A Smooth Transition Autoregressive Model for Electricity Prices of Sweden

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Abstract

In this paper, nonlinear features of the daily electricity prices of Sweden are studied. Following from Chow Test, an evidence of a data break point is found between the warm season and the cold season during the period from May 1st, 2007 to April 30th, 2008. A test of the null hypothesis of linearity against nonlinearity for the first difference of daily prices of Sweden is carried out by using a Multiplier (LM)-type test given by Teräsvirta (1994). Modeling for this electricity prices is considered and a logistic smooth transition autoregressive (LSTAR) model allowing to have a regime switching can characterize nonlinear features of electricity prices of Sweden.

Keywords: Nonlinearity, Smooth Transition, LM-test, Chow test

CONTENT

A STAR model for electricity prices of Sweden

1 Introduction

As the world's first international electricity power exchange market, Nord Pool began operating officially in 1993 in Olso, Norway. Sweden joined in January 1996; Finland was fully integrated in March 1999. West-Denmark became a separate price area in July 1999 (Hjalmarsson, 2000).

Nord pool is a free market, basically, it organizes two markets, a "physical market" (Elspot) and a "financial market" (Eltermin and Eloption). In the Elspot market, hourly power contracts are traded daily for physical delivery in the next day's 24-hour period. The price calculation is based on the balance between bids and offers from all market participants – finding the intersection point between the market's supply curve and demand curve. Every contract in Elspot refers to a load, in megawatt-hours (MWh, 1MWh equals 1,000kWh), during a given hour, and a price per MWh.

Since electricity is flow commodity with limited storability and transportability, the demand will change in different seasons. The price is strongly dependent on the electricity demand. For example, in winter, the demand is larger and the price is higher.

In Sweden, approximately half of the total installed capacity is hydropower. Nuclear power has the next highest capacity share of approximately 30 percent. Except for a small amount of renewable generation capacity, oil and gas comprise the others parts

Haldrip (2006) and Lucia (2002) have touched the issue about Nordic Pool spot price system, but both of them used linear model to analyze the characters of the price system. Since there are significantly difference between cold season and warm season, nonlinearity model maybe is a better choice. Despite that linearity is now a standard procedure in the characterization of the time series properties of any process, nonlinearity has been applied in some areas like unemployment fluctuations (Skalin and Terävirta, 2002), inflation (Arango and Gonzalez, 2001) and stock market returns (Bredin, 2008).

The aim of the paper is to study the features of Sweden electricity prices, to test whether there have data break point between the cold season and the warm season and then to use time series smooth transition autoregressive (STAR) models to specify the changes of the daily electricity prices. The reminder of this paper is set out as follows. Section 2 presents the data, section 3 shows the models and section 4 presents the conclusions and the final remarks.

2 Data

The data employed in this study is daily observations of Sweden electricity prices from May 1st 2007 to April 30th 2008 (366 observations). The data was obtained from the Nordic Pool's FTP sever files. The sample period includes two different parts: warm season and cold season. Warm season is from 1st May to 30th September of the same year, and cold season is from 1st October to 30th April of the next year.

Table 1 presents summary statistics for the four different series P_t, log (P_t), P_t-P_{t-1}, log (Pt)-log (P_{t-1}). From this table, we can see the significant difference exists between the cold season and the warm season. Noting that warm season had a daily mean about 41% lower than that of cold season. However, the standard deviation of log-price changes show that warm season is about 1.6 times as volatile as cold season (0.099 or 189% annualized and 0.1566 or 298% annualized). The ratio of standard deviation over mean of the daily price is 0.24 for warm season and 0.17 for cold season, indicating a significantly higher stability of the mean price for cold seasons as compared to warm seasons. Changes of prices have very similar standard deviations for both warm and cold seasons.

The reason why the price in winter is higher and more stabile than in summer is that the demand in winter is higher and about 50% of installed capacity of this country is hydropower, however, in winter the lakes and rivers are freezing.

