Testing Nonlinearity in Unemployment in OECD Countries: Using Panel Logistic Smooth Transition Autoregressive (PLSTAR) model

Essay in Statistics
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August,2007

Abstract

This paper aims to find empirical evidence of nonlinearity in unemployment rates in OECD countries. The model involved is the first-order panel smooth transition autoregressive (PSTAR(1)) model which allows nonlinear dynamic structure for each individual. The main-body test is a joint one with null hypothesis of unit root under linearity against a nonlinear stationary model. The empirical example is illustrated by applying the joint test to the unemployment rates of 18 OECD countries from 1956 to 1998, and our finding supports that the unemployment rates of OECD countries display a regime-switching type of nonlinearity and rejects hysteresis hypothesis after shock during the period .

Keywords: PLSTAR, nonlinearity, unit root, hysteresis, asymmetry

1 Introduction

Two hypotheses of unemployment rates have been discussed frequently when researchers consider researching the features of unemployment rates: hysteresis and asymmetry. Hysteresis hypothesis implies shocks bring permanent influences on unemployment. It simply indicates the non-stationary process; whereas its alternative hypothesis, that is the natural rate of unemployment, refers to a situation that the unemployment rate tends to revert to long-run equilibrium level after a shock. Such existence of hysteresis was suggested in Blanchard and Summers [1987] and Mitchell [1993]. Camarero et al. [2006] point out that the distinction between the two hypotheses is no so clear-cut. There is a stage regarded as persistence which implies a slow speed of adjustment towards the equilibrium level in the long-run. In this sense the unemployment rate is a mean-reverting process, and persistence can be regarded as a special case of the natural rate of unemployment. And another special situation is that full persistence is taken as hysteresis, see in Skalin and Teräsvirta [2002]. A popular way of testing these hypotheses is unit root test. The hysteresis hypothesis indicates the unemployment rate has a unit root (the $I(1)$ process), while the natural rate hypothesis would be in line with an $I(0)$ process, the stationary one. Persistence, then according to Camarero et al. [2006], is $I(0)$ around m shifting means. Such characteristic would be properly estimated in a nonlinear model.

Asymmetry on the other hand, is not necessarily related to hysteresis or multiple equilibria. It is, as Rothman [1998] point out that when modeling business cycling, the unemployment rate increases quickly in recessions but declines relatively slowly during expansions, which can be regarded as nonlinear phenomenon.

Linear time series models do not well represented these features, thus in this paper, we try to find evidence for proper nonlinear models. Researchers have managed to employ various nonlinear time series models, parametric and nonparametric models, for modelling unemployment rates. In recent years, among above models, those allowing for state-dependent or regime-switching behavior have been most popular. As in Van Dijk et al. [2002], the Smooth Transition Autoregressive (STAR) model is one of them. This paper provides the tests based on PLSTAR model, a special case of STAR model.

The purpose of this paper is to analyze a set of panel data—unemployment rates in Organization for Economic Co-operation and Development (OECD) countries, and to find empirical evidence of nonlinearity. Our data comes from Camarero et al. [2006]. The sample contains 19 OECD members with annual data from 1956 to 1998. Figure 1 shows the magnitude of unemployment rate in each country over time. From the graphic, it is most likely that the series of Belgium is persistence, while the one of France appear to

