D-level Essay in Statistics

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Test for both unit root and linearity within the panel STAR framework: An application to exchange rates in European Union panel

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Abstract

In this paper, we introduce a dynamic Panel Logistic Smooth Transition Autoregressive (PLSTAR) model, within which a test to detect both unit root and linearity against the non-linear but globally stationary process can be carried out. We present the inconsistent LSDV estimator in the auxiliary version of PLSTAR model. When our assumptions hold, the biases and the asymptotic distributions of the estimators are derived under the null hypothesis. At last, we illustrate our model and theorems which are used to carry out the test by the empirical application of foreign exchange rates in EU panel, providing some evidence of integral non-linear but globally stationary feature.

Key Words: Panel; Smooth Transition Autoregressive model (STAR); LSDV estimator; Central limit theorem; Non-linear but globally stationary process; Exchange rates

1. Introduction

Nonstationarity and nonlinearity are among the most important characteristics of economic time series data. Recently, there are increasing time series econometrical literatures focused on the interplay of these two characteristics since the classical unit root tests for instance Dickey-Fuller (DF) test, are lack of power if the data presents some non-linear feature. These literatures separate into two branches: the first analyze unit root in the context of nonlinear models like threshold autoregressive model (TAR) and smooth transition autoregressive model (STAR), see Cancer and Hansen (2001) and Kapetanios *et al*. (2003); the second analyze linearity when the unit root is known, for example in Kilic (2004), they use LM-type linearity tests in the presence of a unit root.

However, not much focus has been put on this topic in the dynamic panel framework, except for He and Sandberg (2006). In their paper, they made inference for unit roots in the panel smooth transition autoregressive model (panel STAR). Although their alternative hypothesis was panel STAR model, their test concentrated on the unit root statistic alone, implicitly disregarding the linearity test statistic. To this end, we extend their test to get the linearity statistic, so that we can test the null of both unit root and linearity.

Plenty of empirical analysis have shown that real exchange rates have a nonlinear adjustment process toward purchasing power parity (PPP), for example Michael *et al*. (1997) and Kapetanios *et al.* (2003). However, these empirical analysis undertaken a group of countries individually, but not the panel case. Although panel data model is quite complicated to analyze, it has considerable significance especially in the nowadays highly integrated organizational economy, like European Union (EU) and Organization for Economic Co-operation and Development (OECD) etc. Since our test is constructed in the dynamic panel framework, we will investigate exchange rates in the EU panel in order to check if exchange rates still possess the nonlinear but stationary feature for the whole organization.

The current paper is organized as follow: Section 2 provides the theoretical testing procedure, containing model, hypothesis, estimation, bias and asymptotic distribution these five parts, as well as the assumption and theorems; Section 3 presents the empirical application and gives detailed explanation; Section 4 draws concluding remarks for the whole paper; additionally simple proofs of theorems are presented in Appendix.

2. Testing the null of unit root and linearity against the alternative of a non-linear but globally stationary panel STAR process

2.1 Model

According to Escribano and Jorda´ (2001), the Smooth Transition Autoregressive (STAR) model is defined as follow:

$$y_t = \pi' \cdot X_t + F(z_{t-d}, r, c)\Theta' X_t + v_t \quad t = 1, 2, \cdots, T \tag{1}$$

where y_t is a scalar; $X_t = (1, y_{t-1}, \cdots, y_{t-p})' = (1, \tilde{X}_t')'$; $\pi' = (\pi_0, \pi_1, \cdots, \pi_p) = (\pi_0, \tilde{\pi}')$; $\Theta' = (\Theta_0, \Theta_1, \cdots, \Theta_p) = (\Theta_0, \tilde{\Theta}')$; v_t is a martingale difference sequence with constant variance. It is common practice to assume that the process in (1) is stationary and ergodic. We have two choices of smooth transition function $F(z_{t-d}, r, c)$, one is exponential form[1] and the other is logistic form:

$$F(z_{t-d}, r, c) = [\{1 + \exp(-r(z_{t-d} - c))\}^{-1} - \frac{1}{2}] \tag{2}$$

z_{t-d} is usually chosen to be y_{t-d} ($1 \le d \le p$), although it could be any other predetermined or exogenous variable.

Extend the STAR model in (1) to panel framework, and only consider the autoregressive model of order one, we will get the panel STAR(1) model,

$$y_{it} = \pi_{i,10} + \pi_{11} y_{i,t-1} + (\pi_{i,20} + \pi_{21} y_{i,t-1}) F(z_{t-d}, r, c) + v_{it}, \; i = 1, \dots, n, \; t = 1, \dots, T \tag{3}$$

In the above equation, the subscript i denotes cross-section dimension and subscript t denotes time-series dimension. This model includes both linear and non-linear heterogeneous fixed effects ($\pi_{i,10}; \pi_{i,20}$), and both linear and non-linear homogeneous autoregressive coefficients ($\pi_{11}; \pi_{21}$). Furthermore, we consider the case $z_{t-d} = t$ in the transition function defined in (2). Then the logistic transition function has a form:

$$F(t, r, c) = [\{1 + \exp(-r(t - c))\}^{-1} - \frac{1}{2}] \tag{4}$$

where we assume the slope parameter $r > 0$, which indicates $F(t)$ is increasing in t, and $c \in (0, T)$ is the location parameter. From these restrictions we know the logistic function in (4) is bounded between $(-1/2, 1/2)$.