Table 1: the descriptive statistics for the daily system price and other related time series(20070501-20080430)

series	number of observation	Mean	Median	Minimum	Maximum	Stand Deviation	Skewness	Kurtosis
			Panel A all seasons (20070501-20080430)					
P_t	366	326.8	325.5	142.4	550.6	94.82758	0.0364,	1.859169
P_t-P_{t-1}	365	0.4398	-2.3900	-117.3000	142.6000	37.91562	0.6772	5.231862
Log(P_t)	366	5.744	5.785	4.958	6.311	0.3082654	-0.3569	2.016582
Log(P_t)-log(P_{t-1})	365	0.001839	-0.006296	-0.355300	0.622900	0.1258321	0.7304	5.613087
			Panel B Warm season (20070501-20070930)					
P_t	153	241.2	230.9	142.4	412.0	53.61197	0.7768,	3.300963
P_t-P_{t-1}	152	1.033	-0.980	-85.110	142.600	38.10343	0.8709,	4.821876
Log(P_t)	153	5.462	5.442	4.958	6.021	0.2146573	0.2844	2.562724
Log(P_t)-log(P_{t-1})	152	0.004337	-0.004862	-0.328600	0.622900	0.1557187	0.7604	4.420209
			Panel C cold season (20071001-20080430)					
P_t	213	388.3	390.1	234.2	550.6	65.83965	-0.2935,	2.486616
P_t-P_{t-1}	212	0.0117	-2.6150	-117.3000	141.7000	37.864	0.5351,	5.519445
Log(P_t)	213	5.946	5.967	5.456	6.311	0.1792881	-0.6552,	2.76421
Log(P_t)-log(P_{t-1})	212	3.576e-05	-6.443e-03	-3.553e-01	3.823e-01	0.09919314	0.3779	5.62696

Note: This table displays descriptive statistics for the daily spot system price (denoted P_t) and other related series as indicated in the first column. The sample period is from 1st May, 2007 to 30th April, 2008. Panel A displays the results for the whole set of data, panel B for warm season from 1st May. to 30th Sep of the same year and panel C for the cold season from 1st Oct. to 30th Apr. of the next year.

Using the model $y_t = (c_{10} + \beta_{11} y_{t-1} + \beta_1 y_{t-2} + \cdots + \beta_{1p} y_{t-p}) + (c_{20} + \beta_{21}\Delta y_{t-1} + \beta_{21}\Delta y_{t-2} + \cdots + \beta_{2p}\Delta y_{t-p}) \times (\frac{1}{1+e^{-\gamma(\Delta y_{t-1} - c_0)}}) + u_t$, where Δy_t stands for P_t–$P_{t-1}$and log (P_t)-log(P_{t-1}).

