Testing for linear cointegration against nonlinear cointegration:

Theory and application to Purchasing power parity

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Abstract

In this thesis, we study a smooth-transition type of nonlinear cointegration among a dynamic system. Base on the Logistic Smooth Transition Autoregressive (LSTAR) models, the definition of cointegration which is extended form Engle and Granger (1987)'s definition of linear cointegration is introduced. Then statistical test for linear cointegration against nonlinear cointegration is derived. The statistical properties of this test are investigated and the asymptotic distribution of statistics is obtained by Monte Carlo simulation. We apply this testing theory into an empirical example which concern dollar/lira real exchange rate. We find that there is nonlinearly cointegrating relation in this purchasing power parity (PPP) dynamic system and the PPP puzzle in Hamilton (1994) can be explained by nonlinear cointegration defined in this essay.

Key words: Nonlinear Cointegration, OLS F test, LSTAR model, Monte Carlo Experiment.

Content list

1. Introduction

Cointegration which is introduced in Engle and Granger (EG) (1987)'s articles is an econometric property of time series variables and it has been well applied in modeling macroeconomic time series in econometrics. The EG's definition of cointegration have been well developed in linear models for modeling relationship among economic variable and also have been well applied in empirical problems.

However, economic relationships often present in such nonlinear form as structural change and regime-switching. Thus, studies for extension of this definition of cointegration into nonlinear system are necessary. Recently, many researches which attempted to extend EG's cointegration into nonlinear form have been studied. The well known example of this approach is to manifests itself in the concept of threshold cointegration and its smooth versions studied by Balke and Fomby (1997) and to employ a similar model to investigate the purchasing power parity by Enders and Falk (1998).

In this essay, my goal is to test the null hypothesis of linear cointegration against nonlinear cointegration. Firstly, we raised a definition of nonlinear cointegration which is developed from the EG's definition of linear cointegration and propose a testing procedure to test that there exist linear cointegration against there exist nonlinear cointegration in the dynamic system. For a nonlinear form, we adopt one of the most popular nonlinear dynamic models, the smooth transition autoregressive (STAR) model, which nests a linear autoregressive model and contains a regime-switching structure. For a detailed discussion of STAR models, see Terasvirta (1994). Therefore, the cointegrating vector can be specified in a form of a smooth transition function if there is a regime-switching in the system. For testing procedure, we apply the method which is raised by He and Sandberg (2005a) to reduce the model into linear form and employ the auxiliary regression to set up testing procedure. And then, the table of critical F value will be produced by Monte Carlo simulation experiment.

At last, we will apply the definition and testing procedures into the empirical problem which concern the theory of purchasing power parity (PPP). From Hamilton (1994), we can see that the null hypothesis of no linear cointegration of EG's (1987) can not be rejected at 5% significant level when applying a residual-based testing procedure to the dollar/lira real exchange rate. In this essay, we suggest that the PPP puzzle could be solved by the nonlinear cointegration which is introduced in previous section.

This essay is organized as follow: Section 2 introduces the definition of this smooth transition type nonlinear cointegration; Section 3 presents the model; Section 4 introduces the procedure of testing for linear cointegration against nonlinear cointegration; Section 5 presents the result of asymptotic distribution properties by Monte Carlo simulation; Section 6 shows the studying results of an empirical example about PPP; Concluding remark will be illustrated at last section.

2. Definition of nonlinear cointegration

Engle and Granger (1987) have raised a definition of cointegration which contains a linear combination of nonstationary variables and describes a long-run equilibrium relation existing in a linear dynamic system. In this section, I will generalize this concept to nonlinear cointegration which based on a nonlinear stationary system.

Definition:

Let $y_t = (y_{1t}, ..., y_{nt})'$ be the n-dimensional random vector and for each

$i = 1, 2, ..., n$, $y_{it} \sim linear\ I(1)$, where the definition of $I(1)$ follows from the Engle and Granger (1987)'s definition.

Then y_t is said to be nonlinear cointegration if there exists a time-varying

vector $\alpha_t = (\alpha_{1t}, \alpha_{2t}, ..., \alpha_{nt})'$ such that $\alpha_t ' y_t \sim I(0)$.