After plugging the logistic transition function in (4) into the panel STAR(1) model in (3), we will obtain the Panel Logistic Smooth Transition Autoregressive of order one model (PLSTAR(1) for short),

[1] The exponential smooth transition function is usually chosen as $F(z_{t-d}, r, c) = [1 - \exp\{-r(z_{t-d} - c)^2\}]$.
For the selection rule of two forms of smooth transition function, see Escribano and Jorda´ (2001) for a further discussion.

$$y_{it} = \pi_{i,10} + \pi_{11} y_{i,t-1} + (\pi_{i,20} + \pi_{21} y_{i,t-1})[\frac{1}{1+\exp(-r(t-c))} - \frac{1}{2}] + v_{it}, \quad (5)$$

This panel logistic STAR model has a distinctive feature of non-linear but globally stationary.

2.2 Hypothesis

In order to carry out a test contains both unit root and linearity in the full model described in (5), we will divide our null hypothesis into two parts, one is to test the unit root with the null hypothesis of $\pi_{11} = 1$, and the other is to test linearity with the null hypothesis of either $r = 0$ or $\pi_{i,20} = \pi_{21} = 0$.

However, it will lead to identification problem either adopt the hypothesis of $r = 0$ or $\pi_{i,20} = \pi_{21} = 0$ for linearity. A proper solution proposed by Luukkonen, Saikkonen and Terasvirta (1988) is applying Taylor approximation to the smooth transition function. Here we only consider the first order Taylor expansion of transition function $F(t,r,c)$ around $r = 0$:

$$F(r) = F(r)\big|_{r=0} + \frac{\partial F(r)}{\partial r}\big|_{r=0} \cdot r + \Delta = \frac{1}{4} r(t-c) + \Delta \quad (6)$$

where Δ is the remainder. This equation is introduced into model (3), after merging terms and re-parameterizing, we obtain an auxiliary version of PLSTAR(1) in (5),

$$y_{it} = \alpha_i + \rho y_{i,t-1} + \delta_i t + \phi t y_{i,t-1} + v_{i,t}^{\ *} \quad (7)$$

where $v_{i,t}^{\ *}$ is adjusted to be the same with $v_{i,t}$ in (3). Note that the auxiliary model has not change the structure of the original full model since it preserves the heterogeneous parameters (α_i and δ_i), and the homogeneous parameters (ρ and ϕ).

Therefore, we can base on the auxiliary model in (7) to re-construct our null hypothesis for both unit root and linearity:

$$H_0^{aux} : \alpha_i \in R \text{ for all i;} \quad \rho = 1; \delta_i = 0; \phi = 0 \quad (8)$$

In this auxiliary version of null hypothesis, $\rho = 1$ test unit root, and $\delta_i = 0; \phi = 0$ test linearity. Nevertheless, even if $\delta_i s$ do not equal to 0, the null model will be random walk with drift, which can also be recognized as linear model. Another concern is that $\delta_i s$ are not easy to be estimated in our model, see Section 2.3 LSDV Estimation. Therefore, we focus on ϕ for the linearity test.

In all, this is a test of the null—a panel of linear unit root process, against the alternative—a non-linear but globally stationary panel STAR process.

2.3 LSDV Estimation

According to the Monte Carlo simulation results from Judson and Owen (1999), although the bias of Least Square Dummy Variable (LSDV) estimator is sizeable, the bias corrected LSDV estimator is the best choice for the dynamic panel data model. Therefore, we will provide LSDV estimation for the homogeneous parameters in our auxiliary model in this part.

If set $u_{i,t} = u_i + v_{i,t} = \alpha_i + \delta_i t + v_{i,t}$, we consider the auxiliary model in (7) as an one-way error component model[2] for the disturbance $u_{i,t}$, where $u_i = \alpha_i + \delta_i t$ denote the unobservable individual specific effect and $v_{i,t}$ denotes the remainder disturbance. Furthermore, regarding α_i and δ_i as fixed parameters which have correlations with the covariates, a fixed effect model[3] is utilized, which has the particular advantage of removing the individual-specific heterogeneity.

In our auxiliary model, LSDV estimation for homogeneous parameters is illuminated with the following steps.

Firstly, rewrite the auxiliary model in (7) into vector form,

$$y = X\beta + u \qquad u = Zr + v \tag{9}$$

where $y_{NT\times 1} = (y_{11}, y_{12}, \cdots, y_{1T}, \cdots, y_{N1}, \cdots, y_{NT})'$; $X_{NT\times 2} = (y_{-1}^{'}, ty_{-1}^{'})$; $\beta = (\rho, \phi)'$;
$y_{-1} = (y_{10}, y_{11}, \cdots, y_{1,T-1}, \cdots, y_{N0}, \cdots, y_{N,T-1})$
$ty_{-1} = (1y_{10}, 2y_{11}, \cdots, Ty_{1,T-1}, \cdots, 1y_{N0}, \cdots, Ty_{N,T-1})$;
$r = (\alpha_1, \delta_1, \alpha_2, \delta_2, \cdots, \alpha_N, \delta_N)'$; $v_{NT\times 1} = (v_{11,} v_{12}, \cdots, v_{1T}, \cdots v_{N1,} v_{N2}, \cdots, v_{NT})'$;
Note that Z is a matrix of individual dummies, $Z_{NT\times 2N} = I_N \otimes J$, where $J_{T\times 2} = (\iota_T, \tau_T)$, ι_T is a vector of 1 of dimension T and $\tau_T = (1, 2, \cdots, T)'$.

Secondly, obtain matrix P as the projection matrix of $Z: P = Z(Z'Z)^{-1}Z'$, and another matrix Q: $Q = I_{NT} - P$. Matrix P and Q have the following properties: (i) $PZ = Z$, from which can further derive $QZ = 0$;(ii) P and Q are both symmetric and idempotent matrices, i.e. $P' = P, P^2 = P, Q' = Q, Q^2 = Q$.