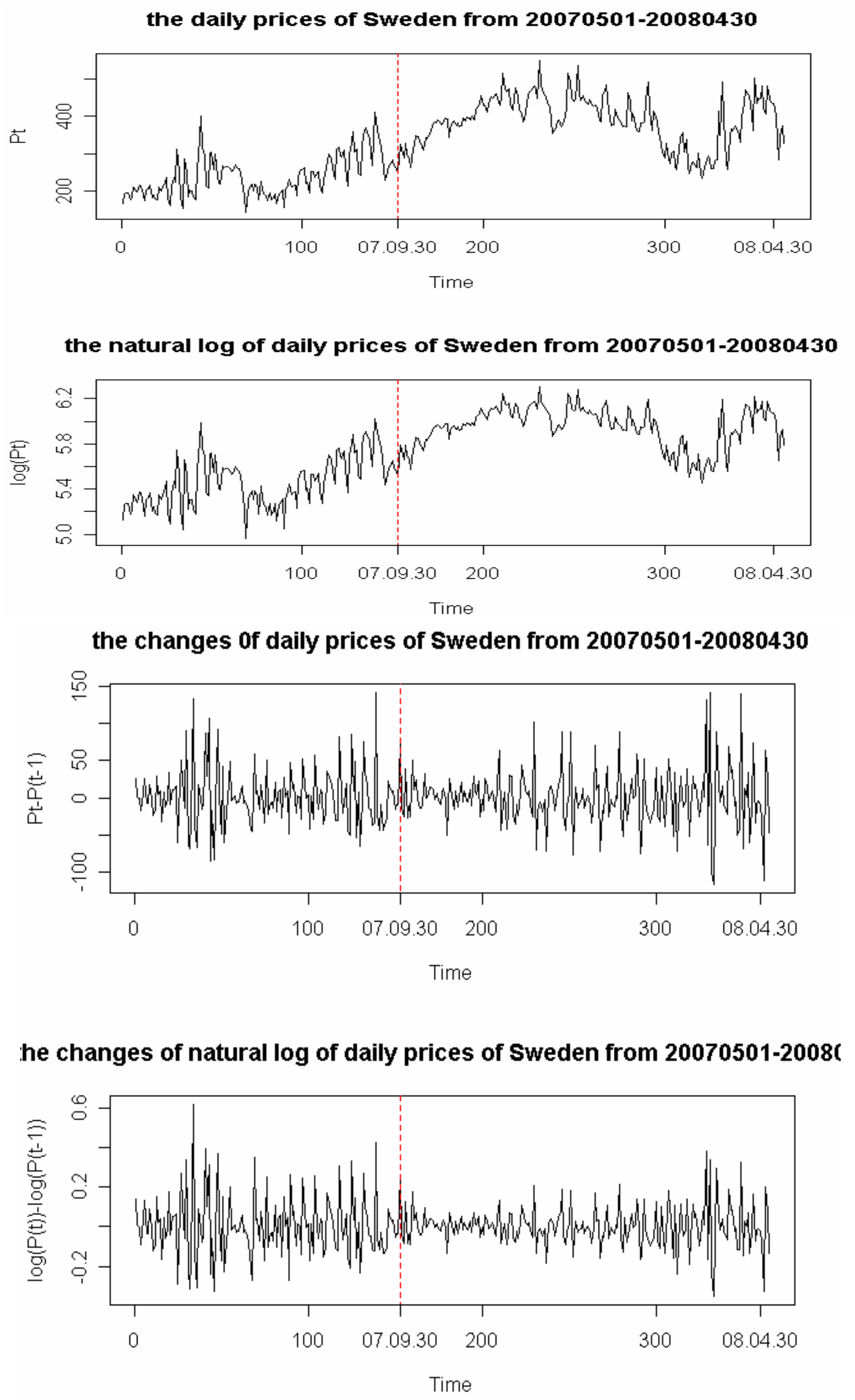

Figure 1. the Daily System price Time Series
(2007.05.01-2008.04.30)

3 Models

3.1 General

As discussed in section 1 and section 2, since the series exhibit non-linear features such as regime switching, smooth rather than abrupt changes are expected. A STAR model allows that the changes of electricity price alternatives smoothly between two regimes. The first thing we should do is to test whether there have data break points during the process. Then we will test linearity against the alternative that the process is nonlinearity for the changes of the daily prices series. If linearity is rejected against such an alternative, we specify, estimate, and evaluate smooth transition autoregressive model for the electricity and discuss the implications of the estimated model.

3.2 Test data breaks

Let a model for a time series process $\{y_t\}$(t = 1,2, …, T) at a point (T_0) and before it (t =1,2,…,T_0) be a pth order AR(p) model as follow.

$$y_t = \beta_{10} + \beta_{11} y_{t-1} + \beta_{12} y_{t-2} + \cdots + \beta_{1p} y_{t-p} + u_t \qquad (t \le T_0) \tag{1}$$

Similarly, let a model after a point (T_0) of a structural change (t=T_{B+1},T_{B+2}, T)

$$y_t = \beta_{20} + \beta_{21} y_{t-1} + \beta_{22} y_{t-2} + \cdots + \beta_{2p} y_{t-p} + u_t \qquad (T_0 \le t \le T) \tag{2}$$

For the whole period, suppose another AR (p) model

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \cdots + \beta_p y_{t-p} + u_t \qquad (t \le T) \tag{3}$$

In these models, the null hypothesis that a structural change does not exist and the alternative hypothesis that it does are shown as follows.

$$H_{10}: \ \beta_{10} = \beta_{20},\ \ \beta_{11} = \beta_{21},\ \ \beta_{12} = \beta_{22},\ \ \cdots,\ \ \beta_{1p} = \beta_{2p}$$

H_{11} : at least one equation above does not hold.