Where, time-varying vector should satisfy following condition:

(1) The first item α_{1t} should be a nonzero-constant such that is can be normalized

   as $\alpha_t = (1, \alpha_{2t}, ..., \alpha_{nt})'$;

(2) α_{it}, $i = 2, ..., n$ are well defined function of a random variable S_t such that for each i, α_{it}

   has a logistic smooth transition form: $\alpha_{it} = \alpha_i + G_{it}$

(3) Where, $G_{it} = \left(1 + \exp\{-\gamma_i (S_{it} - c_i)\}\right)^{-1}$ α_i, γ_i and c_i are parameters, $\gamma_i \geq 0$

   The transition variables S_{it} are weakly stationary or deterministic variables.

Here, α_t is called nonlinear cointegrating vector.

3. The Model:

In this essay, we should consider the trinomial system since we will apply the result to the ppp data. The model form as follow:

$$\begin{cases} y_{1t} = \alpha_{2t} y_{2t} + \alpha_{3t} y_{3t} + \varepsilon_{1t} \\ \quad y_{2t} = y_{2t-1} + \varepsilon_{2t} \\ \quad y_{3t} = y_{3t-1} + \varepsilon_{3t} \end{cases} \quad \varepsilon_t = \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix} \sim N \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ & \sigma_2^2 & \sigma_{23} \\ & & \sigma_3^2 \end{pmatrix} \right\}$$

Where $\alpha_{it} = \alpha_i + G(\gamma_i, \Delta y_{it-d}, c_i)$, $i = 2, 3$

The transition function $G(\gamma_i, \Delta y_{it-d}, c_i)$ is defined as follows:

$$G\left(\gamma_i, \Delta y_{it-d}, c_i\right) = \left(1 + \exp\left\{-\gamma_i\left(\Delta y_{it-d} - c_i\right)\right\}\right)^{-1} - \frac{1}{2}, \quad i = 2,3$$

The transition function is called the logistic smooth transition function and the model is called nonlinear cointegration with logistic smooth transition function. Where, γ_i is a parameter which determines the speed of transition from one regime to another at time c and the speed of transition will be faster with the increasing of γ_i. And we choose Δy_{it-d} as transition variable.

About the delay parameter d of transition variable Δy_{it-d}, it should be larger than 1 at least since if d=0, then there will be specification problem (see Tsay (1989)). In application, the delay parameter d needs to be determined from the data. In this essay, we should apply the methods which have been discussed by Tsay. Tsay (1989) suggests that first select the order of autoregression p and then determine d by varying it and choosing the value minimizing the p value of his linearity test. In the testing procedure, we fixed the delay parameter at 1.

4. Testing procedure:

In this essay, my goal is to test the null hypothesis of a linear cointegration against nonlinear cointegration. The null hypotheses can be expressed as the following parameter restriction:

$$H_0 : \gamma_2 = 0, \ \gamma_3 = 0$$

From the model which was mentioned above, we can see that $\gamma_2 = 0, \ \gamma_3 = 0$ implies that the transitions function $G\left(\gamma_i, \Delta y_{i,t-1}, c_i\right) = 0$. That is the model will degenerate into linear model.

Then we will apply Taylor expansion of gamma around 0 in transition function $G\left(\gamma_i, \Delta y_{i,t-1}, c_i\right)$.

There are two merits of this method to apply Taylor expansion to get the auxiliary regression. First we don't need to estimate the model before the testing procedure. Secondly the computing technical will be simplified. However, we should keep in mind that the first-order expansion will lead to low power if the transition takes place only in the intercept (see Luukkonen et al.(1988) and He and Sandberg (2005a)).
The first and third-order Taylor expansion are as follows:

$$G_{1,i,t}\left(\gamma_i, \Delta y_{i,t-1}, c_i\right) = \frac{\gamma_i\left(\Delta y_{i,t-1} - c_i\right)}{4} + r_1\left(\gamma_i\right) \tag{4.1}$$

$$G_{3,i,t}\left(\gamma_i, \Delta y_{i,t-1}, c_i\right) = \frac{\gamma_i\left(\Delta y_{i,t-1} - c_i\right)}{4} + \frac{\gamma^3\left(\Delta y_{i,t-1} - c_i\right)^3}{48} + r_3\left(\gamma_i\right) \tag{4.2}$$

