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STATISTICS

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Testing the unit root hypothesis in Smooth Transition Autoregressive (STAR) models

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Abstract

In this paper, we propose a Nonlinear Dickey-Fuller (NDF) F test of a unit root in a first order Logistic Smooth Transition Autoregressive models LSTAR(1) which contains nonlinear dynamic structures. The NDF F test statistics is derived under the null hypothesis of a random walk with or without drift and the alternative model is a stable LSTAR(1) model. It is shown that the limiting distribution of the NDF F test is nonstandard. Empirical distributions of small samples are investigated by Monte Carlo experiment. An empirical example is illustrated by applying the NDF F test to the France's unemployment rate[1] from 1955 to 1999, and a regime-switching type of nonlinearity evidence is found during the period.

Keyword: Unit root, Unit root Test, Dickey-Fuller F test, STAR model, Smooth transition function, Monte Carlo experiment

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[1] Unemployment rate: The number of people who is out of work in 100 labors.

Content List

I. Introduction

Recently, in many empirical studies of economic time series there exist strong evidence that many time series display nonlinear features. For example, in the unemployment rate time series reported from OECD[2] countries, when time evolves, the data break always occurs according to the adjustment of national politics or enormous economical effectors such as the oil price change, and the whole period's data show non-linearity, (see Joakim Skalin and Timo Teräsvirta (2002)). In order to capture such nonlinear features, we need to employ nonlinear dynamic models. Many nonlinear dynamic models have been introduced in the literature, we adopt one of the most popular nonlinear models, the Smooth Transition Autoregressive (STAR) model, (see Simon M. Potter (1999)). The STAR models allow nonlinear structures between the data regimes with a smooth regime transition function and they are already successfully applied to a wide range of time series. See, for example, Teräsvirta and Anderson (1992) for the model application to industrial production time series data.

Testing linearity against nonlinearity is a necessary testing procedure when researchers wish to consider a nonlinear modeling. A unit root test in the nonlinear time series model is a joint test that under the null hypothesis testing of both unit root and linearity. It is well know that classical Dickey-Fuller (1976) tests are found of lacking power when alternative model shows non-linearity. Leybourne *et al.*(1998) and Harvey and Mills(2002) have discussed the situation where the model with a logistic smooth transition in intercept and trend. In this paper we derive a test for testing hypothesis of unit root and linearity in the models containing the smooth transition in intercept and dynamics part with a logistic smooth transition function. He and Sandberg (2005) have proposed the nonlinear Dickey-Fuller ρ and t test statistics, and alternatively we derive a non-linear Dickey-Fuller F test statistic in this paper.

We organize the paper into following sections: Section II presents the model, and in Section III we introduce the procedure for testing unit root against the stable STAR model. In section IV we present the asymptotic properties of the F test statistic, and in section V we investigate finite sample properties of the F test by Monte Carlo experiment. In section VI an empirical example about the OECD unemployment rate of France from 1955 to 1999 is illustrated by applying the proposed testing procedure of the NDF F test. Concluding remarks can be found in the final section. All proofs of Theorems in this paper are given in appendix.

II. The Models

We consider the following two STAR models, the first does not have a constant term whereas the second one does:

Case 1: $y_t = \pi_{11} y_{t-1} + \pi_{21} y_{t-1} F(t;\gamma,c) + \mu_t$; μ_t ~iid (0, σ_u^2) 0< σ_u^2 < ∞ (1)

Case 2: $y_t = \pi_{10} + \pi_{11} y_{t-1} + (\pi_{20} + \pi_{21} y_{t-1}) F(t;\gamma,c) + \mu_t$ μ_t ~iid (0, σ_u^2) 0< σ_u^2 < ∞ (2)

2 Organisation for European Economic Co-operation (OEEC), led by Frenchman Robert Marjolin, to help administer the Marshall Plan for the reconstruction of Europe after World War II. Later its membership was extended to non-European states, and in 1961 it was reformed into the Organisation for Economic Co-operation and Development.

The transition function $F(t;\gamma,c)$ in (1) and (2) is defined as follows:

$$F(t;\gamma,c)=\frac{1}{(1+\exp\{-\gamma(t-c)\})}-\frac{1}{2}.$$

This transition function is called the logistic smooth transition function and our models are first order logistic smooth transition models LSTAR(1). In the function, γ is a parameter that determines the speed of transition from one regime to another at time c, the larger the γ is, the steeper of the transition function will be. In Figure 1, we set c fixed in three cases where γ =20, 10, 5. Then the smooth transition function is a bounded continuous non-decreasing transition function in t such that F(t) : $R_+ \rightarrow [-1/2,\ 1/2]$ when t is from 1 to 44.