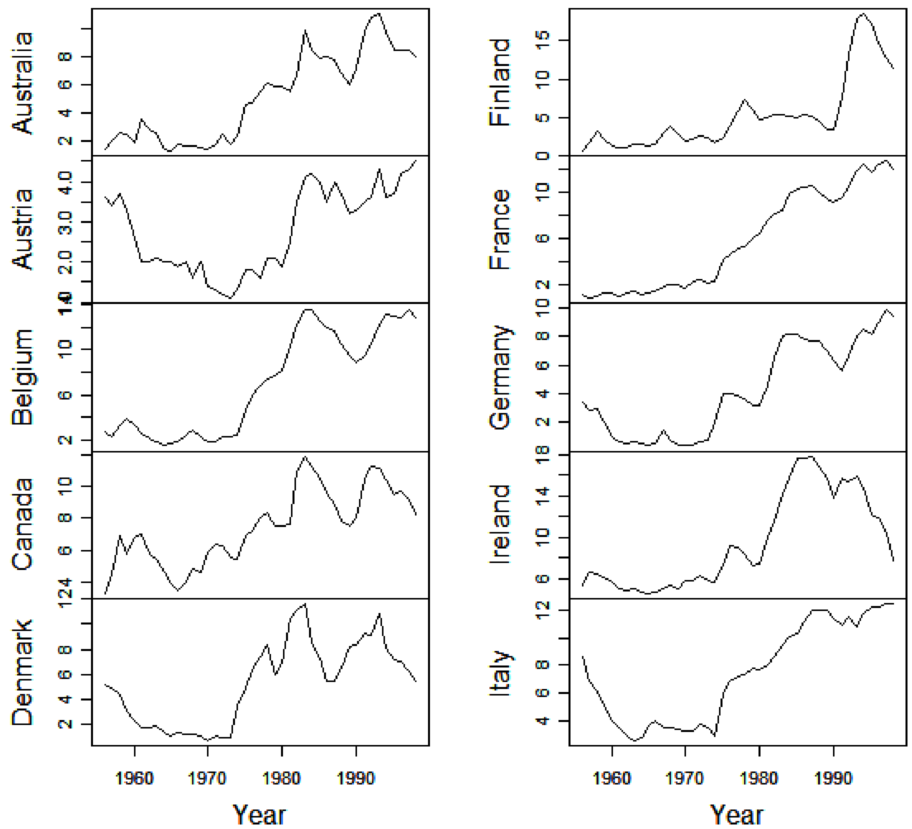

Figure 1: Unemployment rate in OECD countries

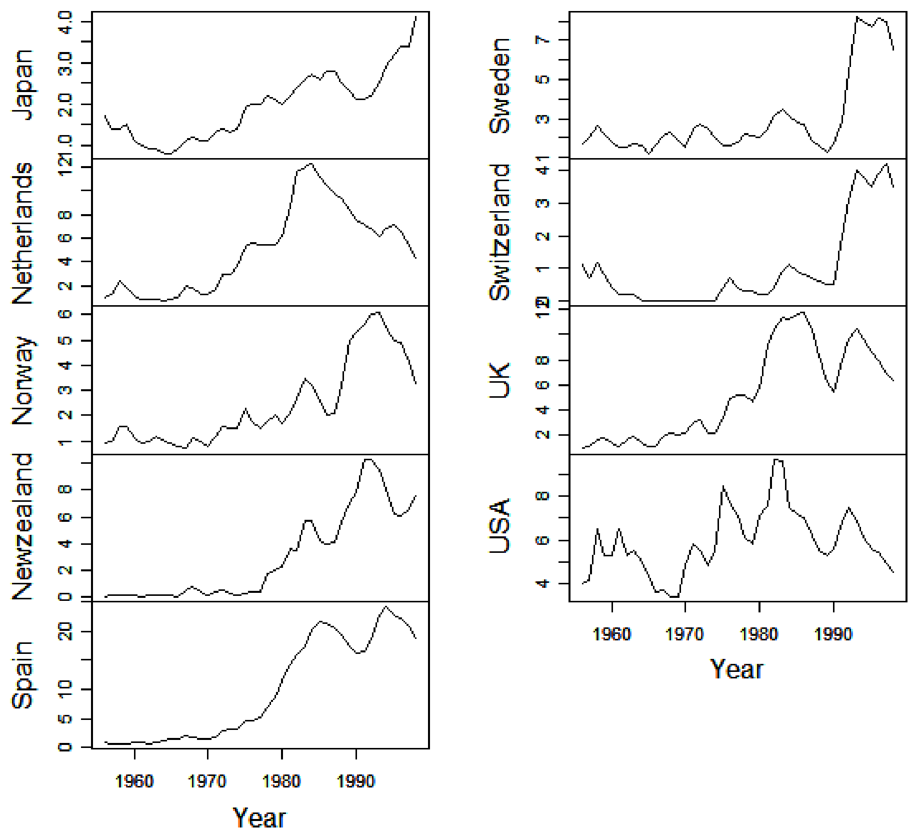

Figure 1: Unemployment rate in OECD countries (continued)