Put the above equations into the model which is under the alterative hypothesis, and then we can get the auxiliary regression as follow:

$$y_{1,t} = \left(y_{2,t} \cdot h_{2,t}\right)' \varphi_2 + \left(y_{3,t} \cdot h_{3,t}\right)' \varphi_3 + \mu_t^* \tag{4.3}$$

Where the parameters are defined as follows:

For order I, m=1:

$$h_{2,t} = \left(1, \Delta y_{2,t-1}\right)'; \quad h_{3,t} = \left(1, \Delta y_{3,t-1}\right)'; \quad \varphi_2 = \left(\varphi_{20}, \varphi_{21}\right)'; \quad \varphi_3 = \left(\varphi_{30}, \varphi_{31}\right)'.$$

For order III:

$$h_{2,t} = \left(1, \Delta y_{2,t-1}, \Delta y_{2,t-1}^2, \Delta y_{2,t-1}^3\right)'; h_{3,t} = \left(1, \Delta y_{3,t-1}, \Delta y_{3,t-1}^2, \Delta y_{3,t-1}^3\right)';$$

$$\varphi_2 = \left(\varphi_{20}, \varphi_{21}, \varphi_{22}, \varphi_{23}\right)'; \varphi_3 = \left(\varphi_{30}, \varphi_{31}, \varphi_{32}, \varphi_{33}\right)'.$$

Then the corresponding auxiliary null hypotheses are:

$$H_{0m}^* : \varphi_{mi} = 0, \ i > 0; \ m = 1, 3$$

5. Asymptotic Distribution by Monte Carlo Simulation

(1) We should define the data generating function. Under the null hypotheses, the model can be reduced as linear form, and then we can get the data generating function as follow:

$$\begin{cases} y_{1,t} = \alpha_2 y_{2,t} + \alpha_3 y_{3,t} + \mu_{1,t} \\ \quad y_{2,t} = y_{2,t-1} + \mu_{2,t} \\ \quad y_{3,t} = y_{3,t-1} + \mu_{3,t} \end{cases} \tag{5.1}$$

where $u_{it} \sim iid \ N(0,1) \ i = 1, 2, 3$

We can generate the data with desired sample size according to above function.

(2) Given the auxiliary regression, we employ OLS to estimate the parameter and compute the value of F statistic according to following formula:

$$b_T = \left(X'X\right)^{-1} X' y_{1,t} \quad \text{Where, } X = \left(h_{2,t}, h_{3,t}\right),$$

$$s^2 = \left(y_{1t} - X \cdot b_T\right)'\left(y_{1t} - X \cdot b_T\right)/\left(T - k\right),$$

$$F = \left(Rb_T - r\right)'\left[s^2 R\left(X'X\right)^{-1} R'\right]^{-1}\left(Rb - r\right)/m,$$

Where, R is a diagonal matrix and r is a vector which value is fixed according to the null hypotheses.

Then repeat step (1) and (2) 10000 times to simulate the asymptotic distribution of OLS F statistic. Let T=1,000,000 and replications are 10,000 to get asymptotic distribution for F statistic. The simulation results are reported in Table 1 and Table 2

Table 5.1

T	99.0%	97.5%	95.0%	90.0%	10.0%	5.0%	2.5%	1.0%
			Probability that F statistic is greater than entry					
25	0.134	0.228	0.336	0.524	4.496	5.765	7.154	8.878
50	0.147	0.233	0.339	0.524	4.184	5.239	6.174	7.502
100	0.146	0.240	0.352	0.519	4.004	4.884	5.791	6.991
250	0.140	0.233	0.354	0.531	3.986	4.883	5.806	6.914
500	0.148	0.243	0.368	0.537	3.919	4.689	5.505	6.534
∞	0.144	0.257	0.375	0.579	3.869	4.763	5.522	6.798

Critical values for F test, Order I

Table 5.2

T	99.0%	97.5%	95.0%	90.0%	10.0%	5.0%	2.5%	1.0%
			Probability that F statistic is greater than entry					
25	0.3198	0.4331	0.5512	0.7259	3.5186	4.3578	5.1298	6.3121
50	0.2836	0.3833	0.4855	0.6281	2.6709	3.1516	3.5840	4.3395
100	0.2898	0.3822	0.4772	0.6046	2.4570	2.8770	3.2947	3.7899
250	0.2802	0.3662	0.4525	0.5937	2.3051	2.6796	3.0474	3.4498
500	0.2623	0.3571	0.4484	0.5887	2.2805	2.6459	3.0131	3.4619
∞	0.2778	0.3868	0.4887	0.5951	2.2375	2.5803	2.8797	3.4817