At last, multiply both sides of model (9) by Q and apply its properties, $Qy = QX\beta + QZr + Qv = QX\beta + Qv$. After setting $\tilde{y} = Qy, \tilde{X} = QX, \tilde{v} = Qv$, we can obtain the LSDV estimator of β,

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[2] In the panel data regression, there are two kinds of model when concerning the disturbance, one-way error component model and two-way error component model. If the disturbance only contains the unobservable individual effect and the remainder disturbance, it is a one-way error component model; if additionally contains the unobservable time effect, it is a two-way error component model. For more details see Baltagi (1995).

[3] Another choice is random effect model, in which u_i are assumed as random, has no correlation with covariates.

$$\hat{\beta} = \begin{pmatrix} \hat{\rho} \\ \hat{\phi} \end{pmatrix} = (\tilde{X}'\tilde{X})^{-1}(\tilde{X}'\tilde{y})$$
$$= [(QX)'(QX)]^{-1}[(QX)'(Qy)] = [X'Q'QX]^{-1}(X'Q'Qy) \qquad (10)$$
$$= (X'QX)^{-1}(X'Qy)$$

2.4 Bias

The LSDV estimator is inconsistent, may largely due to the fact that the error term u has correlation with the covariates X, which contains the lag of y. Although in Baltagi (1995) argued that when time T goes to infinity, LSDV estimator will be consistent.

In order to derive the bias of our LSDV estimator, we need to introduce some assumptions beforehand, which will be used along the paper.

In He and Sandberg (2006), they constructed two sets of assumptions, and the difference lies in the homogeneous individual errors in the first set and the heterogeneous assumption in the second one. In their paper, they also argued that the heterogeneous error assumption set will obtain the same results as the first assumption set does. As a matter of fact, one may find in practice that the homogeneous error assumption is too restricted to obtain, but the heterogeneous error is generally the case. Therefore, we adopt the second assumption set in our paper—heterogeneous error assumption and then formulate another version of their theorems in the following parts.

- *Assumption:*

(A1). $\{v_{it}\}$ is independent distributed with $E(v_{it}) = 0$ and $E(v_{it}^2) = \sigma_i^2 < \infty$ hold for all i and t. $\lim_{n\to\infty} n^{-1}\sum_{i=1}^{n}\sigma_i^2 = \sigma_v^2 < \infty$

(A2). $E\left|v_{it}^4\right|^{4+\delta} < \infty$ holds for $\delta > 0$, $\lim_{n\to\infty} n^{-1}\sum_{i=1}^{n} u_{4i} = u_4 < \infty$, where $u_{4i} = Ev_{it}^4$.

(A3). The individual effect $\alpha_i = 0$ for all i.

Assumption (A1) and (A2) indicate that v_{it} are independent across i with the same mean but heterogeneous variance, which enables us to derive the asymptotic distributions of our test statistics. Assumption (A3) requires the individual effects equal to 0, which is a quite restrictive assumption but necessary to simplify the problem.[4]

[4] For more details, see Appendix in He and Sandberg (2006).

With these assumptions, we can derive the bias of our LSDV estimator. In Theorem1 in He and Sandberg (2006), they have figured out the bias $B_1(T)$ of $(\hat{\rho}-1)$. Using the same method and extending it to the estimator $\hat{\phi}$, we will obtain a complete theorem for the bias.

- *Theorem 1:*

Consider the auxiliary model in (7), when *Assumption* holds and under the null hypothesis in (8), for fixed T>2 , the LSDV estimator satisfies:

$$\lim_{N\to\infty}\begin{pmatrix}\hat{\rho}-1\\ \hat{\phi}\end{pmatrix}=\begin{pmatrix}B_1(T)\\ B_2(T)\end{pmatrix}$$

where $B_1(T)=-\frac{1}{4}\frac{23T^2-21T-74}{(T^2-2)(T+2)}$, $B_2(T)=-\frac{7}{2(T^2-2)}$.

Proof: see Appendix.

Theorem 1 proves the LSDV estimator is inconsistent. An interesting finding is that the biases are both negative and only depending on time T. The bias $B_1(T)$ of $(\hat{\rho}-1)$ is $o(T^{-1})$, which enables $\lim_{N,T\to\infty}(\hat{\rho}-1)=0$ and the bias $B_2(T)$ of $\hat{\phi}$ is $o(T^{-2})$, which enables $\lim_{N,T\to\infty}\hat{\phi}=0$. This also suggests when $\mathrm{T}\to\infty$, LSDV is consistent.

2.5 Asymptotic Distribution

In this part, the asymptotic distributions for the bias corrected LSDV estimators under the null hypothesis are presented when the time dimension is fixed. We will find all kinds of the test statistics have the asymptotic standard normal distribution as the following theorems state.

- *Theorem 2:*

Under the null hypothesis H_0^{aux}, consider model (7) when *Assumption* holds, for fixed T>2, the asymptotic distributions of LSDV estimators of both $(\hat{\rho}-1)$ and $\hat{\phi}$, adjusted by their own bias, are given by:

$$\sqrt{n}(\hat{\rho}-1-B_1(T))\xrightarrow{d}N(0,\sigma_{\hat{\rho}}^2(T,\kappa_4))$$

$$\sqrt{n}(\hat{\phi} - B_2(T)) \xrightarrow{d} N(0, \sigma^2_{\hat{\phi}}(T, \kappa_4))$$

where $\kappa_4 = u_4 / \sigma_v^4$ and $\sigma^2_{\hat{\rho}}(T, \kappa_4)$, $\sigma^2_{\hat{\phi}}(T, \kappa_4)$ are defined in the Appendix.[5]

If the asymptotic standard normal distribution is requested in Theorem 2, the test statistics above will be transformed by dividing by their own standard deviation, i.e. $\frac{\sqrt{n}(\hat{\rho}-1)}{\sigma_{\hat{\rho}}(T,\kappa_4)} - \frac{\sqrt{n}B_1(T)}{\sigma_{\hat{\rho}}(T,\kappa_4)}$ for the unit root test statistic and $\frac{\sqrt{n}\hat{\phi}}{\sigma_{\hat{\phi}}(T,\kappa_4)} - \frac{\sqrt{n}B_2(T)}{\sigma_{\hat{\phi}}(T,\kappa_4)}$ for the linearity test statistic.

