficient number of servers may not be the solution to increase the efficiency of the service by each service unit. Considering salespersons in a server as an individual server, contradicts the queuing system; that is, no queue is formed in a server in that situation. In other words, the entire sales checkout units are functional unnecessarily whereas numbers of customers are limited.

As far as Quality of Service is concerned, the customer is always satisfied with the complete and efficient service in a shorter time span. ICA supermarket in Borlange is efficiently following all the components of good quality service. Moreover, the analysis proves an efficient and a good quality service of sales checkout units provided to the customers for purchasing items at ICA supermarket Borlange.

APPENDIX

Questionnaire

This questionnaire is only for Employees serving at Sales checkout unit of ICA supermarket. Therefore, all information is intended specifically to the Service Unit.

Service provided to the customer at the spot (Quality of Service)

Please mark \surd into and fill into the following questions:

1. At what time the customer arrived at the sales checkout unit?

2. Is there any other customer waiting for his turn, while the service unit is already serving another customer?

No

Yes

3. If yes, then how many?

1

3

5

2

4

6 or more

4. Mode of payment for purchase of items at checkout unit:

Cash

Credit card

5. The checking out operation service given to the customer for sales item was _____ for the customer.

Sufficient

Moderately Sufficient

Insufficient / Incomplete

6. At what time the customer left the sales checkout unit after a successful purchase of items?

Spreadsheet

Friday

	Arrival Time	Customer Type (Swede / Non-Swede)	No. of People in a Queue	Payment Mode (Cash / Credit)	Departure Time	Service Time	
1	16:50:00	S		cash	16:51:00	0:01:00	
2	16:51:00	S		cash	16:53:30	0:02:30	
3	16:53:00	S		card	16:55:00	0:02:00	
4	16:54:00	S		cash	16:55:00	0:01:00	
5	16:55:00	S		cash	16:56:00	0:01:00	
6	16:55:00	S		card	16:56:00	0:01:00	
7	16:56:00	S		card	16:56:45	0:00:45	
8	16:56:00	S		card	16:57:00	0:01:00	
9	16:57:00	S	0	Card	16:59:10	0:02:10	
10	16:59:10	S	1	Cash	16:59:40	0:00:30	
11	16:58:00	S		cash	16:59:00	0:01:00	
12	17:00:00	S	0	Card	17:01:00	0:01:00	
13	17:01:00	S	0	Card	17:03:00	0:02:00	
14	17:03:00	S	1	Card	17:03:30	0:00:30	
15	17:03:00	S		cash	17:03:40	0:00:40	
16	17:03:30	S	2	Card	17:04:00	0:00:30	
17	17:04:00	S		card	17:05:00	0:01:00	
18	17:04:00	S	0	Cash	17:05:00	0:01:00	
19	17:05:00	S	0	Card	17:06:00	0:01:00	
20	17:07:00	S	2	enquiry	17:17:00		
21	17:07:00	S	0	Cash	17:09:00	0:02:00	
22	17:09:00	S	1	Cash	17:10:00	0:01:00	
23	17:10:00	S	2	Cash	17:11:00	0:01:00	
24	17:10:00	S	left without transaction				
25	17:11:00	S	1	Card	17:12:00	0:01:00	
26	17:12:00	S	2	Card	17:12:50	0:00:50	
27	17:12:50	S	2	Cash	17:14:00	0:01:10	
28	17:14:00	S	3	Card	17:14:30	0:00:30	
29	17:14:30	S	3	Cash	17:15:00	0:00:30	
30	17:15:00	S	4	Card	17:18:00	0:03:00	
31	17:15:00	S	5	Card	17:16:00	0:01:00	
32	17:15:00	S	2	cash	17:16:00	0:01:00	
33	17:16:00	S	0	cash	17:17:00	0:01:00	
34	17:18:00	NS	1	Card	17:19:00	0:01:00	
35	17:17:00	S	2	cash	17:17:30	0:00:30	
36	17:19:00	S	2	Card	17:20:00	0:01:00	
37	17:18:00	S	0	card	17:18:30	0:00:30	
38	17:20:00	S	2	Card	17:21:00	0:01:00	
39	17:21:00	S	2	Card	17:21:40	0:00:40	
40	17:19:00	S	0	cash	17:19:30	0:00:30	
41	17:20:00	S	2	cash	17:21:00	0:01:00	
42	17:21:40	S	1	Cash	17:23:00	0:01:20	
43	17:21:00	S	0	cash	17:21:30	0:00:30	
44	17:21:00	S	0	card	17:23:00	0:02:00	
45	17:23:00	S	1	Cash	17:23:30	0:00:30	
	17:08				17:09	0:01:04	

mean arrival time = 0.0122 Total time of data collection = 0:33 min
 Arrival Rate = 82
 mean serving time = 1.0667
 Service Rate = 87

The missing cells in column 'No. of people in a Queue' are due to inefficiency of the observer. This is ignored because this information has no concern with the research result.