Critical values for F test, Order III

Figure 5.1

Histogram of F_v
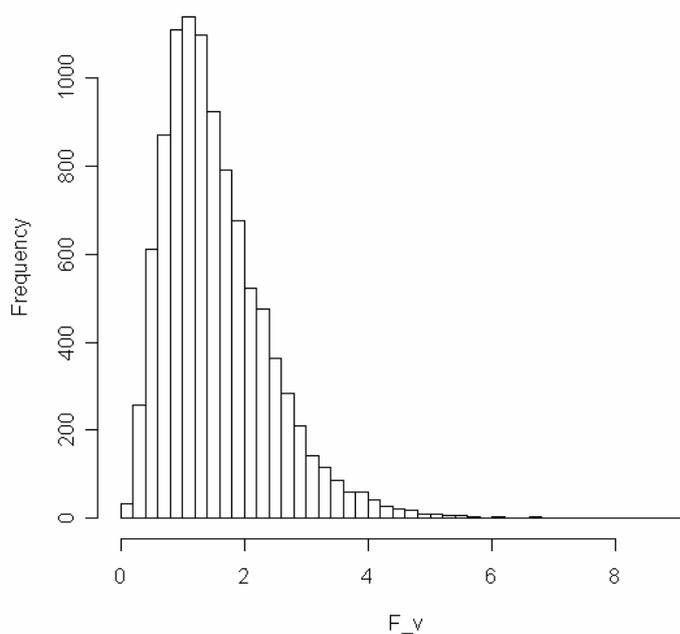
Simulation, time=10000, when T=50

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Figure 5.2
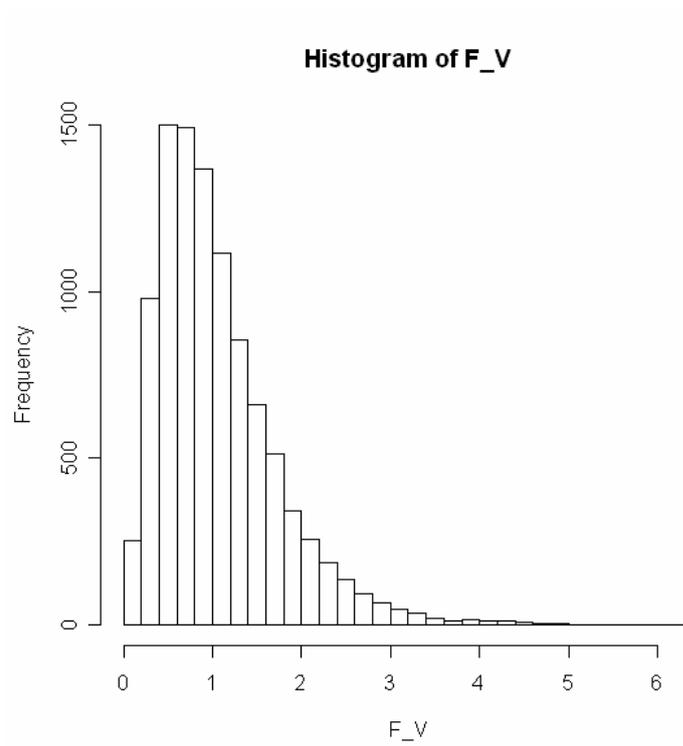
Histogram of F_V

Stander F distribution with df (6,42)

We compare the histograms of the OLS F test with standard F (6, 42) test in the above graphs. We can see that the standard F (6, 42) distribution turn out to be heavier skewed to left.

6. Empirical Example – Purchasing Power Parity

6.1 Theory of PPP

The purchasing power parity (PPP) theory which is a classical economic example of cointegration uses the long-term equilibrium exchange rate of two currencies to equalize their purchasing power. The basic idea of purchasing power parity is first introduced by Gustav Cassel in 1920. It is based on the law of one price: the theory that, in an ideally efficient market, identical goods should have only one price.