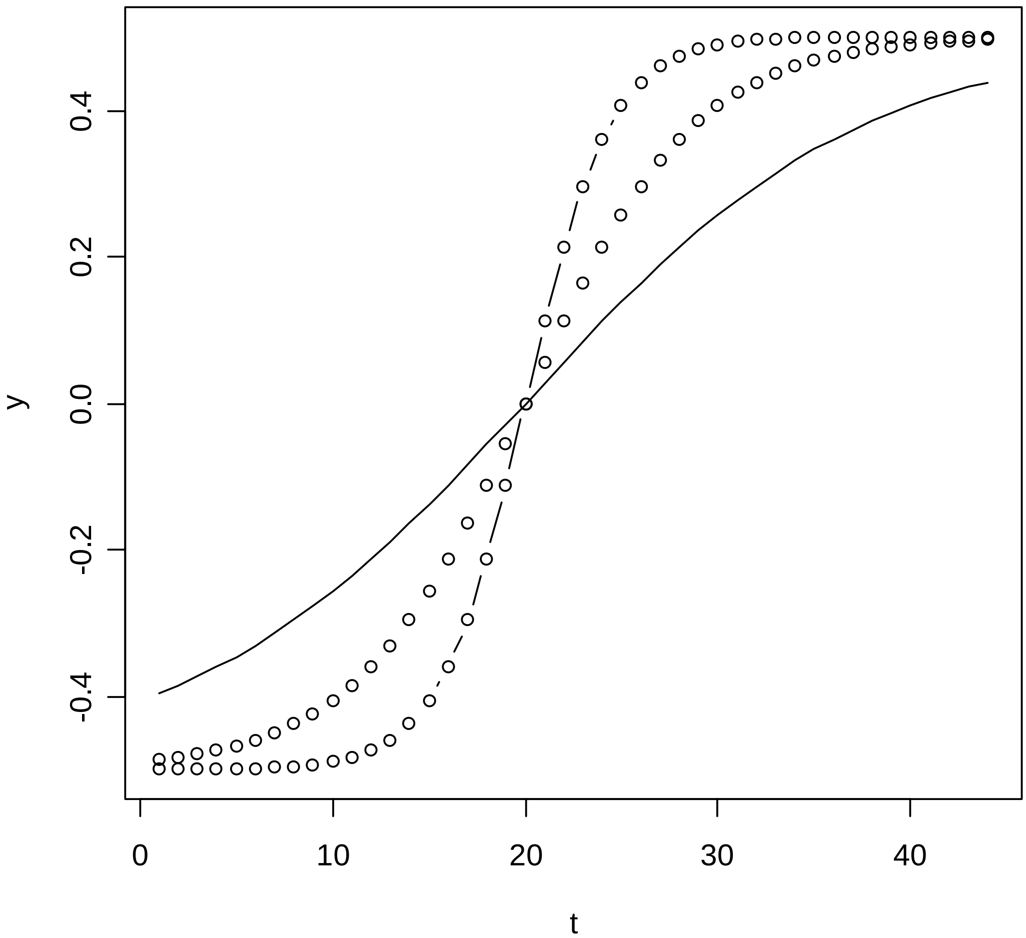

Figure 1: Logistic smooth transition function for γ =20 (dashed—dotted line), γ =10 (dotted line), γ =5 (solid line)

The models also show that the level and dynamics are initially at equilibrium and as time evolves a new long run equilibrium took place. Meanwhile, set t and c fixed, $F(t;\gamma,c)$ is the function of γ and $\lim_{\gamma\to 0} F(\gamma)=0$ which leads the resulting model become linear.

III. Testing procedure:

Our goal is to test the null hypothesis of a random walk without drift against the stable nonlinear LSTAR(1) model. The null hypothesis can be expressed as the following parameter restriction:

Case1 $H_0^*: \gamma=0, \pi_{11}=1$

Case2 $H_0: \gamma=0, \pi_{10}=0, \pi_{11}=1$

As γ =0 implies that the transition function $F(t;\gamma,c)$ =0, then γ =0 represent that the model is linear, $\pi_{10}=0, \pi_{11}=1$ represent a random walk without drift. However γ =0 will lead to an identification problem under the null hypotheses, to remedy the problem, we follow the approach which Luukkonen, Saikkonen and Teräsvirta (1988) use, we apply Taylor expansion of γ around 0 in $F(t;\gamma,c)$. However, we should keep in mind that the first-order expansion will lead to low power if the transition takes place only in the intercept (see Luukkonen *et al*.(1988) and He and Sandberg (2005a)).Therefore, the third-order Taylor expansion are more robustness in power. The first and third –order Taylor expansion are as follows:

$$F_1(t;\gamma,c)=\frac{\gamma(t-c)}{4}+r_1(\gamma)$$
$$F_3(t;\gamma,c)=\frac{\gamma(t-c)}{4}+\frac{\gamma^3(t-c)^3}{48}+r_3(\gamma)$$

Substituting the above equations into H_a, after merging the terms, we get the following auxiliary regressions:

$$Case1 \,\&\, Order1: y_t=(y_{t-1}s_{1t})'\varphi_1+u_{1t}^*$$
$$Case1 \,\&\, Order3: y_t=(y_{t-1}s_{3t})'\varphi_3+u_{3t}^*$$
$$Case2 \,\&\, Order1: y_t=s_{1t}'\lambda_1+(y_{t-1}s_{1t})'\varphi_1+u_{1t}$$
$$Case2 \,\&\, Order3: y_t=s_{3t}'\lambda_3+(y_{t-1}s_{3t})'\varphi_3+u_{3t}$$

Where the parameters are defined as follows:

$s_{1t}=(1,t)'; \lambda_1=(\lambda_{10},\lambda_{11})', \varphi_1=(\varphi_{10},\varphi_{11})'$, $s_{3t}=(1,t,t^2,t^3)'$, $\lambda_3=(\lambda_{30},\lambda_{31},\lambda_{32},\lambda_{33})'$,

$\varphi_3=(\varphi_{30},\varphi_{31},\varphi_{32},\varphi_{33})'$

Then the corresponding auxiliary null hypotheses are:

$H^{*}_{0m}: \varphi_{m0} = 1, \varphi_{mj} = 0,\ j \geq 1; m = 1,3$

$H_{0m}: \lambda_{mi} = 0, \forall i; \varphi_{m0} = 1, \varphi_{mj} = 0,\ j \geq 1; m = 1,3$

We should keep in mind that under the null hypothesis, u^{*}_{mt} and u_{mt} is equal to u_t ; m=1,3

IV. Distributions of the Dickey-Fuller F tests

In this section, we derive unit-roots test statistics based on the above equations and their distribution properties. In order to get the limiting distributions for the test statistics are nuisance parameter-free, we should present the theorem under the assumption that the error term in equation (1) and (2) is i.i.d. About the inference when the nuisance parameters depending on the mixing assumption, see Phillips (1987); Phillips and Perron (1988); He and Sandberg, (2005 a).

Assumption :Let $u_t \sim iid(0,\sigma^2)$ with $E(u_t^4) < \infty$

Under Assumption, we derive two theorems that will be used to deduct the D-F F tests statistic distribution. We first consider the case that does not contain intercept.