We note in Theorem 2, that the asymptotic variance $\sigma^2_{\hat{\rho}}(T, \kappa_4)$ and $\sigma^2_{\hat{\phi}}(T, \kappa_4)$ are both functions of time T and the nuisance parameter κ_4. Apparently, if we add one more assumption—(A4) normality distribution of the error term, the nuisance parameter will be eliminated, which leads to Theorem 3.

- *Theorem 3:*

Under null hypothesis H_0^{aux} , consider model (7) when *Assumption* and (A4) $\{v_{it}\}$ normally distributed hold, for fixed T>2, the asymptotic distribution of both $(\hat{\rho}-1)$ and $\hat{\phi}$, adjusted by their own bias, is given by:

$$\sqrt{n}(\hat{\rho} - 1 - B_1(T)) \xrightarrow{d} N(0, \sigma^2_{\hat{\rho}}(T))$$

$$\sqrt{n}(\hat{\phi} - B_2(T)) \xrightarrow{d} N(0, \sigma^2_{\hat{\phi}}(T))$$

where $\sigma^2_{\hat{\rho}}(T)$, $\sigma^2_{\hat{\phi}}(T)$ are defined in the Appendix.

Similar with the comments to Theorem 2, we will transform the test statistics in order to get asymptotic standard normal distribution, $\frac{\sqrt{n}(\hat{\rho}-1)}{\sigma_{\hat{\rho}}(T)} - \frac{\sqrt{n}B_1(T)}{\sigma_{\hat{\rho}}(T)}$ for the unit root test statistic and $\frac{\sqrt{n}\hat{\phi}}{\sigma_{\hat{\phi}}(T)} - \frac{\sqrt{n}B_2(T)}{\sigma_{\hat{\phi}}(T)}$ for the linearity test statistic.

In the above theorems, we have considered the case of time T fixed; however, there are quantities of examples when T goes to infinity in practice. Therefore, we proceed to consider the case of $T \to \infty$.

[5] It is because they are too complicated equations to present in the context.

- *Corollary 3:*

Under null hypothesis H_0^{aux} , consider model (7) when *Assumption* and (A4) $\{v_{it}\}$ normally distributed holds, also when $\mathrm{T} \to \infty$, the asymptotic distribution of both $\left(\hat{\rho}-1\right)$ and $\hat{\phi}$, adjusted by their own bias, is given by:

$$\sqrt{\frac{n}{C_1}}T(\hat{\rho}-1)+\frac{23}{4}\sqrt{\frac{n}{C_1}} \xrightarrow{d} N(0,1)$$

$$\sqrt{\frac{n}{C_2}}T^2\hat{\phi}+\frac{7}{2}\sqrt{\frac{n}{C_2}} \xrightarrow{d} N(0,1)$$

where $C_1 = \frac{52803853}{709632} \approx 74.41$ and $C_2 = (105)^2(\frac{392843}{13305600} - \frac{382943}{6652800} + \frac{11556935}{259459200}) \approx 181.98$

Simple proof of Corollary 3 is provided in the Appendix.

3. Empirical application: exchange rates in EU panel

From a theoretical point of view, as Michael *et al*. (1997) stated, "Equilibrium models of real exchange rate determination in the presence of transactions costs imply a nonlinear adjustment process toward purchasing power parity (PPP)." See also Kapetanios *et al*. (2003), they also argued that if any test fails to reject the unit root in real exchange rates, it may be due to the lack of power of the test, because such kind of result implies potentially unbounded gains from arbitrage in traded goods, which is contradicted with the theoretical prediction. In the empirical analysis in Harvey and Leybourne (2007), they showed the results for real exchange rates in 15 European countries, several countries are nonlinear: Austria, Belgium, Denmark etc., but several are linear with unit root: Finland, Greece, Italy etc..

However, all the above literatures consider individual countries, while our empirical analysis is going to provide a different perception, which is to inspect the fact that exchange rates also have nonlinear but globally stationary as an integral feature for a panel of countries. Therefore, we will proceed to apply our test for both unit root and linearity to the exchange rates in the EU panel.

3.1 Data

As we know, European Union has 27 memberships until now (2007), in which 12 East European countries are newly entered since 2004. As a result, these countries are not included in our analysis. We collected data for the left of European Union member countries, which are listed in Table 1, as well as their own currencies.

From Table 1, we can see that although there are 15 countries, actually 14 individual currencies are included in our research (N=14). The foreign exchange rates are measured by the rate of their own currencies to US dollars, namely bilateral exchange rate. As to the time dimension, we apply monthly data from 04/01/1981 to 12/01/2000 (T=237).[6]