A purchasing power parity exchange rate equalizes the purchasing power of different currencies in their home countries for a given basket of goods. It is often used to compare the standards of living between countries, rather than a per-capita gross domestic product (GDP) comparison at market exchange rates. There is obvious difference between purchasing power parity exchange rate and market exchange rate, because the market exchange rate fluctuates frequently according to the need of the market. While many economists believe that purchasing power parity exchange rates are characterized by a long-run equilibrium.

In this essay, we will discuss the PPP theory base on the data set of dollar/lira real exchange rate. Given fundamental idea of the PPP theory, we know that: $Pt=Pt' \times St$ where Pt denotes the domestic price index Pt' is the foreign price index and St is the exchange rate between these two countries. Thus, we take logarithm s of both sides of $Pt=Pt' \times St$ and turn it into plus form: $pt=pt'+st$. It is to say that this dynamic data set should be modeled as cointegration model by cointegrating vector $(1,-1,-1)$. However, from the research in Hamilton's, it expresses that the theory of purchasing power parity is not satisfied in the example of United States and Italy. This is widespread disagreement of the purchasing power parity and this problem is called PPP puzzle. We will explain the PPP puzzle in more details in section 6.3. Then, in last two sections, we apply the testing procedure which is introduced in previous section to test there is linear cointegration against nonlinear cointegration in this dynamic system and estimate the cointegration model by NLS so that we can check the residuals of model whether satisfy the condition which we expected.

Another problem about the price level of certain country should be mentioned. It is nature to use price level to measure the purchasing power in each country. We can say that the purchasing power of certain country decrease if the price level growth. But in fact there is no uniform price level can be provided to measure the purchasing power among different countries. Thus it is necessary to compare the cost of baskets of goods and services consumed by people from different countries using a price index and the consumer price index organized monthly is the best choice to assess the purchasing power.

6.2 Data description

In this data set, P_t denotes a price index in United States (in dollar per good) P_t' is a price index in Italy (in lira per good) and S_t is the rate of exchange between these two countries.

For analysis objectives, the raw data are transformed at first as presented:

$$p_t = 100 \times \left[\log(P_t) - \log(P_{1973:1}) \right]$$

$$p'_t = 100 \times \left[\log(P'_t) - \log(P'_{1973:1}) \right]$$

$$s_t = 100 \times \left[\log(S_t) - \log(S_{1973:1}) \right]$$

Such transition is to make sure that above three time series have same starting values, zero at January 1973. Multiplying by 100 roughly represents the percentage difference between the value at current time point and the starting value at January 1973.

Figure 6.1 plots the monthly data of the price level p_t in United States, the price level p'_t in Italy

and the exchange rate s_t between dollar in United States and lira in Italy from January 1973 to

October 1989 after the transition.

Figure 6.1
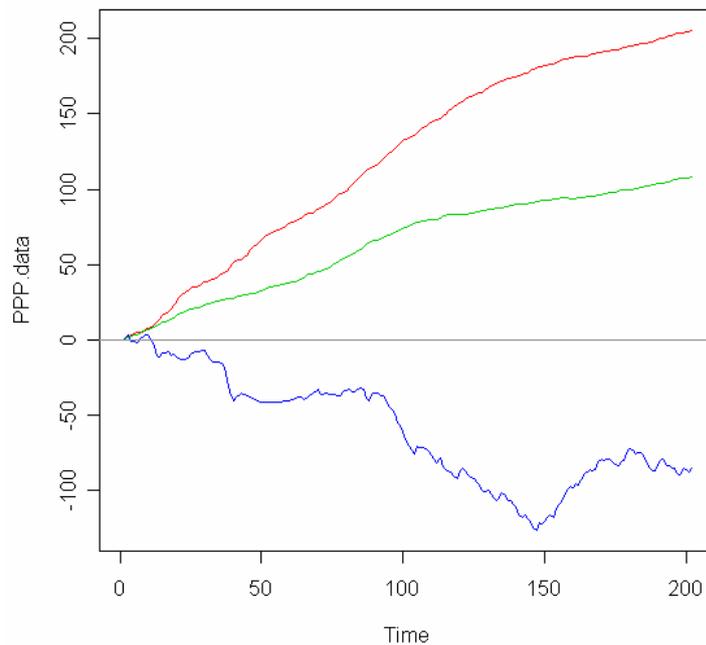
**Monthly data of p_t , s_t and p_t' from 1973 to 1989 after transition**