Theorem1:

Consider models $y_t = (y_{t-1} s_{mt})' \varphi_m + u^{*}_{mt}$ hold, and assume that $(u^{*}_t)^{\infty}_{t=1}$ fulfills assumption 1, then for m=1,3 , such that

$$\hat{\psi}^{*}_m - \psi^{*}_m \xrightarrow{p} 0, \Upsilon^{*}_{m}(\hat{\psi}^{*}_m - \psi^{*}_m) \xrightarrow{d} (\Psi^{*}_m)^{-1}\Pi^{*}_m,$$

$$(s^{*}_{m})^2 \Upsilon^{*}_{m} (\sum x^{*}_{mt}(x^{*}_{mt})')^{-1} \Upsilon^{*}_{m} \xrightarrow{L} \sigma^2 (\Psi^{*}_m)^{-1}$$

Where the parameter restrictions are as follows:

$$\Upsilon^{*}_{1} = diag\{T^{*}_1\}, T^{*}_1 = [\,T \quad T^2\,]\ ,\ (r^{*}_1)' = [1\ 0]$$

$$\Upsilon^{*}_{3} = diag\{T^{*}_3\}, T^{*}_3 = [\,T \quad T^2 \quad T^3 \quad T^4\,],\ (r^{*}_3)' = [1\ 0\ 0\ 0]$$

$$\hat{\psi}^{*}_m = (\hat{\varphi}'_m),\ \psi^{*}_m = (\varphi'_m),\ x^{*}_{mt} = \left[\, y_{t-1} s_{mt} \,\right]$$

$$\Psi^{*}_m = \sigma^2_u C_m \qquad \Pi^{*}_m = E_m$$

$$C_m = \left[\, c_{ij} \,\right]_{(m+1)*(m+1)}, E_m = \left[e_i \right]_{(m+1)*1,}$$

$$c_{ij} = \int_0^1 r^{i+j-2} W(r)^2 dr,\ e_i = \frac{\left(W(1)^2 - (i-1)\int_0^1 r^{i-2} W(r)^2 dr - 1/i\right)}{2}$$

Based on Theorem1, we have Nonlinear D-F F test statistics as follows:

$$F^{*}_m = (\hat{\psi}^{*}_m - \psi^{*}_m)'(R^{*}_m)'\{(s^{*}_m)^2 R^{*}_m [\sum x^{*}_{mt}(x^{*}_{mt})']^{-1}(R^{*}_m)'\}^{-1} R^{*}_m (\hat{\psi}^{*}_m - \psi^{*}_m)/2$$

$$= (\hat{\psi}^{1}_m - \psi^{1}_m)'(R^{1}_m)' \Upsilon^{*}_{m} * \{(s^{*}_m)^2 \Upsilon^{*}_{m} R^{*}_m [\sum x^{*}_{mt}(x^{*}_{mt})']^{-1}(R^{*}_m)' \Upsilon^{*}_{m}\}^{-1} * \Upsilon^{*}_{m} R^{*}_m (\hat{\psi}^{*}_m - \psi^{*}_m)/2$$

$$\xrightarrow{L}[(\Psi_m^*)^{-1}\Pi_m^*]'[\sigma_u{}^2(\Psi_m^*)^{-1}]^{-1}[(\Psi_m^*)^{-1}\Pi_m^*]/2=(\Pi_m^*)'(\Psi_m^*)^{-1}\Pi_m^*/2\sigma_u{}^2$$

Theorem 2 is for the second case that contains smooth transition function both in intercept and dynamics.

Theorem2:

Consider models $y_t = s_{mt}^{'}\lambda_m + (y_{t-1}s_{mt})^{'}\varphi_m + u_{mt}$ hold, and assume that $(u_t)_{t=1}^{\propto}$ fullfill assumption 1, then for m=1,3:

$$\Upsilon_m(\hat{\psi}_m-\psi_m)\xrightarrow{d}\Psi_m^{-1}\Pi_m,\hat{\psi}_m-\psi_m\xrightarrow{p}0,$$

$$s_m^2\Upsilon_m(\sum x_{mt}x_{mt}^{'})^{-1}\Upsilon_m\xrightarrow{L}\sigma_u{}^2\Psi_m^{-1}$$

Where the parameters are defined as the follows:

$$\Upsilon_1 = diag\{T_1\}, T_1 = [T^{1/2}\ T^{3/2}\quad T\quad T^2]\ ,\ r_1^{'} = [0\ 0\ 1\ 0]$$

$$\Upsilon_3 = diag\{T_3\}, T_3 = [T^{1/2}\quad T^{3/2}\quad T^{5/2}\quad T^{7/2}\quad T\quad T^2\quad T^3\quad T^4],\ r_3^{'} = [0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ ]$$

$$\hat{\psi}_m = (\hat{\lambda}_m^{'}, \hat{\varphi}_m^{'}),\ \ \psi_m = (\lambda_m^{'}, \varphi_m^{'}),\ \ x_{mt} = \begin{bmatrix} s_{mt} \\ y_{t-1}s_{mt} \end{bmatrix}$$

$$\Psi_m = \begin{bmatrix} A_m & \sigma_u \mathrm{B}_m \\ \sigma_u \mathrm{B}_m^{'} & \sigma_u^2 C_m \end{bmatrix}\quad \Pi_m = \begin{bmatrix} \sigma_u D_m \\ \sigma_u^2 E_m \end{bmatrix}$$

$$A_m = \left[a_{ij}\right]_{(m+1)*(m+1)}\quad,\quad B_m = \left[b_{ij}\right]_{(m+1)*(m+1)}\quad,\quad C_m = \left[c_{ij}\right]_{(m+1)*(m+1)}\quad,\quad D_m = \left[d_i\right]_{(m+1)*1,}$$

$$E_m = \left[e_i\right]_{(m+1)*1,}$$

$$a_{ij} = T^{-(i+j-1)} * \sum_{t=1}^{T} t^{i+j-2}, b_{ij} = \int_0^1 r^{i+j-2}W(r)dr,\ c_{ij} = \int_0^1 r^{i+j-2}W(r)^2 dr,$$

$$d_i = W(1) - (i-1)\int_0^1 r^{i-2}W(r)dr,\quad e_i = \frac{\left(W(1)^2 - (i-1)\int_0^1 r^{i-2}W(r)^2 dr - 1/i\right)}{2}$$