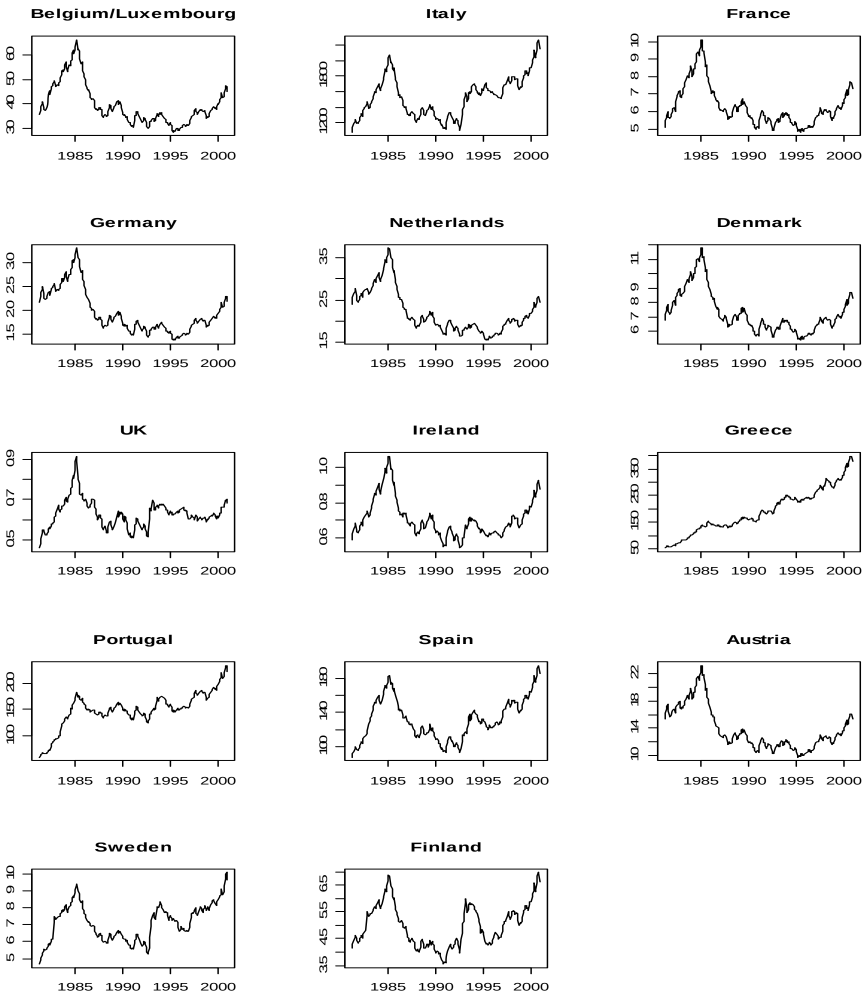
The time series plots are shown in Figure 1, from which we can say these countries almost share the similar behavior, and obviously not linear, only with an exception Greece which has an upward trending. Therefore, we have an intuition from the plots that our non-linear panel STAR model would be a good choice for these data.

[6] Data are provided by Economic Research, Federal Reserve Bank of St.Louis, with the following website: http://research.stlouisfed.org/fred2/categories/15.

Table 1. 15 European Union member countries and their own currencies

Country	Currency	Country	Currency
Belgian/Luxembourg	Belgian/Luxembourg Francs	Ireland	Irish Pound
Italy	Italian Lira	Greece	Greek Drachma
France	French Franc	Portugal	Portuguese Escudo
Germany	German Marks	Spain	Spanish Peseta
Netherlands	Dutch Guilder	Austria	Austrian Schilling
Denmark	Danish Kroner	Sweden	Swedish Kronor
UK	Pound sterling	Finland	Finnish Markkas

Figure 1. Time series plots of real bilateral exchange rate



3.2 Testing results

Although we reckon this panel has a non-linear feature from the time series plots, we need to test the linearity and unit root analytically to get further evidence.

At the first step, we need to figure out the LSDV estimates for the homogeneous parameters ρ and ϕ in our auxiliary panel STAR model in (7). According to Section 2.3 LSDV Estimation, the LSDV estimates and their own bias are summarized in Table 2. As to the unobservable individual heterogeneities α_i and δ_i, we have showed that they are removed in the fixed effect model. Therefore, we are not able to estimate them in our model.

We can see from the value of the LSDV estimators, around 0.9645 of $\hat{\rho}$, which is close to 1 in our null hypothesis, and around 0.0002 of $\hat{\phi}$, which is very close to 0 in the null. One may guess after comparing the value of estimates and of the null hypothesis, it will be hard to reject the null hypothesis of unit root and linearity. Nevertheless, we will show it is not the case when the theorems are applied.

Since we did not estimate the whole model, it is not feasible to check the model residuals. As a result we simply assume the residuals independently follow normal distributions with zero means and heterogeneous variances, which enables us to apply *Theorem 3* (T is fixed) and *Corollary 3* (T sufficiently large) state in the Section 2.5.

Table 3 displays the testing results from both *Theorem 3* and *Corollary 3.* We can see that values of the test statistics and p-values in both *Theorem 3* and *Corollary 3* do not have distinguished difference. Therefore, similar interpretation of the results can be deduced as follow: when T(=237) is considered as fixed, unit root can be rejected at the significance level of 12% and linearity is highly significant; when T is consider as sufficiently large, unit root can be rejected at the significance level of 13% and linearity still can be rejected easily.

Our result of rejecting linearity in favor of a nonlinear panel model but not easily rejecting the unit root hypothesis is not implausible for the following reasons: First, it is a well known fact that some nonlinear models do generate “unit-root-like” behavior, i.e. the unit root hypothesis is not able to be rejected at the conventional significance level, for example the testing results in Harvey and Leybourne (2007) exhibit 10% level for individual countries; Second, Åsbrink (1997) argued in his book that “STAR models often have locally explosive roots which are needed to model locally nonstationary behavior in the data”, therefore although our result shows the nonlinear but globally stationary STAR process is more acceptable against the linear unit root process, it may probably exits some locally unit roots in the series.

We can conclude from all the above discussion that for the whole EU panel, exchange rates also have a non-linear speed of convergence to equilibrium, which is coincide with the theoretical assumption from the macro-economic and finance.