(Green is p_t' , Red is p_t' , Blue is s_t)

6.3 PPP puzzle in Hamilton's book

Given the fundamental idea of PPP theory we have that the real exchange and purchasing power is equilibrium in a longer term. It is to say $P_t = P_t' \times S_t$ where Pt denotes the domestic price index Pt' is the foreign price index and St is the exchange rate between these two countries. Thus, we take

logarithm s of both sides of $P_t = P_t' \times S_t$ and turn it into plus form: $p_t = p_t' + s_t$. It is to say that this dynamic data set should be modeled as cointegration model by cointegrating vector (1,-1,-1). However when we consider this model into dollar/lira real exchange rate data set, we found that it is not the case. Under that data set and given the cointegrating vector, we compute the $z_t = p_t - p_t' - s_t$ and plot the trend figure as follow:

Figure 6.2
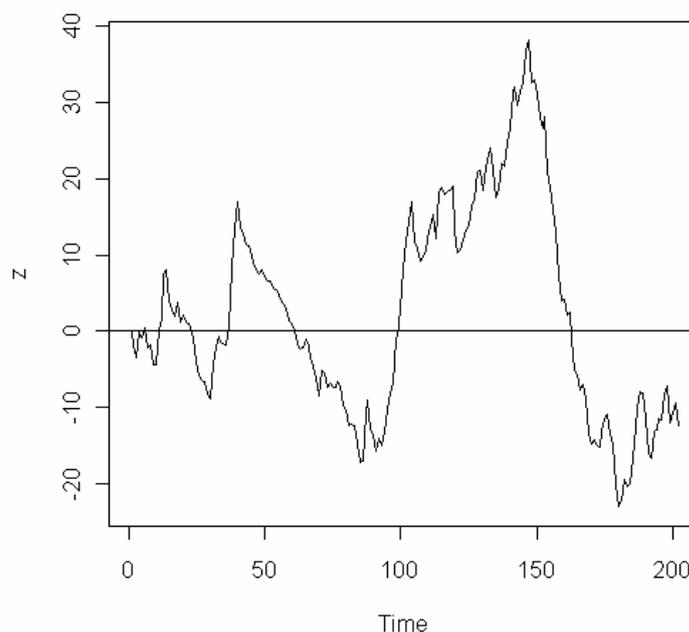
Time series plot of residuals for cointegration model

From figure 6.2, we could suspect that z_t is not nonstationary. Then after the testing by Dickey Fuller t test and Philips Peron test, we can get the common result that is we can reject the null hypothesis, z_t is stationary. Thus, the result by testing is opposite to the PPP theory.

6.4 Testing for nonlinear cointegration against nonlinear cointegration in PPP

After studying the Hamilton (1994)'s empirical example of purchasing power parity between United States and Italy from 1973 to 1989, we apply the definition of nonlinear cointegration and go further analysis to check whether there is any cointegrating connection in the system.

I) Test p_t, p'_t, s_t are all I(1) process:

H_0 : p_t, p'_t, s_t are all nonstationary.

We apply augmented Dickey-Fuller test to prove that we can not reject the null hypothesis of

nonstationary. The results of the estimation and the calculation of the test statistics are stated in details in Hamilton (1994)'s.

Next step, we apply the testing procedure in section 4 to check if there exist nonlinear cointegration vector such that a nonlinear cointegration which was defined in section 2 can be expressed in this dynamic system.

II) Test linear cointegration against nonlinear cointegration:

$$H_0 : \gamma_1 = \gamma_2 = 0$$

Given the third order auxiliary regression, we apply OLS to fit restricted regression and non-restricted regression respectively:

Restricted regression:

$$p_t = \beta_1 p'_t + \beta_2 s_t + \mu_t$$

Non-restricted regression:

$$p_t = \varphi_{20} p'_t + \varphi_{21} p'_t \Delta p'_{t-1} + \varphi_{22} p'_t \Delta p'^2_{t-1} + \varphi_{23} p'_t \Delta p'^3_{t-1} + \varphi_{30} s_t + \varphi_{31} s_t \Delta s_{t-1} + \varphi_{30} s_t \Delta s^2_{t-1}$$

$$+ \varphi_{30} s_t \Delta s^3_{t-1} + \mu^*_t$$

Compute the sum square of the residuals for these two regressions respectively, and then get the value of F statistics:

$$F = \frac{(RSS_0 - RSS_1)/m}{RSS_1/(T-k)} = 24.33$$

Compare with critical value of F is 3.45 in table 5.2, we can reject the null hypothesis which is there is linear cointegration in this dynamic system at 1% level.