The classical test for this kind of structural change is typically attributed to Chow (1960). This test was popular for many years and was extended to cover most econometric models for interest. For a recent treatment, see Andrews and Fair (1988).

$$KF = \frac{K(SSR_R - SSR_1 - SSR_2)/K}{(SSR_1 + SSR_2)/(T-2K)} \sim \chi^2(K) \tag{4}$$

Where K is the number of parameters; SSR_R is the sum of the squared residuals of (3); SSR_1 and SSR_2 are the sum of the squared residuals of (1) and (2). Chow KF statistics follows Chi-square distribution with the degree of freedom K.

In this study, let the T_0= 153, which point is the boundary of the warm season and the cold season. After estimation of the AR (p) model (here choose p = 7), the main test results at the 5% level are summarized in Table 2. Null hypothesis H_{10} is rejected for series y_t and $\log(y_t)$ series.

Table 2 : the results of Chow test about data breaks

series	Number of observation (T)	K	KF-value	Reject or Accept H_{10}
P_t	365	8	24.28	R
$Log(P_t)$	365	8	26.50	R
$\chi^2_{.95}(8) = 16.9$				

3.3 Test linearity again nonlinearity

3.3.1 LSTAR model

The LSTAR model of order p [STAR (p)] is as below

$$y_t = (c_{10} + \beta_1' x_t) + (c_{20} + \beta_2' x_t) \times G(s_t, \gamma, c_0) + \varepsilon_t \tag{5}$$

$$\varepsilon_t \sim iid(0, \sigma^2) \tag{6}$$

$$x_t = (y_{t-1}, ..., y_{t-p}), \quad \beta_j = (\beta_{j1,...,}\beta_{jp}), j = 1, 2 \tag{7}$$

$$G(s_t; \gamma, c_0) = (1 + \exp\{-\gamma(s_t - c_0)\})^{-1} \tag{8}$$

The main property of this model is the “Smooth transition” between regimes instead of a sudden jump from one regime to the other. G is a transition function bounded by zero and unity. s_t is the transition variables and c_0 is the threshold value. γ represents the speed of the transition process. When γ is large, the shape of the transition function G(.) is very steep in the neighborhood of the threshold value c_0 (see appendix A). As $\gamma \to \infty$, and $s_t > c_0$ then G=1; when $s_t \le c_0$, G=0; so equation (5) become a TAR (p) model. When $\gamma \to 0$, equation (5) becomes an AR (p) model.

The Lagrange Multiplier (LM)-type test linearity against nonlinearity were described by Granger and Teräsvirta (1993, Ch.6), Luukkonen (1988) and Teräsvirta (1994) is used here. After expanding the transition function (8) into a third order Taylor series around $\gamma = 0$, merging terms and reparameterizing.

$$y_t = (c_{10} + \beta_1' x_t) + (c_{20} + \beta_2' x_t)[\frac{1}{4} r(s_t - c_0) - \frac{1}{48} r^3 (s_t - c_0)^3] + \eta_t \tag{9}$$

$$= c^* + \beta_1^{*\prime} x_t + \beta_2^{*\prime} x_t s_t + \beta_3^{*\prime} x_t s_t^2 + \beta_4^{*\prime} x_t s_t^3 + \eta_t \tag{10}$$

Test linearity against non-linearity Hypothesis

$$H_{20} : \beta_2^{*\prime} = \beta_3^{*\prime} = \beta_4^{*\prime} = 0 \Leftrightarrow \gamma = 0 \tag{11}$$

$H_{21} : \beta_2^{*\prime}, \beta_3^{*\prime}, \beta_4^{*\prime}$ not be zero at the same time

As Teräsvirta (1994) and van Dijk and Teräsvirta (2002) shown that

$$LM_3 = T(SSR_0 - SSR_1)/SSR_0 \sim \chi^2(3p) \tag{12}$$

SSR_0 is the sum of the residual squares of the estimated model under the null hypothesis of linearity of y_t on x_t $SSR_0 = \sum_{t=1}^{T} \hat{\varepsilon}_t^2$. SSR_1 is the sum of the squared residual squares of the estimated model under the auxiliary regression of y_t on x_t and $x_t s_t^i$, $i = 1,2,3.$ Compute the squared residuals $\hat{e}_t$ and the sum of the squared residuals $SSR_1 = \sum_{t=1}^{T} \hat{e}_t^2$ .The tests statistic is distributed as χ^2 with 3p degree of freedom.