However, we must keep in mind that this non-linear convergence speed is rather slow because of the locally explosive behaviors.

In all, the whole empirical analysis not only leads us to understand the testing procedure proposed in our paper, but also confirms the integral feature of the real exchange rates for the whole panel.

Table 2. LSDV Estimation of PLSTAR model

	$\widehat{\rho}$	$\widehat{\phi}$
LSDV estimates	0.9645383959	0.0002012179
Bias	-0.02406566	-6.28434e-05

Table 3 Results from *Theorem 3* and *Corollary 3*

	Results from Theorem 3 (T fixed)	Results from Corollary 3 ($T \to \infty$)
Test statistic for unit root:	-1.177983	-1.135988
p-value	0.1194017	0.1279808
Test statistic for linearity	4.129107	4.079246
p-value	1.820875e-05***	2.259100e-05***

NOTE: Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

4. Concluding remarks

This paper mainly develops a test of the null of a linear unit root process against the alternative of a non-linear but globally stationary panel logistic STAR process. As to the details, we introduce the panel logistic smooth transition autoregressive of order one model and its auxiliary version; clearly derive the Least Square Dummy Variable estimator as well as its bias for homogeneous parameters in the auxiliary model, which is accepted as the best estimation choice for the dynamic panel model; propose the theorems of asymptotic distributions ($N \to \infty$) of the test statistics for both unit root and linearity under the null hypothesis, when the assumption holds. Furthermore, we provide an empirical application to the exchange rates in EU panel, the result of which gives sustainable evidence to the theoretical assumption in macroeconomic and finance.

However, there are some problems left in the paper deserve more consideration and discussion, which should be analyzed in further work. First, it is not precise to apply the lower order of Taylor expansion to the smooth transition function, which in our paper we only adopt the first order in Section 2.2.[7] Second, since it is hard to define the autoregressive order in panel framework, we can not be sure of the assumed order one autoregressive model in the empirical analysis.[8] Third, it is better to account for the cross-correlation in the residuals than the independent case as we assumed in *Assumption*. As argued in Camarero *et al*. (2006): "This is of special relevance for highly integrated area, as the EU, where it is unrealistic to maintain the independence hypothesis when formulating panel unit root test." Fourth, the poolability for the restricted parameters and heterogeneity for the unrestricted parameters should be under investigation as stated in the model specification. Finally, we can further estimate the individual effects in the auxiliary model, and then check the assumptions regarding residuals and fixed effects before applying the theorems.

[7] However, if apply some higher order of Taylor expansion, we will add quadratic or higher order terms into the full model, which will bring estimation problem in return.

[8] As Fok *et al.*(2005) stated in their paper: "if the disaggregate data are generated by an AR(1) model with different parameters, the aggregate series will not follow a simple AR(1)", therefore, it will be useless to examine the individual autoregressive order and then base on its result to postulate the panel has the same order of autoregressive.

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Appendix

Proof of Theorem 1:

Under the null hypothesis H_0^{aux} in (8), LSDV estimator in (10) has another form,

$$\begin{pmatrix} \hat{\rho}-1 \\ \hat{\phi} \end{pmatrix} = [\sum_{i=1}^{N} \begin{pmatrix} y'_{i,-1} \\ (ty_{i,-1})' \end{pmatrix} Q_T (y_{i,-1},, ty_{i,-1})]^{-1} [\sum_{i=1}^{N} \begin{pmatrix} y'_{i,-1} \\ (ty_{i,-1})' \end{pmatrix} Q_T v_i]$$

where $y_{i,-1} = (y_{i,0}, y_{i,1}, \cdots, y_{i,T-1})'$; $ty_{i,-1} = (y_{i,0}, 2y_{i,1}, \cdots, Ty_{i,T-1})'$;

$Q_T = I_T - P_T = I_T - J(J'J)^{-1}J'$; J is defined in (9); $v_i = (v_{i,1}, v_{i,2}, \cdots, v_{iT})'$.

Follows from the proof of Theorem 1 in He and Sandberg (2006) that by the Slutzky theorem,

$$\lim_{N\to 0} \begin{pmatrix} \hat{\rho}-1 \\ \hat{\phi} \end{pmatrix} = [\lim_{N\to 0} \frac{1}{N} \sum_{i=1}^{N} \begin{pmatrix} y'_{i,-1} \\ (ty_{i,-1})' \end{pmatrix} Q_T (y_{i,-1},, ty_{i,-1})]^{-1}$$

$$\times [\lim_{N\to 0} \frac{1}{N} \sum_{i=1}^{N} \begin{pmatrix} y'_{i,-1} \\ (ty_{i,-1})' \end{pmatrix} Q_T v_i]$$

with

$$\lim_{N\to 0} \frac{1}{N} \sum_{i=1}^{N} \begin{pmatrix} y'_{i,-1} \\ (ty_{i,-1})' \end{pmatrix} Q_T (y_{i,-1}, ty_{i,-1}) = \sigma_v^2 \begin{pmatrix} \frac{1}{15}(T^2-4) & \frac{1}{30}(T^2-4)(T+1) \\ \frac{1}{30}(T^2-4)(T+1) & \frac{1}{420}(T^2-4)(11T^2+14T-1) \end{pmatrix}$$

$$\lim_{N\to 0} \frac{1}{N} \sum_{i=1}^{N} \begin{pmatrix} y'_{i,-1} \\ (ty_{i,-1})' \end{pmatrix} Q_T v_i = -\sigma_v^2 \begin{pmatrix} \frac{1}{2}(T-2) \\ \frac{1}{60}(17T+19)(T-2) \end{pmatrix}.$$

Theorem 2:

Following from the proof of Theorem 2 in He and Sandberg (2006) and extending it to the asymptotic variance of $\hat{\phi}$, then we can get

$$\sigma_{\hat{\rho}}^2(T, \kappa_4) = q_{11}^2 EG_{i,1t}^2 + 2q_{11}q_{12} EG_{i,1t}G_{i,2t} + q_{12}^2 EG_{i,2t}^2 ,$$

$$\sigma_{\hat{\phi}}^2(T, \kappa_4) = q_{12}^2 EG_{i,1t}^2 + 2q_{12}q_{22} EG_{i,1t}G_{i,2t} + q_{22}^2 EG_{i,2t}^2$$

where $q_{11} = \dfrac{\mathrm{EW_{i,2t}}}{\mathrm{EW_{i,1t}EW_{i,2t}\text{-}(EW_{i,3t})^2}} = \dfrac{15(11T^2+14T-1)}{4(T^2-4)(T^2-2)\sigma_v^2}$,

$$q_{12} = -\frac{\mathrm{EW_{i,3t}}}{\mathrm{EW_{i,1t}EW_{i,2t}\text{-}(EW_{i,3t})^2}} = -\frac{105(T+1)}{2(T^2-4)(T^2-2)\sigma_v^2},$$

$$q_{22} = \frac{\mathrm{EW_{i,1t}}}{\mathrm{EW_{i,1t}EW_{i,2t}\text{-}(EW_{i,3t})^2}} = \frac{105}{(T^2-4)(T^2-2)\sigma_v^2},$$

and

$$EG_{i,1t}^2 = (\mathrm{T}^2\text{-}2)^{-2}(\mathrm{T}^2\text{-}1)^{-1}(\mathrm{T}+2)^{-1}\mathrm{T}^{-1}(\mathrm{T}\text{-}2)[\frac{\mathrm{u}_4}{110880}(\mathrm{T}\text{-}3)(6543\mathrm{T}^7\text{-}13599\mathrm{T}^6\text{-}36602\mathrm{T}^5+170566\mathrm{T}^4$$
$$\text{-}25173\mathrm{T}^3\text{-}707131\mathrm{T}^2\text{-}29344\mathrm{T}+589812)+\frac{\sigma_v^4}{3326400}(392843\mathrm{T}^9\text{-}1987498\mathrm{T}^8+3181711\mathrm{T}^7+5981726\mathrm{T}^6$$
$$\text{-}34139267\mathrm{T}^5+37748138\mathrm{T}^4+74407229\mathrm{T}^3\text{-}182674806\mathrm{T}^2\text{-}70246356\mathrm{T}+159249240)]$$

$$EG_{i,1t}G_{i,2t} = (\mathrm{T}^2\text{-}2)^{-2}(\mathrm{T}^2\text{-}1)^{-1}(\mathrm{T}+2)^{-1}\mathrm{T}^{-1}(\mathrm{T}\text{-}2)[\frac{\mathrm{u}_4}{221760}(\mathrm{T}^2\text{-}9)(3647\mathrm{T}^7+1535\mathrm{T}^6+3726\mathrm{T}^5\text{-}10450\mathrm{T}^4$$
$$\text{-}179845\mathrm{T}^3\text{-}226905\mathrm{T}^2+219080\mathrm{T}+348188)+\frac{\sigma_v^4}{6652800}(382943\mathrm{T}^{10}\text{-}1478555\mathrm{T}^9\text{-}1151547\mathrm{T}^8+11553117\mathrm{T}^7$$
$$\text{-}1552641\mathrm{T}^6\text{-}12256869\mathrm{T}^5+21309007\mathrm{T}^4\text{-}120784417\mathrm{T}^3\text{-}220115082\mathrm{T}^2+156524484\mathrm{T}+282032280)]$$

$$EG_{i,2t}^2 = (\mathrm{T}^2\text{-}2)^{-2}(\mathrm{T}^2\text{-}1)^{-1}(\mathrm{T}+2)^{-1}\mathrm{T}^{-1}(\mathrm{T}\text{-}2)[\frac{\mathrm{u}_4}{2882880}(\mathrm{T}\text{-}3)(55489\mathrm{T}^9+125238\mathrm{T}^8+298513\mathrm{T}^7$$
$$+975550\mathrm{T}^6\text{-}4756977\mathrm{T}^5\text{-}18293686\mathrm{T}^4\text{-}2616677\mathrm{T}^3+34056714\mathrm{T}^2+19476036\mathrm{T}\text{-}6731784)$$
$$+\frac{\sigma_v^4}{259459200}(115569935T^{11}\text{-}40537573\mathrm{T}^{10}\text{-}55120525\mathrm{T}^9+286295547\mathrm{T}^8+63649665\mathrm{T}^7+923928009\mathrm{T}^6$$
$$+843787177\mathrm{T}^5\text{-}12232771967\mathrm{T}^4\text{-}10684267460\mathrm{T}^3+21359553264\mathrm{T}^2+17174416608\mathrm{T}\text{-}5452745040)]$$

Theorem 3:

We can easily derive from Assumption (A4) normal distribution of error that $\kappa_4 = u_4/\sigma_v^4 = E(u_{it}^4)/\sigma_v^4 = 3\sigma_v^4/\sigma_v^4 = 3$. Therefore, the asymptotic variances are no longer the function of κ_4, but only depending on time T.

$$\sigma_{\hat\rho}^2(T) = \frac{1}{709632(\mathrm{T}^2\ \text{-}\ 2)^4(\mathrm{T}+2)^3\ (\mathrm{T}\text{-}2)}\times$$
$$[52803853\mathrm{T}^{10}\text{-}\ 33761490\mathrm{T}^9\text{-}\ 295736530\mathrm{T}^8+78337770\mathrm{T}^7\text{-}\ 438526236\mathrm{T}^6\text{-}\ 538473642\mathrm{T}^5$$
$$+3583336934\mathrm{T}^4+1400993790\mathrm{T}^3\text{-}\ 42710039]$$