Wednesday

	Arrival Time	Customer Type (Swede / Non-Swede)	No. of People in a Queue	Payment Mode (Cash / Credit)	Departure Time	Service Time
1	16:25:00	S	1	card	16:26:00	0:01:00
2	16:25:00	S	0	Card	16:26:00	0:01:00
3	16:26:00	S	1	Cash	16:27:00	0:01:00
4	16:26:00	S	0	cash	16:27:20	0:01:20
5	16:27:00	S	3	card	16:27:30	0:00:30
6	16:27:00	S	2	cash	16:28:00	0:01:00
7	16:27:00	S	1	Cash	16:28:00	0:01:00
8	16:28:00	S	2	Cash	16:28:30	0:00:30
9	16:28:30	S	3	Cash	16:29:00	0:00:30
10	16:29:00	S	4	Card	16:31:00	0:02:00
11	16:28:00	S	1	cash	16:28:30	0:00:30
12	16:31:00	S	3	Card	16:32:00	0:01:00
13	16:29:00	S	0	cash	16:30:00	0:01:00
14	16:29:00	S	3	Card	16:30:00	0:01:00
15	16:30:00	S	4	Cash	16:31:00	0:01:00
16	16:30:00	NS	1	card	16:31:00	0:01:00
17	16:31:00	S	2	card	16:32:00	0:01:00
18	16:32:00	S	3	cash	16:32:30	0:00:30
19	16:32:00	S	5	card	16:33:00	0:01:00
20	16:33:00	S	5	card	16:33:30	0:00:30
21	16:33:00	S	5	Cash	16:36:00	0:03:00
22	16:36:00	S	6	Cash	16:37:00	0:01:00
23	16:34:00	S	4	cash	16:34:30	0:00:30
24	16:34:00	S	3	card + cash	16:35:00	0:01:00
25	16:35:00	S	2	cash	16:35:30	0:00:30
26	16:37:00	S	4	Cash	16:38:00	0:01:00
27	16:36:00	S	1	card	16:37:00	0:01:00
28	16:37:00	S	0	cash	16:37:30	0:00:30
29	16:37:00	S	0	card + cash	16:40:00	0:03:00
30	16:38:00	S	1	Card	16:38:30	0:00:30
31	16:38:30	S	1	Cash	16:40:00	0:01:30
32	16:40:00	S	0	card	16:40:30	0:00:30
33	16:40:00	NS	0	card	16:42:00	0:02:00
34	16:40:00	S	0	card	16:43:00	0:03:00
35	16:41:00	S	1	Cash	16:43:00	0:02:00
36	16:43:00	S	2	Card	16:44:00	0:01:00
37	16:43:00	NS	1	card	16:44:00	0:01:00
38	16:44:00	S	1	Cash	16:45:00	0:01:00
39	16:44:00	S	1	cash	16:45:00	0:01:00
40	16:45:00	S	2	Card	16:46:00	0:01:00
41	16:45:00	NS	0	cash	16:46:00	0:01:00
42	16:46:00	S	1	cash	16:47:00	0:01:00
43	16:47:00	S	2	card	16:48:00	0:01:00
44	16:48:00	S	2	card	16:48:40	0:00:40
45	16:48:00	S	1	cash	16:49:00	0:01:00
46	16:49:00	S	1	card	16:49:35	0:00:35
47	16:49:00	S	2	cash	16:50:00	0:01:00
48	16:50:00	S	1	cash	16:50:30	0:00:30
49	16:50:00	NS	0	cash	16:51:00	0:01:00
	16:36				16:37	0:01:04

mean arrival time = 0.0085 Total time of data collection = 0:25
 Arrival Rate = 118 min
 mean serving time = 1.0667
 Service Rate = 125

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