Based on Theorem1, we have Nonlinear D-F test statistics as follows:

$$F_m = (\hat{\psi}_m - \psi_m)' R_m^{'}\{s_m^2 R_m(\sum x_{mt}x_{mt}^{'})^{-1}R_m^{'}\}^{-1}R_m(\hat{\psi}_m - \psi_m)/2$$

$$= (\hat{\psi}_m - \psi_m)' R_m^{'}\Upsilon_m * \{s_m^2\Upsilon_m R_m(\sum x_{mt}x_{mt}^{'})^{-1}R_m^{'}\Upsilon_m\}^{-1} * \Upsilon_m R_m(\hat{\psi}_m - \psi_m)/2$$

$$\xrightarrow{L}(\Psi_m^{-1}\Pi_m)'\{\sigma_u{}^2\Psi_m^{-1}\}^{-1}(\Psi_m^{-1}\Pi_m)/2 = \Pi_m^{'}\Psi_m^{-1}\Pi_m/2\sigma_u{}^2$$

V. Monte Carlo Experiment

To find out the finite-sample distributions of the test, we generate data from the model

$y_t = y_{t-1} + \mu_t$ where μ_t ~n.i.d.(0,1) with desired sample sizes. To get the asymptotic distributions for the Nonlinear Dickey-Fuller F test, we let T=1,000,000 to simulate a Brownian motion W(r), the number of replications are 10,000. Finite sample critical values are reported in Table 1 to Table 4

TABLE 1

Critical values for the NDF F test; Case1, Order 2

	Probability that F test is greater than entry							
T	99%	97.5%	95%	90%	10%	5%	2.5%	1%
25	0.0180	0.0326	0.0707	0.1373	2.5608	3.3741	4.3583	6.5653
50	0.0093	0.0344	0.0753	0.1510	2.3827	3.0328	3.7304	4.9331
100	0.0147	0.0354	0.0524	0.1165	2.6033	3.4503	4.3621	5.3290
250	0.0165	0.0369	0.0644	0.1386	2.4480	3.0523	3.8390	5.7825
500	0.0180	0.0440	0.0812	0.1522	2.6688	3.3631	4.1448	5.6598
∞	0.0427	0.0476	0.0569	0.0950	2.6946	3.6027	3.8758	4.0915

TABLE 2

Critical values for the NDF F test; Case1, Order 4

	Probability that F test is greater than entry							
T	99%	97.5%	95%	90%	10%	5%	2.5%	1%
25	0.2111	0.2917	0.4382	0.6315	4.4386	5.8039	7.0416	8.6338
50	0.1741	0.3256	0.4478	0.6617	4.3110	5.4232	6.5863	8.4232
100	0.1850	0.3007	0.4562	0.6560	4.4515	5.1960	6.2109	7.9776
250	0.1567	0.2586	0.4340	0.6063	4.1787	4.8901	5.6025	6.7303
500	0.1816	0.2839	0.4322	0.6453	4.4517	5.3794	6.1353	7.0345
∞	0.2279	0.3781	0.4554	0.5938	4.1860	5.8483	6.2879	6.7039

TABLE 3

Critical values for the Nonlinear D-F F test; Case 2, Order 2

	Probability that F test is greater than entry							
T	99%	97.5%	95%	90%	10%	5%	2.5%	1%
25	1.2681	1.5086	1.8299	2.2519	9.0499	10.9624	12.3598	15.6200
50	1.2360	1.5645	1.8089	2.2370	7.6721	8.8355	10.0181	12.1867
100	1.3031	1.5181	1.7936	2.1358	7.4440	8.7062	10.3310	11.2303
250	1.3429	1.4833	1.8339	2.3572	7.5173	8.9643	10.2704	11.9400
500	1.3054	1.5704	1.8891	2.2434	7.2648	8.3361	9.3852	10.4372
∞	1.6440	1.8515	2.0155	2.3750	7.5639	8.2807	8.5646	8.6974

TABLE 4

Critical values for the NDF F test; Case 2, Order 4

Probability that F test is greater than entry								
T	99%	97.5%	95%	90%	10%	5%	2.5%	1%
25	2.4583	2.9405	3.3632	3.9933	9.8465	10.8789	11.8609	13.2205
50	3.5896	3.8791	4.3369	4.8877	11.8657	13.1100	14.3658	16.0986
100	4.0925	4.5167	4.9422	5.5757	12.7301	14.0162	15.0412	16.6797
250	4.4219	4.8419	5.3583	6.0085	13.2671	14.4580	15.9058	17.4119
500	4.0384	4.7302	5.3296	6.1675	13.6606	14.9531	16.9386	19.2176
∞	4.3998	4.5921	5.0342	5.9195	13.7299	16.2652	17.7931	21.9990

Figure 2 and Figure 3 show the histograms of Nonlinear Dickey-Fuller test statistic (case 2, order 2, T=50) and the histograms of standard F (2,48) test statistics.