be hysteresis. Besides, asymmetric can be found in many countries. The characteristics of the panel data remains unknown, which will be discussed in the latter part.

Using the panel data is based on the following consideration: First, from Figure 1, it is evident that the unemployment rates of the sample countries display somewhat similar dynamic behavior, e.g. trend and regime switching. Studying such dynamic behavior by a panel nonlinear model would help ascertain the common features and the causal forces. Second, testing unit root on one hand and finding nonlinearity on the other using the same data, could be applicable, see in Skalin and Teräsvirta [2002]. Besides, many of the classical unit root tests in univariate settings suffer from low power against near unit root alternatives, as stated in He and Sandberg [2005], a remedy is to consider a set of panel data. The test method we use comes from He and Sandberg [2006], which is a joint test for unit root and linearity.

The rest of the paper is organized as follows. In Section 2 we introduce the PLSTAR model. In Section 3 the test's method along with its hypothesis and statistics are presented. Then raw data would be applied to the tests. Section 4 executes the test procedure, first find the impropriety for simply linear $AR(1)$ model, and then take the joint test for unit root and nonlinearity. Conclusions are given in Section 5.

2 The model

The first-order panel smooth transition autoregressive (PSTAR(1)) model for a panel time series y_{it}, which represent observations of n individuals over T time period, is given by

$$y_{it} = \pi_{i,10}+\pi_{11}y_{i,t-1}+(\pi_{i,20}+\pi_{21}y_{i,t-1})F(s_t;\gamma,c)+u_{it}, i = 1,\ldots,n, t = 1,\ldots,T. \quad (1)$$

where $F(s_t;\gamma,c)$ is the transition function that is continuous and bounded. Usually there are three choices for the transition variable s_t. (i) s_t could be the level or difference of lagged endogenous variable, for certain integer $p > 0$. That is to say y_{t-p} or Δy_{t-p}. These situations are discussed in Teräsvirta [1994] and Skalin and Teräsvirta [2002]. Or it could be a function of lagged endogenous variables. (ii) s_t equals to an exogenous variable. (iii) In Lin and Teräsvirta [1994], s_t is defined as time trend t.

There are also different choices for the function of $F(t;\gamma,c)$. A popular one is

$$F(s_t;\gamma,c) = (1+\exp\{-\gamma(s_t-c)\})^{-1} - \frac{1}{2}, \gamma > 0. \quad (2)$$

Model (1) with (2) is the PLSTAR(1) model. From the above equation, we can see that $F(s_t;\gamma,c)$ is bounded between $-\frac{1}{2}$ and $\frac{1}{2}$. It is monotone in s_t when γ and c are fixed. γ is a slope parameter. It is obvious that the smoothness in regime-switching is determined by the smoothness of changing value in transition function. And the latter depends on γ. So γ indicates the rapidness of transition. In the special case when $\gamma = 0$, the model will reduce to linear AR model.

In this paper, we will consider $s_t = t$ in the transition function. Therefore, the model is redefined as

$$y_{it} = \pi_{i,10}+\pi_{11}y_{i,t-1}+(\pi_{i,20}+\pi_{21}y_{i,t-1})F(t;\gamma,c)+u_{it}, i = 1,\ldots,n, t = 1,\ldots,T. \quad (3)$$

where $F(t;\gamma,c) = (1+\exp\{-\gamma(t-c)\})^{-1} - \frac{1}{2}$. Evidently, different values of t also influences the regime-switching process. It can be shown that when $\gamma \to \infty$, $\lim F(\gamma) = -0.5$ if $t \in [0,c]$ and $\lim F(\gamma) = 0.5$ if $t \in (c,T]$. $F(t;\gamma,c)$ will change from one extreme value to another at the very point of $t = c$.