Note $\sigma_{\hat\rho}^2(T)$ is $o(T^{-2})$ with a constant $C_1 = \frac{52803853}{709632} \approx 74.41$.

$$\sigma_{\hat{\phi}}^2(T)=\frac{(105)^2(\mathrm{T}\text{-}2)}{(T^2-4)^2(T^2-2)^4(\mathrm{T}^2\text{-}1)(\mathrm{T}+2)\mathrm{T}}\times\{\frac{1}{4}(T+1)^2\times[\frac{3}{110880}(\mathrm{T}\text{-}3)(6543\mathrm{T}^7\text{-}13599\mathrm{T}^6$$

$$\text{-}36602\mathrm{T}^5+170566\mathrm{T}^4\text{-}25173\mathrm{T}^3\text{-}707131\mathrm{T}^2\text{-}29344\mathrm{T}+589812)+\frac{1}{3326400}(392843\mathrm{T}^9\text{-}1987498\mathrm{T}^8$$

$$+3181711\mathrm{T}^7+5981726\mathrm{T}^6\text{-}34139267\mathrm{T}^5+37748138\mathrm{T}^4+74407229\mathrm{T}^3\text{-}182674806\mathrm{T}^2\text{-}70246356\mathrm{T}+159249240)]$$

$$-(T+1)[\frac{3}{221760}(\mathrm{T}^2\text{-}9)(3647\mathrm{T}^7+1535\mathrm{T}^6+3726\mathrm{T}^5\text{-}10450\mathrm{T}^4\text{-}179845\mathrm{T}^3\text{-}226905\mathrm{T}^2+219080\mathrm{T}+348188)$$

$$+\frac{1}{6652800}(382943\mathrm{T}^{10}\text{-}1478555\mathrm{T}^9\text{-}1151547\mathrm{T}^8+11553117\mathrm{T}^7\text{-}1552641\mathrm{T}^6\text{-}12256869\mathrm{T}^5+21309007\mathrm{T}^4$$

$$\text{-}120784417\mathrm{T}^3\text{-}220115082\mathrm{T}^2+156524484\mathrm{T}+282032280)]+[\frac{3}{2882880}(\mathrm{T}\text{-}3)(55489\mathrm{T}^9+125238\mathrm{T}^8$$

$$+298513\mathrm{T}^7+975550\mathrm{T}^6\text{-}4756977\mathrm{T}^5\text{-}18293686\mathrm{T}^4\text{-}2616677\mathrm{T}^3+34056714\mathrm{T}^2+19476036\mathrm{T}\text{-}6731784)$$

$$+\frac{1}{259459200}(115569935T^{11}\text{-}40537573\mathrm{T}^{10}\text{-}551205525\mathrm{T}^9+2862955547\mathrm{T}^8+636496665\mathrm{T}^7+923928009\mathrm{T}^6$$

$$+843787177\mathrm{T}^5\text{-}12232771967\mathrm{T}^4\text{-}10684267460\mathrm{T}^3+21359553264\mathrm{T}^2+17174416608\mathrm{T}\text{-}5452745040)]]\}$$

Note $\sigma_{\hat{\phi}}^2(T)$ is $o(T^{-4})$ with a constant $C_2=(105)^2(\frac{392843}{13305600}-\frac{382943}{6652800}+\frac{11556935}{259459200})\approx 181.9778$.

Corollary 3:

Since the asymptotic variances of $\sigma_{\hat{\rho}}^2(T)$ is $\Delta(T^{-2})$, and $\sigma_{\hat{\phi}}^2(T)$ is $\Delta(T^{-4})$, if $\mathrm{T}\to\infty$, the standard form of test statistics in Theorem 3 is

$$\frac{\sqrt{n}(\hat{\rho}-1)}{\sigma_{\hat{\rho}}(T)}-\frac{\sqrt{n}B_1(T)}{\sigma_{\hat{\rho}}(T)}\xrightarrow{T\to\infty}\frac{\sqrt{n}(\hat{\rho}-1)}{\sqrt{\mathrm{C}_1\mathrm{T}^{-2}}}-\frac{\sqrt{n}B_1(T)}{\sqrt{\mathrm{C}_1\mathrm{T}^{-2}}}\xrightarrow{T\to\infty}\sqrt{\frac{n}{\mathrm{C}_1}}\mathrm{T}(\hat{\rho}-1)-\sqrt{\frac{n}{\mathrm{C}_1}}\mathrm{T}B_1(T)$$

$$\frac{\sqrt{n}\hat{\phi}}{\sigma_{\hat{\phi}}(T)}-\frac{\sqrt{n}B_2(T)}{\sigma_{\hat{\phi}}(T)}\xrightarrow{T\to\infty}\frac{\sqrt{n}\hat{\phi}}{\sqrt{\mathrm{C}_2T^{-4}}}-\frac{\sqrt{n}B_2(T)}{\sqrt{\mathrm{C}_2T^{-4}}}\xrightarrow{T\to\infty}\sqrt{\frac{n}{\mathrm{C}_2}}T^2\hat{\phi}-\sqrt{\frac{n}{\mathrm{C}_2}}T^2B_2(T)$$

Furthermore, $B_1(T)$ of $\left(\hat{\rho}-1\right)$ is $\Delta(T^{-1})$ and $B_2(T)$ of $\hat{\phi}$ is $\Delta(T^{-2})$, as a consequence, the standard bias adjustments in corollary 3 are simply constants.