Figure 2: Histogram of F test in Case2&order 2 when T=50

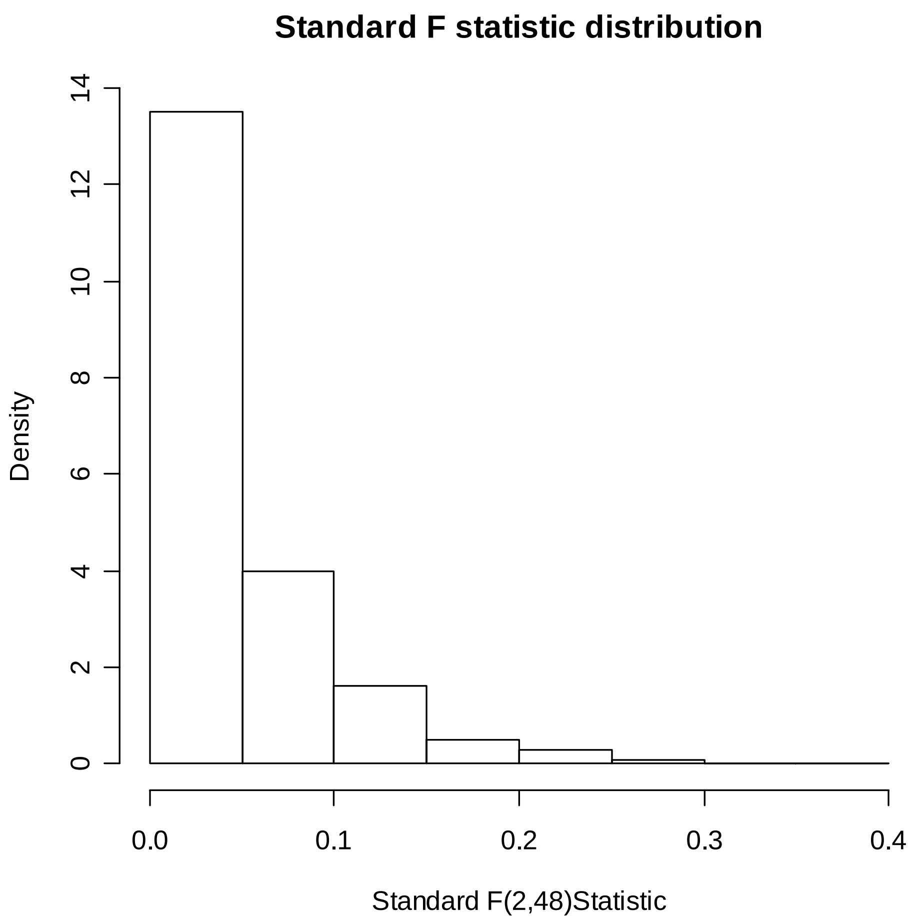

Figure 3: Histogram of Standard F(2,50-2) statistics

We compare the histograms of the Nonlinear Dickey-Fuller F test with standard F (2,48) test in the above graphs. We can see that the standard F(2,48) distribution turn out to be more heavy skewed to left.

VI. Empirical Example

The data in our empirical example is the OECD unemployment rate data of France from 1955 to 1999. It is well known that unemployment rates in OECD countries are nonstationary. From Figure 4, we can see that the time series of France's data shows a obvious data break in the year 1975, which divide the data into two data period. Then we suppose that the STAR model, which contains a smooth transition function, should be a good choice in this case.

However, although we suppose the time series shows Non-linearity, we fit the data with order one autoregressive model and test its unit root with traditional Dickey-Fuller F test first, we take this step into consideration because we will compare the result with the result we get under our nonlinear Dickey-Fuller F test in the later case.

Figure 4: France's unemployment rate, 1945- 1999

In this case, first without testing the data's linearity, we construct a AR(1) process. The following model is fitted by OLS regression to the data, and time t is from1956 to 1999

$$y_t = 0.2274 + 0.9971 y_{t-1} + u_t$$

The sum of residual squares is 15.04, and the sample residual graph of u_t is as follows.