Other forms of transition function and PLSTAR have been used in similar researches, like Skalin and Teräsvirta [2002] and Van Dijk et al. [2002].

3 Test

3.1 Auxiliary hypothesis

He and Sandberg [2006] carry out the unit root test with test for linearity embedded. Therefore, the null hypothesis based on model(1) is, $H_0 : \pi \in \mathbb{R}$ for all i, $\pi_{11} = 1$, and $\gamma = 0$ or $\pi_{i,20} = \pi_{21} = 0$, against a stable PLSTAR(1) model with $\gamma > 0$ alternatively. This is a joint test, $\pi_{11} = 1$ refers to a unit root and $\gamma = 0$ or $\pi_{i,20} = \pi_{21} = 0$ for linearity. It is rather complex to do the test with model(1) directly. Thus, it is necessary to give rise to an approximate function of $F(t; \gamma, c)$, which would bring much convenience in figuring out the conditions of stability and linearity, thus formulate the null hypothesis in an equivalent way. Luukkonen et al. [1998] suggest approximate the function with Taylor expansion around $\gamma = 0$. The first-order Taylor approximation of the transition function(2) is given by

$$F(t; \gamma, c) = \frac{t - c}{4} + R_n(t; \gamma, c) \tag{4}$$

where $R_n(t; \gamma, c)$ is the remainder term. So Model(1) can be rewritten as

$$\begin{aligned} y_{it} &= \pi_{i,10} - \frac{c}{4}\pi_{i,20} + (\pi_{11} - \frac{c}{4}\pi_{21})y_{i,t-1} + \frac{\pi_{i,20}}{4}t + \frac{\pi_{21}}{4}ty_{i,t-1} \\ &\quad + u_{it} + (\pi_{i,20} + \pi_{21}y_{i,t-1})R_n(t; \gamma, c) \\ &= \alpha_i + \rho y_{i,t-1} + \delta_i t + \phi t y_{i,t-1} + u^*_{it}, \quad i = 1, \ldots, n, t = 1, \ldots, T. \end{aligned} \tag{5}$$

where $\alpha_i = \pi_{i,10} - \frac{c}{4}\pi_{i,20}$, $\rho = \pi_{11} - \frac{c}{4}\pi_{21}$, $\delta_i = \frac{\pi_{i,20}}{4}$, $\phi = \frac{\pi_{21}}{4}$, and $u^*_{it} = u_{it} + (\pi_{i,20} + \pi_{21}y_{i,t-1})R_n(t; \gamma, c)$. When $\gamma = 0$, the remainder term $R_n(t; \gamma, c) = 0$, which means $u^*_{it} = u_{it}$, and the distribution properties of error is not affected by the approximation or equivalent null hypothesis. Besides, since $\gamma = 0$ simply means there would be no term containing t or product of t, it is evident that $\gamma = 0$ is equivalent to $\delta_i = 0$, $\phi = 0$, which also implies $\pi_{i,20} = \pi_{21} = 0$. Thereafter, $\rho = 1$ holds for every c only when $\pi_{11} = 1$ and $\pi_{21} = 0$ are both true at the same time, on other words, $\rho = 1$ itself could be the test statistics for both unit root and linearity. Comparing with the original null hypothesis, we get the auxiliary null hypothesis stated as below

$$H_0^{aux1} : \alpha_i \in \mathbb{R} \text{ for all } i,\ \rho = 1, \delta_i = 0, \phi = 0 \tag{6}$$

And in the latter part of the thesis, ρ would be used as the test statistics. Van Dijk et al. [2002] has similar format of auxiliary regression without the unit root part and do the test in a straightforward manner with univariate dataset.