3.3.2 Artificial example for LM-type test

Let c_{10}, c_{20}, c_0 equal to zero, p=1 and $s_t = y_{t-1}$, changing γ, use (5) to generate series, and then use (12) to test (100 replications). The result is in Table 3 is the successful frequency.

Table 3: LM-test on data generated with nonlinear model (5)
(100 replications)

	γ=0.5	γ=1	γ=2	γ=5	γ=10	γ=100
T=50	2	7	9	11	12	15
T=100	17	24	29	32	35	38
T=200	47	51	65	75	78	86
T=500	97	99	100	100	100	100
T=1000	100	100	100	100	100	100

As shown above, when γ and the number of observations are more large, it is more easy to test the nonlinearity successfully.

3.3.3 The results for the really data

Since the method above requires stationary variables, we test for unit roots on the four series (P_t, log (P_t), P_t-P_{t-1}, log (P_t)-log (P_{t-1})) both under ADF and KPSS procedure. Accordingly, since stationary is needed, we use the first difference of the daily prices or the first difference of the log prices. According the values of AIC and ACF, we choose the lap p =7, the linearity and nonlinearity test using (12) is shown in Table 4, under the 5% level. According the result of Table 4, the null hypothesis of H_{20} is rejected for both the series $\Delta P_t = P_t - P_{t-1}$ and $\Delta \log(P_t) = \log(P_t) - \log(P_{t-1})$

Table 4 : the results of LM-type test linear against nonlinear

series	Number of observation	P	LM_3-value	Reject or Accept H_{20}
$P_t - P_{t-1}$	365	7	89.6	R
$\text{Log}(P_t)$- $\text{Log}(P_{t-1})$	365	7	96.1	R
$\chi^2_{.95}(3\times 7) = 32.7$				

3.4 Estimation

The final model is estimated by non-linear least squares (NLS) including estimation of the parameters γ and c_0. Estimation of the slope parameter γ is not easy. Teräsvirta (1994) and others suggest that the transition function $G(s_t, \gamma, c)$ should be standardized to make γ scale-free, which implies dividing the exponent in $G(s_t, \gamma, c)$ by the standard deviation of s_t. Grid search method is used here and the final γ and c_0 and other parameters are chosen according the AIC.

$$G(s_t, \gamma, c) = (1 + \exp\{-\frac{\gamma}{\hat{\sigma}_{s_t}}(s_t - c)\})^{-1} \tag{13}$$

As the number of observations extends to infinite, the parameters extend to the real value, which shows this method is quite good.

Here we only give the results of the first difference of the daily price system, p= 7 and $s_t = \Delta y_{t-1}$. The first difference of the natural log price system is shown in the appendix B.

$$\begin{aligned}
\Delta y_t = \; & (\underset{(0.0000)}{-0.1095}\Delta y_{t-2} \underset{(0.0002)}{-0.0893}\Delta y_{t-3} \\
& \underset{(0.0001)}{-0.1004}\Delta y_{t-5} \underset{(0.0031)}{-0.0638}\Delta y_{t-6} + \underset{(0.0008)}{0.0844}\Delta y_{t-7}) \\
& + (\underset{(0.0015)}{-0.1194}\Delta y_{t-1} \underset{(0.0106)}{-0.0896}\Delta y_{t-4}) \\
& \times [1 + \exp\{-202 / \hat{\sigma}(\Delta y_{t-1}) \times (\Delta y_{t-1} - \underset{(<\ 0.0000)}{15.0675})]\}]^{-1} + \hat{u}_t
\end{aligned} \tag{14}$$

$\hat{\sigma}_{\Delta y_{t-1}}$ = 38.1417 S = 4.8920 SK = -1.622 EK = -0.842

JB = 12.14(0.002) AIC =2170.509 S/S_{AR} =0.146

Where figures in parentheses below the parameter estimates denote the p-values of the tests. S is the estimated standard deviation of the residuals, SK is skewness, and EK is excess kurtosis. Jarque-Bera (JB) test rejects normality. S/S_{AR} is the ratio between the residuals standard deviation of the LSTAR model and the AR model. Here the ratio is less than unity (= 0.146), which means that the former outperforms the later.

