Review of the literature on credit risk modeling:

Development of the recent 10 years

Author:

Chengcheng Hao

Ba Zhang

Supervisor:

Kenneth Carling

Md. Moudud Alam

One-year-Master thesis in Applied Statistics, Jun 15th 2009.
Högskolan Dalarna

Abstract

This paper traces the developments of credit risk modeling in recent 10 years. Our work is divided into two parts: selecting articles and summarizing results. On the one hand, by building an ordered logit model on historical Journal of Economic Literature (JEL) codes of articles on credit risk modeling, we select articles highly related to our topic. It is shown that the JEL codes have become the standard to classify researches in credit risk modeling. On the other hand, comparing with the classical review Altman and Saunders (1998), we show that some important changes of research methods have emerged recently. The main finding is that current focuses on credit risk modeling have moved from static individual-level models to dynamic portfolio models.

JEL *classification:* G21; G33; C23; C52

Keywords: Bank Lending; Structural model; Reduced-form model; Default; SME

CONTENT

1. Introduction

In the August of 2007, the subprime lending crisis of United State has been shocking the world's major financial markets. The fundamental reason for the crisis lies in the fact that asset price bubble which is brought about by excessive expansion of the real estate market has increased the risk preferences of investors. But it is also because the lending institutions driven by interest rates have loosened the credit risk management.

The usual risk taxonomy has three components: market risk, credit risk and operational risk. According to the New Basel Accords proposed (hereafter Basel II, see Basel Committee on Banking Supervision (2006)), market risk is the risk of loss in on and off-balance-sheet positions arising from movements in market prices. Credit risk is the risk of loss due to the probability an obligor (borrow, counterparty) is unable or unwilling to pay its credit. And operational risk, which is broadly a residual category, is the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. Credit risk makes up about 50-60% of the total risk in a large bank (Kuritzkes et al. (2003)). Comparing with the situation that bulk of academic research focuses on market risk, credit risk has received less attention in the literature, until recently. In fact, we have seen an explosion of research over the last few years as the well-established tools and the measuring techniques find their way to the realm of credit assets assessment. Credit risk modeling attracts a diverse group of disciplines, from traditional finance (asset pricing) and mathematical statistics to econometrics. Because credit risk is the dominant risk for banks, regulators also pay particular attention to its measurement and management.

For the above mentioned facts it is time to summarize the literature on credit risk modeling, just as the overview given by Altman and Saunders (1998), which includes the developments on credit risk modeling over its past 20 years. However, no other review is done after it. In this paper, we try to trace the development from that time and summarize current knowledge on credit risk modeling. We cover more than 100 articles on credit risk which have appeared recently. Though these are not the full context on this topic, we believe that they include about 75% recent papers (It is argued later). Moreover, we use a criterion of article selection to select the articles of interested and they do represent, in our view, the best of the crop and illustrate the diverse set of issues and their treatments which have occupied the literature.

The paper is organized as follows. In Sections 2 and 3, we present the selection criterion and selection result for the articles we review in this paper. The main body of this review is from Section 4 to Section 9. We analyze the recent contributions in the credit risk modeling from several different aspects. Section 4 discusses the databases used in modeling loans' credit risk. Section 5 examines the different definitions of general risk measures (default and losses given default) used by current articles. Section 6 presents the developments of credit risk modeling under different modeling frameworks. Section 7 mentions the popular commercial models. Section 8 summarizes the statistical tests, test strategies and other topics related to evaluating credit risk models. Section 9 covers the current increasing studies on modeling the credit risk of loan from small and median enterprise (SME). Section 10 is our main conclusions.

2. Selection of Articles

Our literature search begins with electronic full-text databases ELIN and EconPapers, using searching term “credit risk” in the title or keywords.

Elin@Dalarna integrates data from several publishers, full-text databases and e-print open archives. Elin allows a cross-searching its own user interface. It also covers most journals which may contain the contribution on credit risk modeling, such as Journal of Banking and Finance, Journal of Finance, Journal of Empirical Finance, Journal of Financial Economics, Journal of Financial Intermediation, Review of Financial Economics, Journal of Economics, Journal of Economics and Business, International Review of Financial Analysis, Research in International Business and Finance, Global Finance Journal, Finance Research Letters and so on. EconPapers provides access to RePEc, the world’s largest collection of on-line Economics working papers, journal articles and software. It is hosted by the Swedish Business School at Örebro University and is available at http://econpapers.repec.org/.

Our search is in April 2009, and yields 1399 results in these two databases (see Table 1). Because ELIN includes articles in published journals and the recent-5-year articles in EconPapers cover all high quality economics working papers in published proceeding, we infer that the 1399 results include most developments up to now on credit risk modeling, which we want to add to the literature pool.

Table 1 Querying results before applying selection criteria

Type	**Database**	**Query**	**Number of hits**	**With JEL**
Journal:	ELIN	ti:"credit risk" OR kw:"credit risk"	982	138
Working Paper:	EconPapers	#Keywords and Title: "credit risk" # Search as: Phrase #Search: working papers / articles # From the year 2005 to 2009 # Date is: Creation/revision of item # Sort by: Date modified	417	Most
Total	-	-	**1399**	**About 550**

However, it is a challenging task of reviewing and summarizing such a large number of articles within one paper and many papers will be unrelated to credit risk modeling. So before our review work, we want to define clearly which articles to be reviewd. The words “define clearly” here contain two kinds of meaning. First, we should have a standard to justify which ones are most interesting in this review. Second, an effective method to find such interesting articles should be described here. Usually, researchers attain these goals by adding some important keywords or conditions during querying in the database and we do a similar thing but by using Journal of Economic Literature (JEL) classification code[1]. The explanation of JEL classification codes and the reason we use it will be argued later. But, a problem raised here is: how do we know which keywords, JEL codes or conditions are important and relevant to our preference?