3.2 Test statistics

In He and Sandberg [2006], the null hypothesis is a joint one. We have proved above, that $H_0 : \pi_{11} = 1$, and $\gamma = 0$ or $\pi_{i,20} = \pi_{21} = 0$ would lead to $\rho = 1$. Therefore, if reject the hypothesis of $\rho = 1$, we can also get rejection to $H_0 : \pi_{11} = 1$, and $\gamma = 0$ or $\pi_{i,20} = \pi_{21} = 0$. In this situation, the unit root test and test of linearity are taken in one integrate procedure.

Following He and Sandberg [2006], the LS estimator for $(\rho, \phi)'$ is

$$\begin{pmatrix} \hat{\rho} \\ \hat{\phi} \end{pmatrix} = \left[\sum_{i=1}^{n} \begin{pmatrix} y'_{i,t-1} \\ y'_{i,t-1} D_T \end{pmatrix} Q_T \begin{pmatrix} y_{i,t-1} & D_T y_{i,t-1} \end{pmatrix} \right]^{-1} \times \left[\sum_{i=1}^{n} \begin{pmatrix} y'_{i,t-1} \\ y'_{i,t-1} D_T \end{pmatrix} Q_T y_{it} \right] \quad (7)$$

Under H_0^{aux1}, $(\hat{\rho} - 1)$ after adjusted has following asymptotic distribution

$$\sqrt{n}(\hat{\rho} - 1 - B_1(T)) \xrightarrow{d} N(0, \sigma^2_{\hat{\rho}}(T)) \quad (8)$$

where $B_1(T) = p\lim_{n\to\infty}(\hat{\rho} - 1) = -\frac{1}{4}\frac{23T^2-21T-74}{(T^2-2)(T+2)}$ is the bias. And the variance is $\sigma^2_{\hat{\rho}}(T) = \frac{n_1(T)}{n_2(T)}$ with

$$\begin{aligned} n_1(T) = {} & 528038853T^{10} - 337761490T^9 - 295736530T^8 + 78337770T^7 \\ & - 4385262236T^6 - 538473642T^5 + 3583336934T^4 + 1400993790T^3 \\ & - 4271003921T^2 + 1598065812T + 4063557132 \end{aligned}$$

and $n^2(T) = 709632(T^2 - 2)^4(T + 2)^3(T - 2)$.

4 Empirical result

4.1 Test procedure

Rather than going to PLSTAR(1) directly, it is necessary to make some exploration of the dataset first. We follow a "specific to general" strategy to carry out the test. The empirical test is started with a simple model. If evidence of inadequacy is found in the present model, then the next step goes to a more complicate one which contains more regressors or has different format. In Teräsvirta [1994], similar steps are put forward. Our test procedure is specified as follows

1. Fit the panel data to a panel AR(1) model under investigation using White test for heteroscedasticity.
2. Specify individual country with linear AR(1) model and apply Chow test to find out whether there exist structural break.
3. Test the auxiliary null hypothesis against the stable nonlinear PLSTAR(1) model.

4.2 Panel AR(1) model

Consider the dataset in this model

$$y_{it} = \alpha_i + \varphi y_{i,t-1} + \nu_{it}, \;\; i = 1, \ldots, n, t = 1, \ldots, T \tag{9}$$

Using inference provided by Harris and Tzavalis [1999], the least squares (LS) estimator of φ is

$$\hat{\varphi} = [\sum_{i=1}^{n} y'_{i,\,t-1} Q_T y_{i,\,t-1}]^{-1} [\sum_{i=1}^{n} y'_{i,\,t-1} Q_T y_{it}] \tag{10}$$