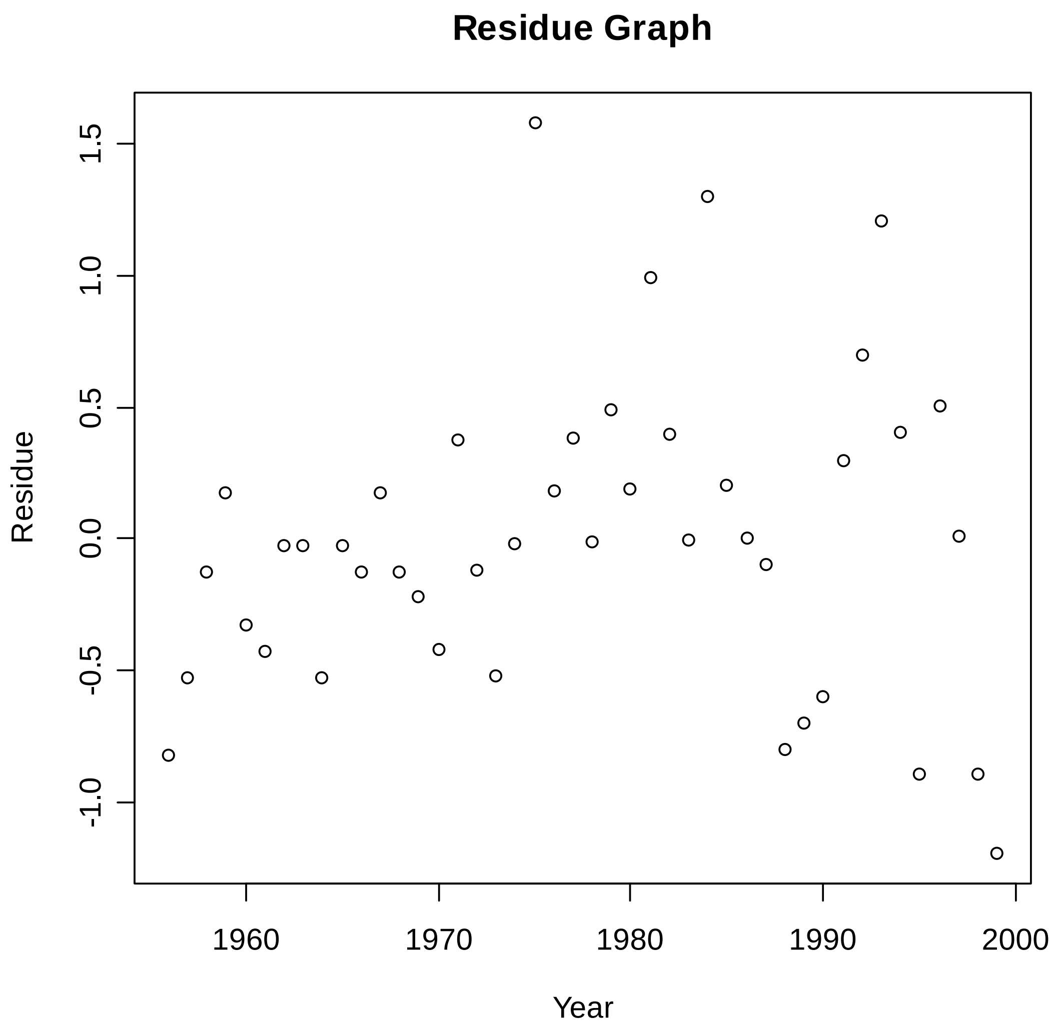

Figure 4: Sample residual graph of the estimated AR(1) model.

Although the residuals does not exhibit a obvious serial correlation, we still need further test of linearity, however, we first perform the traditional Dickey-Fuller F test for the unit root test.
The Dickey-Fuller F test based on the estimated value of F statistic for this specification is:

$$F = (b_T - \beta)' \Upsilon_T' \{s_T^2 \Upsilon_T (\sum x_t x_t')^{-1} \Upsilon_T'\}^{-1} \Upsilon_T (b_T - \beta)/2 = 2.7538$$

$$(b_T = \begin{bmatrix} 0.4710 \\ 0.9353 \end{bmatrix}, \beta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \Upsilon_T = \begin{bmatrix} T^{1/2} & 0 \\ 0 & T \end{bmatrix}, \ \sum x_t x_t' = \begin{bmatrix} T & \sum y_{t-1} \\ \sum y_{t-1} & \sum y_{t-1}^2 \end{bmatrix})$$

We compare F with 4.86 in table B.7, since 2.7538<4.86, the null hypothesis of a unit root is accepted at the 5% level based on the Dickey –Fuller F test. It seems these data can be viewed as random walk without drift. However, from the graph we can see that for the unemployment rate data, the level and dynamics are initially at a equilibrium but after around 1975 there shows a new long-run equilibrium, and it shows a nonlinear structure change. Therefore using the classical DF Ftest may be not valid under the linearity assumption. We need to test the data's linearity at first.

We use the chow test to test the series linearity against a single break, the statistic is

$$F = \frac{SSR_R - (SSR_1 + SSR_2)/k}{(SSR_1 + SSR_2)/(T-2k)}$$

SSR_1 : SSR for the first period

SSR_2 : SSR for the second period

SSR_R : SSR for t=1,…,T

For small sample, the F statistic~ F(k, T-2k) and it is asymptotical to χ^2 (k, T-2k). And the null hypothesis is that the data in period one and period two is in the same linearity(no data break present) Here we set the first period is from 1955 to 1975, the second period is from 1976 to 1995 and get the F statistics :F=0.2369 at the critical value of 20%[3], we reject the null hypothesis and it is proved that there is a data break around 1975, therefore we test the unit root by the nonlinear Dickey-Fuller test in our paper, first, we fit the data to the auxiliary regression model

$$y_t = s_{1t}^{'}\lambda_1 + (y_{t-1}s_{1t})^{'}\varphi_1 + u_{1t}$$

By OLS regression, take t from 2 to 45 (correspond to year 1946 to year 1999), we get the following model:

$$y_t = \text{-0.5518+ 0.0595t+ 0.0834}y_{t-1}\text{-0.006t}y_{t-1}$$

With sum of residue squares is 11.584, the Nonlinear Dickey-Fuller F test statistic value for this specification is:

$$F^* = (\hat{\psi} - \psi)^{'} R^{'} \{s^2 R(\sum x_t x_t^{'})^{-1} R^{'}\}^{-1} R(\hat{\psi} - \psi)/2 = 9.3687$$

We compare it with 8.8355 in Table 5, since 9.3687>8.8355, the null hypothesis of a unit root is rejected at the 5% level.

Then we consider fitting the data to a STAR model:

$$y_t = \pi_{10} + \pi_{11}y_{t-1} + (\pi_{20} + \pi_{21}y_{t-1})F(t;\gamma,c) + \mu_t$$

Then by NLS regression, we get the following model, and t is from 2 to 45 (correspond to year 1946 to year 1999)

$$y_t = 2.3243 + 0.3797y_{t-1} + (2.5870 + 0.6153y_{t-1})[\frac{1}{(1+\exp\{-0.3048(t-20.6567)\})} - \frac{1}{2}]$$

We can see that in this case, the smooth transition function is

$F(t;\gamma,c) = [\frac{1}{(1+\exp\{-0.3048(t-20.6567)\})} - \frac{1}{2}]$, the graph of the smooth transition

[3] We choose 20% critical value as the chow test against a single break is not powerful test against smooth structural change. The figure has show the unemployment rate had already show a up trend at the early 1960s. Therefore our result is correspond to the visual impression. Joakim Skalin and Timo Teräsvirta (2002)

function is as follows:

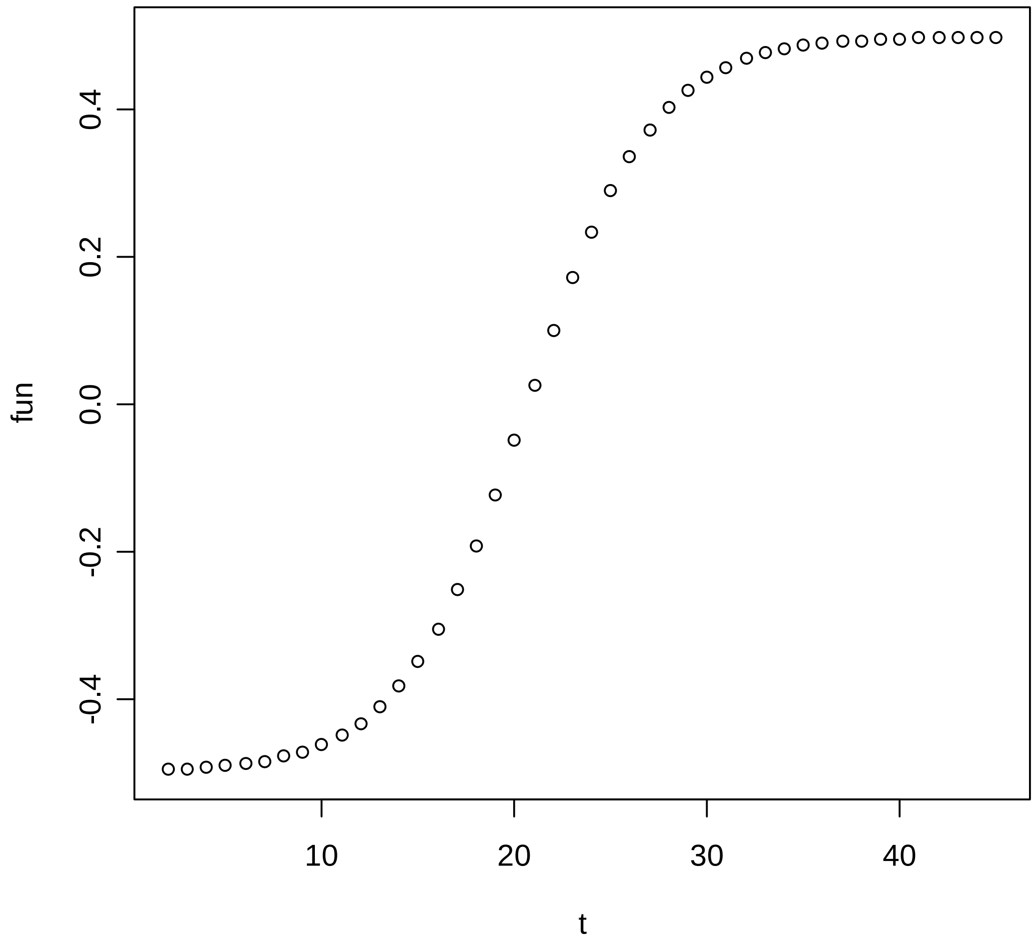

Figure 5: The estimated smooth transition function in the STAR model.

From figure 5 we can see that the transition function is increasing smoothly over time, and it means that the unemployment rate increasing over the observation period, from one extreme regime to the other.

Moreover, the estimation of c is 20.6567, and it proved what we observe from figure 4 that the data break occurs around year 1975. The economical explanation for this break may has relation with the OPEC energy price from 1973, and in 1975 the oil price raise to 10% , which bring a huge economical shock to the economical field including the job market.

The sum of residual squares for STAR model is 9.535904, which is the lowest of the three models. Thus the STAR model fit the asymmetric character of the unemployment best.

VII. Concluding remarks

In this paper we study a nonlinear Dickey-Fuller type of tests in STAR models allowing nonlinear change in level and dynamics. The proposed NDF F test is a joint test for testing the null hypothesis of a linear model with a unit root. The asymptotic distribution of the NDF F test statistic is derived by the Function Central Limit Theorem, and empirical distributions of finite samples for the F tests are obtained in Tables 1-4 by Monte Carlo experiment. We compare the distribution of Nonlinear Dickey-Fuller F test with that of the standard F test and it is seen that the former one is heavier skewed to the left.

The NDF F testing procedure for linearity and unit root is demonstrated by an empirical example of France unemployment rate data during the period of 1955-1999. We carry out the chow test as well as the Nonlinear Dickey-Fuller F test. Results from the empirical example show that we might accept the unit root hypothesis if an illness linearity assumption is used in the data, and if the data is naturally in the nonlinear case we reject the unit root hypothesis. Finally we employ a LSTAR(1) model to fit the data, and the analyzed results from the estimated LSTAR (1) implies that: there is a regime-switching type of nonlinearity evidence during the period of 1955-1999 in France unemployment rate data; and this is corresponded with a common suggestion that the STAR model can fit the asymmetric character of the unemployment data very well.

The F statistic critical tables and the estimation of STAR model are produced in R, and the detailed R program is available from Yushu Li.

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Appendix A: Proofs of Theorem

To prove *Theorem2*, we use Lemma A1 given below:

Lemma A1. If $(u_t)_{t=1}^{\infty}$ satisfy Assumption1, and $\xi_t = \xi_{t-1} + u_t$ with $P(\xi_0 = 0) = 1$, then as T→∞

$$T^{-(p+\frac{q}{2}+1)}\sum_{t=1}^{T} t^p \xi_{t-1}^q \xrightarrow{d} \lambda^q \int_0^1 r^p W(r)^q dr$$

$$T^{-(v+\frac{1}{2})}\sum_{t=1}^{T} t^v u_t \xrightarrow{d} \sigma_u W(1) - v\sigma_u \int_0^1 r^{v-1} W(r) dr$$

$$T^{-(p+1)}\sum_{t=1}^{T} t^p \xi_{t-1} u_t \xrightarrow{d} \frac{\lambda\sigma_u\left(W(1)^2 - p\int_0^1 r^{p-1}W(r)^2 dr - \frac{1}{p+1}\right)}{2}$$

Proof of *Lemma A1* please refer to He and Sandberg(2006)