The estimated value for the threshold value c_0, $\hat{c}_0$=15.0675, shows the intermediate point between increasing or decreasing daily price. $\hat{c}_0$ is within the range of $\{\Delta y_t\}$, which is a good symptom of this model. The estimation of γ, $\hat{\gamma} = (202 / \hat{\sigma}_{\Delta y_{t-1}})$, suggest a quick transition from one regime to the other to the extent that it could mimic a TAR model.

The other estimates can be interpreted by analyzing the limit values that describe the local dynamics of the high (G = 1) and low (G=0) electricity prices. For doing so, we use the roots of the LSTAR model which can be obtained as usual from:

$$\lambda^p - \sum_{j=1}^{p} (\hat{\beta}_{1j} + \hat{\beta}_{2j} \times G)\lambda^{p-j} \tag{15}$$

Where G=0, 1 (see Table 5). In the lower regime it can be seen that the process is convergent with modulus 0.7701 and the upper regime there is also a convergent with modulus 0.7568. The dynamic properties depicted by the LSTAR (7) model means that either the dynamics starts close to the lower regime or the upper regime, the electricity price converts to a positive one very easily.

Table 5: Characterization of extreme regimes polynomials and dominant root

Model	Regime	Root	Modulus
	Lower(G=0)	-0.1234+0.7601i	0.7701
	Upper(G=1)	0.5123+0.5571i	0.7568

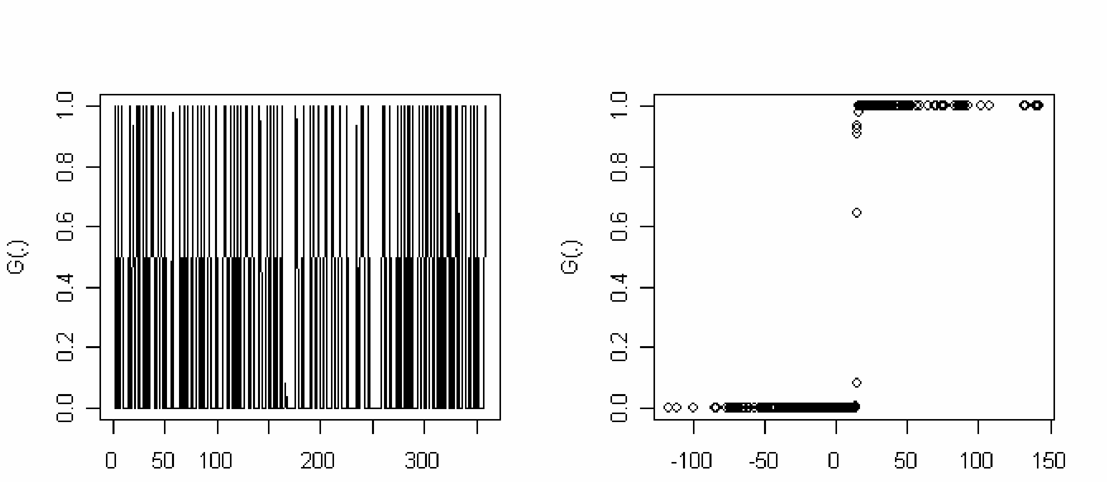

Figure 2. Transition function for the smooth transition autoregressive (STAR) models

Left hand panels plot the transition function $\hat{G}(\Delta y_{t-1};\hat{\gamma},\hat{c}_0)$ against time. Right hand panels plot the transition function $\hat{G}(\Delta y_{t-1};\hat{\gamma},\hat{c}_0)$ against the transition variable Δy_{t-1}.

Figure 2 shows the transition function $\widehat{G}(s_t;\gamma,c_0)$ of the LSTAR model of ΔP_t series. The transition between the two regimes is sharp ($\widehat{\gamma} = 202/\widehat{\sigma}_{\Delta y_{t-1}}$, see appendix A).