By further investigate, we found that for different delay parameter we all can reject the null hypothesis, but we can get the max F value if determine the delay parameter d at 1. Thus, we will determine d as 1 to estimate the nonlinear cointegration model in next section.
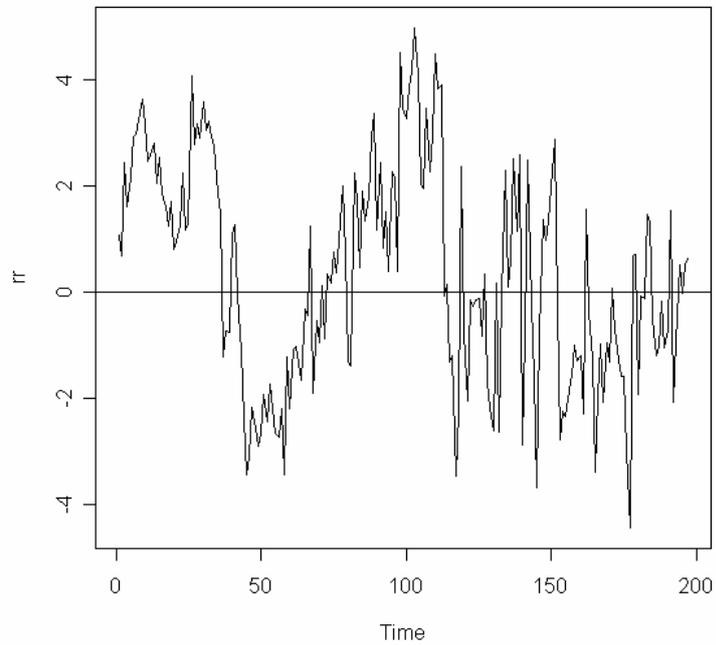
6.5 Model estimate

By NLS regression, we get the estimation of model. The estimation of nonlinear cointegration model as follow:

$$Z_t = p_t - \left(a_1 + \left(1 + \exp\{ -\gamma_1 (\Delta p'_{t-1} - c_1) \} \right)^{-1} \right) p'_t + \left(a_2 + \left(1 + \exp\{ -\gamma_2 (\Delta s_{t-1} - c_2) \} \right)^{-1} \right) s_t$$

where, $\gamma_1 = 0.750, \gamma_2 = 0.145, c_1 = -3.648, c_2 = -24.638, a_1 = -0.398, a_2 = -0.883$

Figure 6.3

Residuals of model time series plot

Figure 6.3 shows the time series plot for residuals of nonlinear cointegration model which we estimated above. After comparing with Figure 6.2 and 6.3, we can see that the residual of cointegration model could be stationary. In other words, the dynamic system of purchasing power parity could be modeled as a cointegration model by the corresponding cointegrating vector. The nonlinear cointegration which we defined in this essay may be a solution for PPP puzzle.

7. Concluding remarks

In this essay, we raised the definition of nonlinear cointegration which generalized Engle and Granger (1987)'s definition of linear cointegration. In this definition of nonlinear cointegration, we are base on the Logistic Smooth Transition Autoregression model to represent the nonlinear cointegration system by triangular representation. Given this definition, we suggest a testing procedure to test linear cointegration against nonlinear cointegration. Under the definition and testing procedure, we get the critical value of F statistic table and asymptotic distribution by Monte Carlo simulation. Compared with standard F distribution, we can see that the OLS F statistics distribution is heavier skewed to left.

In empirical problem application, we consider the Purchasing Power Parity and employ the testing procedure and critical value table which we work out before to analyze dollar/lira real exchange rate. From the testing and model estimation, we can see that there is no linear cointegrating relation in the PPP system, however, there is our nonlinear cointegration evidence supported by our empirical analysis for the PPP example. It is suggested that the PPP system may contain nonlinear features between the economic variables which could cause the PPP puzzle in Hamilton (1994).