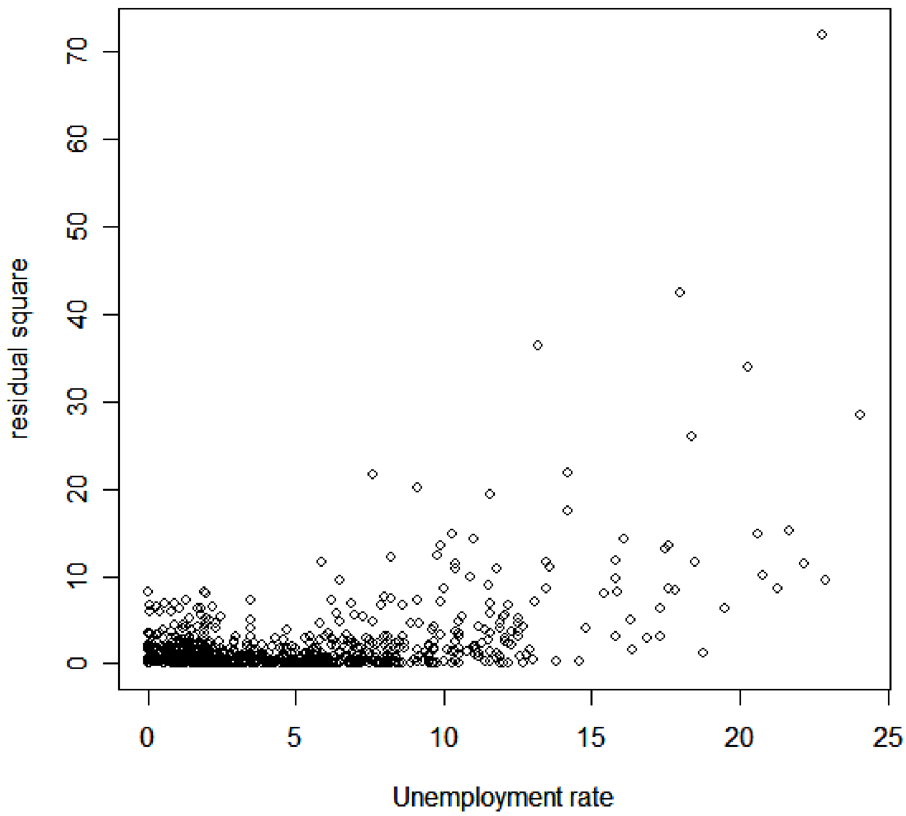

Figure 2: Residuals of each countries against time

where $Q_T = (I_T - (1/T)e_T e'_T)$ and e_T is the unit column vector of length T. The final result is $\hat{\varphi} = 0.763$. α_i is specified as $\frac{1}{T}\sum_{t=1}^{T} y_{it} - \frac{1}{T}\hat{\varphi}\sum_{t=1}^{T-1} y_{i,t-1}$. Then we get the estimated value of residual $\hat{\nu}_{it} = y_{it} - \hat{\alpha}_i - \hat{\varphi} y_{i,t-1}$. Figure 2 shows square of residuals against unemployment rate, from which we can see a clear phenomenon of heteroscedasticity in this simple model.

So for the null hypothesis of homoscedasticity, the White test is applied. Estimate the following regression model with respect to square of residuals

$$\nu_{it}^2 = \tau + \delta y_{i,t-1} + \varepsilon_{it} \tag{11}$$

Calculate R^2 of this model, which is the measurement of goodness-of-fit. Under the null hypothesis, $NR^2 \sim \chi^2$. In our case, $N = n * T = 19 * 42$ and the degree of freedom of χ^2 is 1 since there is one regressor. The result of White test statistic is $19 \times 42 \times 0.0950 = 75.81$. And the critic value of χ^2 with 1 degree of freedom under 5% significant level is 3.841, which is much smaller than the test statistic. Therefore, we can reject the null hypothesis of homoscedasticity.

4.3 Test for structural break in AR(1) model

Existence of structural break is evidence of nonlinearity. If nonlinearities are found in many single time series, taking the classic unit root tests in univariate framework would lead bias towards nonrejection, as is pointed out in Perron [1990]. In this part, we are supposed to find evidence of existing structural break in individual countries using Chow test.