3.5 Evaluations

To test the model fit, need to test no residual autocorrelation (van Dijk and Teräsvirta, 2002, Eitrheim and Teräsvirta, 1996).

$$\widehat{u}_t = \rho\widehat{u}_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim iid(0,\sigma^2) \tag{16}$$

$$H_{30} : \rho = 0; \tag{17}$$

$$H_{31} : \rho \neq 0 \tag{18}$$

$$t = \sqrt{n-1}\frac{\sqrt{SS_x}}{\sqrt{SS_0}}\widehat{\rho} \tag{19}$$

Here $SS_x = \frac{1}{n}\sum(\widehat{u}_{t-1} - \overline{\widehat{u}}_{t-1})^2$, $SS_0 = \sum(\widehat{u}_t - \overline{\widehat{u}}_t)^2 - \widehat{\rho}\sum(\widehat{u}_t - \overline{\widehat{u}}_t)(\widehat{u}_{t-1} - \overline{\widehat{u}}_{t-1})$

If accept H_0, shows that there exist no residual autoregressive.

$\widehat{\rho} = -0.0604$, |t|= 1.138 < $t_{0.025}(\infty)$ =1.64. So there does not exist autocorrelation. Figure 3 shows the residuals of the model (14).

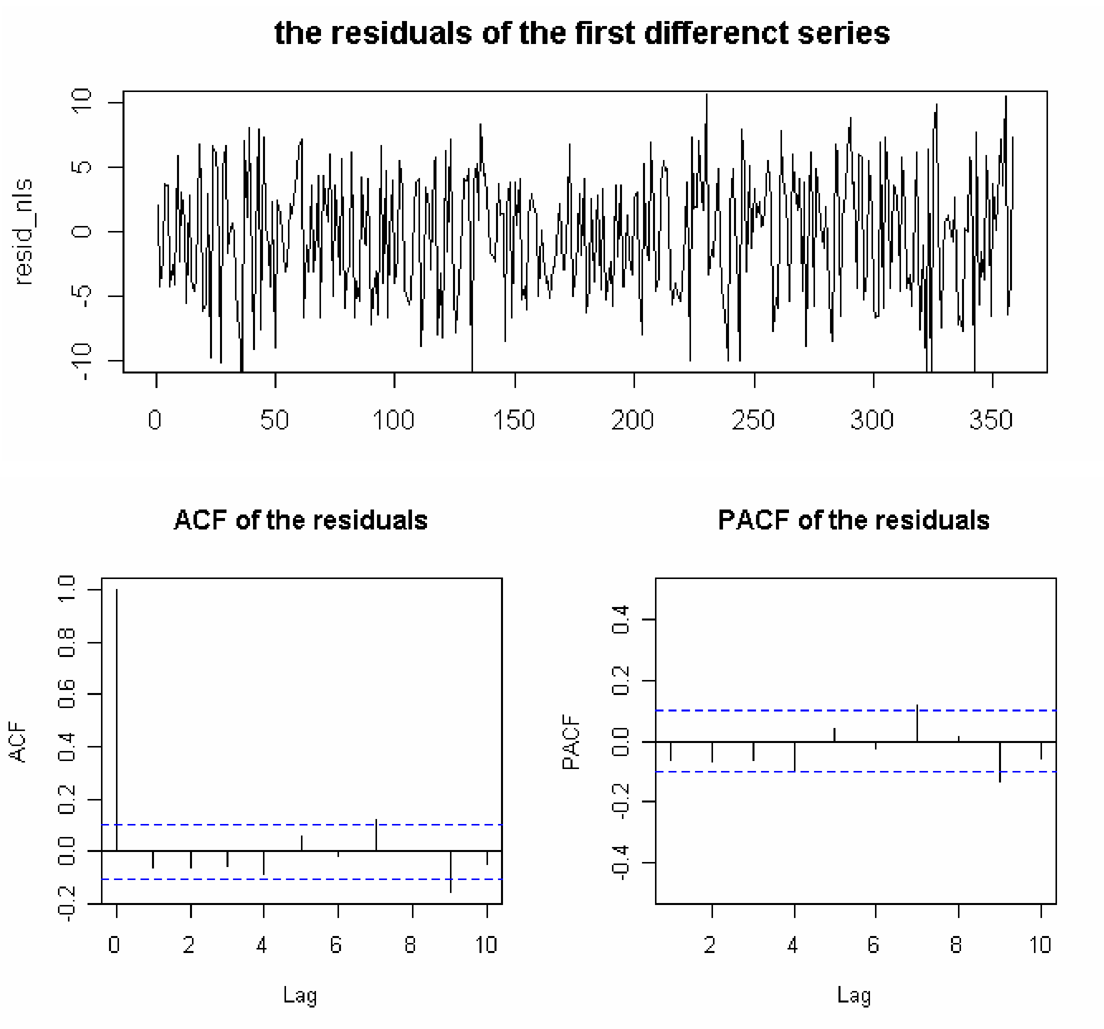

Figure 3. the residuals of the model (14)

4 Conclusions

In this paper, we study the features of the daily electricity prices of Sweden from the period of May 1st, 2007 to Apr 30th, 2008. There are significant evidence that there exist data breaks between the warm season and the cold season. The price in winter is higher but more stable than in summer. We also find that daily prices changes exhibits non-linearity features such as regime switching. After using Lagrange multiplier (LM)-type test (Teräsvirta, 1994), we prove that the ΔP_t and