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Appendix: R code

(1) set up function to generate data and compute f statistics:

```
sim=function(t=25,a=1,b=1,d=0){
# data generation under H0
t=t+5    # check the delay parameter d, and the
first five value is ...
y2=rep(0,t)
y2[1]=rnorm(1)
for(i in 2:t){
ee=rnorm(1)
y2[i]=y2[i-1]+ee
}
y3=rep(0,t)
y3[1]=rnorm(1)
for(i in 2:t){
ee=rnorm(1)
y3[i]=y3[i-1]+ee
}
y1=rep(0,t)
for(i in 1:t){
ee=rnorm(1)
y1[i]=a*y2[i]+b*y3[i]+ee
}
# Compute the F Value
diff_y20=diff(y2)
d1=5-d
d2=t-1-d
diff_y2=c(diff_y20[d1:d2])
yy2=diff_y2*y2[6:t]
yy21=diff_y2^2*y2[6:t]
yy22=diff_y2^3*y2[6:t]

diff_y30=diff(y3)

d1=5-d
d2=t-1-d
diff_y3=c(diff_y30[d1:d2])
yy3=diff_y3*y3[6:t]
yy31=diff_y3^2*y3[6:t]
yy32=diff_y3^3*y3[6:t]

y11=matrix(y1[6:t],t-5,1)

y22=y2[6:t]
y33=y3[6:t]

x=cbind(y22,yy2,yy21,yy22,y33,yy3,yy31,yy
32)
bt=solve(t(x)%*%x)%*%t(x)%*%y11
rr=matrix(c(a,0,0,0,b,0,0,0),8,1)
st=(t(y11-x%*%bt)%*%(y11-x%*%bt))/(t-8)
F_v=t(bt-rr)%*%solve(st[1]*solve(t(x)%*%x)
)%*%(bt-rr)/6

return(F_v)
}
# simulation:
F_v=rep(0,10000)
for(i in 1:10000){
F_v[i]=sim(25,1,1,0)
}
hist(F_v,breaks=40)
q_val=quantile(F_v,probs=c(99,97.5,95,90,10,
5,2.5,1)/100)
q_val
```

(2) Testing for nonlinear cointegration for PPP data set

```
p1=100*(log(PZUNEW)-log(PZUNEW[1]))
p2=100*(log(PC6IT)-log(PC6IT[1]))
s=100*(log(1/EXRITL)-log(1/EXRITL[1]))

PPP.data=ts(cbind(p1,p2,s))
plot(PPP.data,plot.type="single",col=2:4)
abline(h=0, col = "gray60")
```

```
dd=1
diff_p2=diff(p2)
c=length(diff_p2)-dd
diff_p2=diff_p2[1:c]
g=dd+2
pp2=p2[g:length(p2)]
```

```
p22=diff_p2*pp2
p23=diff_p2^2*pp2
p24=diff_p2^3*pp2


diff_s=diff(s)
diff_s=diff_s[1:c]
ss=s[g:length(s)]
s2=diff_s*ss
s3=diff_s^2*ss
s4=diff_s^3*ss
```

```
pp1=p1[g:length(p1)]


lm0=lm(pp1~pp2+ss-1)
lm1=lm(pp1~pp2+p22+p23+p24+ss+s2+s3+s4-1)
rss0=sum(residuals(lm0)^2)
rss1=sum(residuals(lm1)^2)
F_val=((rss0-rss1)/6)/(rss1/(length(pp1)-8))
F_val
```

(3) estimation for model by NLS regression
```
diff_p2=diff(p2)
c=length(diff_p2)-dd
pp2=diff_p2[1:c]
g=dd+2
p22=p2[g:length(p2)]

diff_s=diff(s)
ss=diff_s[1:c]
s2=s[g:length(s)]

pp1=p1[g:length(p1)]

mm1=nls(pp1~(a1+1/(1+exp(-gamma1*(pp2-
c1))))*p22+(a2+1/(1+exp(-gamma2*(ss-c2))))
*s2,data=d,start=list(a1=1,a2=1,gamma1=0.5,
gamma2=0.5,c1=1,c2=1))
summary(mm1)

rr=residuals(mm1)
plot.ts(rr)
abline(h=0)
```