The test procedure goes as follows: suppose we want to test if there is a structural break at time t, which split the whole sample into two subsamples. Estimate the model with whole sample and subsamples respectively, and calculate the Chow test statistic as $F = \frac{[SSR_R - (SSR_1 + SSR_2)]/K}{(SSR_1 + SSR_2)/(T-2K)}$, where SSR_R is the sum of squared residuals for the entire sample, SSR_1 refers to that of the first subsample, SSR_2 for the second one; K is numbers of regressors including constant term and T is the time dimension. F follows the F distribution with K and $T - K$ degrees of freedom.

A great oil crisis took place around 1974, it is most likely that a structural break occurs in many countries at that time. Actually, according to our test and similarly what have Camarero et al. [2006] done in their thesis, a large proportion of the sample countries have been proved to possess such kind of structural break. Here, we will take Germany and Japan for instance, an assumed structural break of 1974 would be estimated in the following model.

$$y_t = \alpha + \varphi y_{t-1} + \nu_t, \ t = 1, \ldots, T \tag{12}$$

Then the Chow test statistics and their $p-value$ are generated as follows:

	Test	*p-value*
Germany	6.8567	0.0029
Japan	2.5562	0.0909

At 10% level, we can draw the conclusion that Germany and Japan have structural break in 1974.

From the above two trials of simple model, it is evident that a linear $AR(1)$ model is not proper for neither the panel data nor the individual ones.

4.4 PLSTAR(1)

As mentioned above, PLSTAR(1) model contains a transition function. By reforming it with first order Taylor expansion, the model is changed into the auxiliary regression in equation(5). The null hypothesis start with a linear model containing a unit root. It would be tested against a stable PLSTAR(1) model with $\gamma > 0$. With the auxiliary regression, we would calculate the magnitude of $\hat{\rho}$ as the test statistics rather than work out γ directly, which avoid the problem of unidentification.

Applied equation(7) to the dataset, the result comes as

$$\begin{pmatrix} \hat{\rho} \\ \hat{\phi} \end{pmatrix} = \begin{pmatrix} 0.577 \\ 0.009 \end{pmatrix}$$

Therefore, $\sqrt{n}(\hat{\rho} - 1 - B_1(T))/\sqrt{\sigma^2_{\hat{\rho}}(T)} = -6.602375$, which also could reject both linearity and a unit root under 5% significant level. Rejecting $\rho = 1$ is equivalent as rejecting $\gamma = 0$, which not only indicates the panel model is nonlinear but also suggests regime-switching of unemployment rate. The unemployment rates would smoothly transit between different equilibria. It supports the assumption of Bianchi and Zoega [1998] that unemployment rate is globally stationary, nonlinear and locally nonstationary. Corresponding to the features of unemployment rate, the result reject hysteresis hypothesis after shock,therefore the panel dataset reveals clue of persistence.

Comparing to traditional panel unit root tests, this approach from He and Sandberg [2006] raises the power of test. Because among the panel framework, most individual time series show nonlinear phenomenon and structural changes. Rather than simply adding up nonlinearities like traditional panel unit root tests, it takes into consideration of the bias towards nonrejection, and allow to operate the test under rather loose conditions.

5 Conclusion

With the above results, we find out that linear models, *i.e.* $AR(1)$ is not proper for fitting the panel data. We also manage to show that there is structural break in the series. Finally, based on PLSTAR model, the hypothesis of linearity and hysteresis are rejected. Shocks may effect the unemployment rate, yet would lead it to a new level of natural rate in the long run.

As mentioned before, we do not exclude other choices of the transition function and the model, like employing other possibilities of s_t or taking different lagged values, or take higher order for the Taylor expansion. Here comes one problem: how to choose a better model. Power of test, results of estimation and forecast, and test of remaining nonlinearity would all be taken into account. In the next step, we will mainly focus on these aspects.

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