Proof of *Theorem 2*

From OLS, we have

$$\hat{\psi}_m - \psi_m = \left[\sum_{t=1}^{T} x_{mt} x_{mt}^{'}\right]^{-1}\left[\sum_{t=1}^{T} x_{mt} u_{mt}\right]$$

$$\Upsilon_m(\hat{\psi}_m - \psi_m) = \Upsilon_m \left[\sum_{t=1}^{T} x_{mt} x_{mt}^{'}\right]^{-1} \Upsilon_m \Upsilon_m^{-1} \left[\sum_{t=1}^{T} x_{mt} u_{mt}\right]$$

$$= \left\{\Upsilon_m^{-1}\left[\sum_{t=1}^{T} x_{mt} x_{mt}^{'}\right]\Upsilon_m^{-1}\right\}^{-1}\left\{\Upsilon_m^{-1}\left[\sum_{t=1}^{T} x_{mt} u_{mt}\right]\right\}$$

As

$$\Upsilon_m^{-1}\left[\sum_{t=1}^{T} x_{mt} x_{mt}^{'}\right]\Upsilon_m^{-1} = \begin{bmatrix} \left[T^{-(i+j-1)} * \sum_{t=1}^{T} t^{i+j-2}\right]_{(m+1)*(m+1)} & \left[T^{-(i+j-\frac{1}{2})}\sum_{t=1}^{T} t^{i+j-2} y_{t-1}\right]_{(m+1)*(m+1)} \\ \left[T^{-(i+j-\frac{1}{2})}\sum_{t=1}^{T} t^{i+j-2} y_{t-1}\right]_{(m+1)*(m+1)} & \left[T^{-(i+j)}\sum_{t=1}^{T} t^{i+j-2} y_{t-1}^2\right]_{(m+1)*(m+1)} \end{bmatrix}$$

$$= \begin{bmatrix} \left[\alpha_{ij}\right]_{(m+1)*(m+1)} & \left[\beta_{ij}\right]_{(m+1)*(m+1)} \\ \left[\beta_{ij}\right]_{(m+1)*(m+1)} & \left[\delta_{ij}\right]_{(m+1)*(m+1)} \end{bmatrix}$$

$$\beta_{ij} = T^{-(i+j-\frac{1}{2})} \sum_{t=1}^{T} t^{i+j-2} y_{t-1} = T^{-(i+j-2+\frac{1}{2}+1)} \sum_{t=1}^{T} t^{i+j-2} y_{t-1} \xrightarrow{d} \sigma_u \int_0^1 r^{i+j-2} W(r) dr = \sigma_u b_{ij}$$

(From *Lemma A1* where $p = i + j - 2$, $q = 1$, $\lambda = \sigma_u$)

$$\delta_{ij} = T^{-(i+j)} \sum_{t=1}^{T} t^{i+j-2} y_{t-1}^2 = T^{-(i+j-2+\frac{}{2}+1)} \sum_{t=1}^{T} t^{i+j-2} y_{t-1}{}^2 \xrightarrow{d} \sigma_u{}^2 \int_0^1 r^{i+j-2} W(r)^2 dr = \sigma_u{}^2 c_{ij}$$

(From *Lemma A1* where $p = i + j - 2$, $q = 2$, $\lambda = \sigma_u$)

From above equations, we can prove that $\Upsilon_m^{-1} \left[\sum_{t=1}^{T} x_{mt} x_{mt}^{'} \right] \Upsilon_m^{-1} \xrightarrow{d} \Psi_m$

$$\Upsilon_m^{-1} \left[\sum_{t=1}^{T} x_{mt} u_{mt} \right] = \begin{bmatrix} \left[T^{-(i-\frac{1}{2})} \sum_{t=1}^{T} t^{i-1} u_t \right]_{(m+1)*1} \\ \left[T^{-i} \sum_{t=1}^{T} t^{i-1} y_{t-1} u_t \right]_{(m+1)*1} \end{bmatrix} = \begin{bmatrix} [\eta_i]_{(m+1)*1} \\ [\theta_i]_{(m+1)*1} \end{bmatrix}$$

$$\eta_i = T^{-(i-\frac{1}{2})} \sum_{t=1}^{T} t^{i-1} u_t = T^{-(i-1+\frac{1}{2})} \sum_{t=1}^{T} t^{i-1} u_t \xrightarrow{d} \sigma_u W(1) - \sigma_u (i-1) \int_0^1 r^{i-2} W(r) dr = \sigma_u d_i$$

(From *Lemma A1* where $\nu = i - 1$)

$$\theta_i = T^{-i} \sum_{t=1}^{T} t^{i-1} y_{t-1} u_t = T^{-(i-1+1)} \sum_{t=1}^{T} t^{i-1} y_{t-1} u_t \xrightarrow{d} \frac{\sigma_u{}^2 \left(W(1)^2 - (i-1) \int_0^1 r^{i-2} W(r)^2 dr - \frac{1}{i} \right)}{2} = \sigma_u{}^2 e_i$$

(From *Lemma A1* where $p = i - 1, \lambda = \sigma_u$)

From above equations, we can prove that

$$\Upsilon_m^{-1} \left[\sum_{t=1}^{T} x_{mt} u_{mt} \right] \xrightarrow{d} \Pi_m$$

From above, we prove that

$$\Upsilon_m (\hat{\psi}_m - \psi_m) = \left\{ \Upsilon_m^{-1} \left[\sum_{t=1}^{T} x_{mt} x_{mt}^{'} \right] \Upsilon_m^{-1} \right\}^{-1} \left\{ \Upsilon_m^{-1} \left[\sum_{t=1}^{T} x_{mt} u_{mt} \right] \right\} \xrightarrow{d} \Psi_m^{-1} \Pi_m$$

It is also easy to show that $s_m^2 \xrightarrow{p} \sigma_{u_m}{}^2$, then we get

$$s_m^2 \Upsilon_m (\sum x_{mt} x_{mt}^{'})^{-1} \Upsilon_m \xrightarrow{L} \sigma_u{}^2 \Psi_m^{-1}$$

Then *Theorem 2* is proved

As Theorem 1, we can combine the character of partitioned matrices and it is easy to get proved

Appendix B:

For the R code of the Monte Carlo experiment and the model fitting, please contact Susu4101@hotmail.com.