$\Delta\log(p_t)$ series are nonlinearity. And these two series can be adequately characterized by a logistic smooth transition autoregressive (LSTAR) model with the result better than the linear models.

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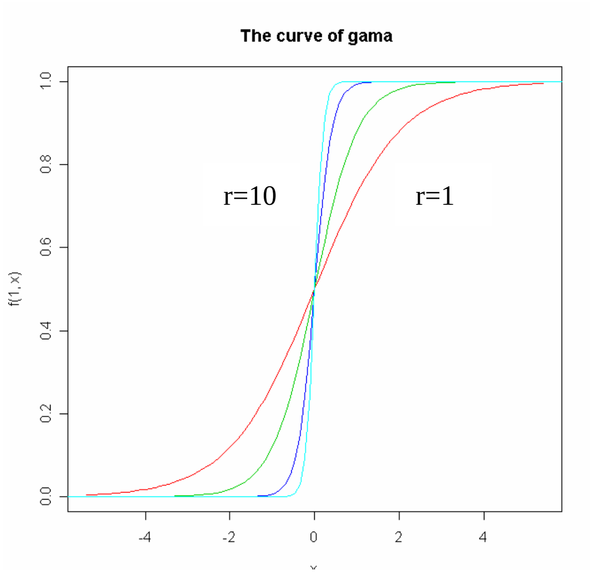
Appendix A: the character of ST function

$$F(\gamma, x) = \frac{1}{1+e^{-\gamma * x}}, \gamma = 1, 2, 5, 10$$



Appendix B: estimation of the first difference of the log price

$$x_t = \log(y_t)$$

$$\Delta x_t = (\underset{(0.0004)}{-0.1002}\Delta x_{t-1} \underset{(4.22e-7)}{-0.2544}\Delta x_{t-2} \underset{(1.26e-5)}{-0.1121}\Delta x_{t-3})$$

$$\underset{(4.12e-5)}{-0.1069}\Delta x_{t-5} \underset{(0.0001)}{-0.0900}\Delta x_{t-6} \underset{(5.27e-6)}{+0.2104}\Delta x_{t-7})$$

$$+(\underset{(0.0018)}{0.1676}\Delta x_{t-2} \underset{(0.0004)}{-0.1024}\Delta x_{t-4} \underset{(0.043)}{-0.1587}\Delta x_{t-7}) \qquad \text{(B.1)}$$

$$\times[1+\exp\{-42/\hat{\sigma}(\Delta x_{t-1})\times(\Delta x_{t-1} \underset{(<2e-16)}{+0.0337})]\}]^{-1} + \hat{u}_t$$

$\hat{\sigma}_{\Delta x_{t-1}}$ = 0.12634 S = 0.26967 SK = -0.1155 EK = -0.8143

JB = 10.69(0.004) AIC =98.6148 S/S_{AR} =0.512