To solve this question, this paper uses the basic idea in statistics and proposes that by applying statistical model, which can be estimated from an observable random sample of the 1399 papers, we are able to draw inferences about our preference among all the articles. The dependent variable, that is, our preference score of the articles, is an ordinal variable, an appropriate model here can be proportional odds model (PO model, also know as ordered logit model or ordered logistic regression). In brief, the selection follows the following steps:

(i) Sample 40 papers without replacement from the 982 ELIN articles and 417 EconPapers articles respectively and get a random sample of size 80.

(ii) Read and evaluate the 80 papers and divide them into the following four preference categories[2].

0= Not interesting:	The paper is not related to credit risk measurement
1= Of little interest:	In the area of credit risk measurement, but little related to credit risk modeling or not related to loan credit risk.
2= Interesting:	Loan credit risk modeling, but with mistakes or missing some important details to understand the methodology).
3= Very interesting:	Worthy to be reviewed, that is, what we are concerned.

(iii) Summarize JEL classification codes from each article in the sample.

(iv) Select and estimate a proportional odds model, which can be written as

$$P(Y \le j) = \frac{\exp(\alpha_j - \boldsymbol{\beta}^{\mathbf{T}}\mathbf{x})}{1 - \exp(\alpha_j - \boldsymbol{\beta}^{\mathbf{T}}\mathbf{x})}$$

where Y represent the preference scores, j=0, 1, 2, 3 and $\mathbf{x}$ is the variables of JEL codes.

(v) Do forecasts and inference.

Now, let us come back to the issue why we use JEL codes rather than keywords as the independent variables of the model. This is because of three reasons. First, economic articles, of course including the ones on credit risk modeling, in the recent 10 years are usually classified according to the system used by the Journal of

[1] An explanation and guide for JEL classification codes can be found at http://www.aeaweb.org/jel/guide/jel.php

[2] Acknowledge our supervisors Kenneth Carling and Md. Moudud Alam. To have a justified article evaluations, they and the authors do this work independently, with twofold blinded ranking and resolution to the disagreed ranks.

Economic Literature. These JEL codes which use 3 digits to represent the categories, subcategories and subsubcategories of the economic study, provide helpful information and an effective way of reference. Thus, it is reasonable that JEL codes of an article are an important factor related to paper selection, and that we can narrow our searching based on JEL codes. Second, compared with keywords, the advantages of JEL codes are as follows. It is a standard classification and the number of kinds of JEL codes is a limited number. These make JEL codes more suitable for considering as variables in model than keywords. The last, although the fact that not all the articles have JEL codes is the biggest disadvantage of this method, our research based on the PO model shows articles without JEL codes have significantly low likelihood to be interesting (regression result shown in Appendix). Based on the consideration of efficiency and feasibility as well, articles without JEL codes will not be reviewed in this article. This is another reason why we limited our review work to the last 10 years: JEL codes are less for the papers published more than 10 years ago.

Thus, we define independent variables as follows,

NoJEL: whether an article has JEL codes. 0=Yes, 1=No
Va1: whether an article has C14 or C15 or C23 or C24 or C32 or C4 or C5. 1=Yes, 0=No;
Va2: whether an article has G21 and (G28 or G33). 1=Yes, 0=No.

According to ordered logistic regression, we hold some conclusions. On the one hand, articles that have "credit risk" in them keywords or title with Va1=1 or Va2=1 have significant higher possibility to be very interesting (also see Appendix 1). In the above 80 samples, 19 (23.8%) articles are very interesting. In contrast, there are 20 papers if we search "Va1=1 OR Va2=1" in the sample, and 14 (70%) among them are in the top of preference rank. Our selection methodology makes the likelihood of finding interesting papers increase from 23% to 70%.

On the other hand, most of the very interesting articles satisfy the condition Va1=1 or Va2=1. There are 19 papers in the top of preference rank, but only 15 have JEL codes. This means 14 (93%) of them are included in the group with Va1=1 or Va2=1.

3. Articles to be reviewed

3.1 Selection result

Summarizing the above two conclusions, a final result is made that we review articles in the following area: "credit risk" in keywords or title AND with JEL codes AND Va1 OR Va2, that is, (C14 OR C15 OR C23 OR C24 OR C32 OR C4 OR C5) OR (G21 AND (G28 OR G33)). Table 2 summarizes the end result of this selection process. We find 59 published articles in ELIN and 86 working papers finished between the year 2005 to 2009 in EconPapers. The total number of articles in the literature pool decreases from 1399 to 145.

Table 2 Querying results after applying selection criteria

Database	Query	Result
ELIN	(kw:"credit risk" OR ti:"credit risk") AND kw:jel AND (kw:(C14 OR C15 OR C23 OR C24 OR C32 OR (C40 OR C41 OR C43 OR C44 OR C45 OR C46 OR C49) OR (C51 OR C52 OR C53 OR C59)) OR kw:(G21 AND (G28 OR G33)))	59
EconPapers	#Keywords and Title: "credit risk" #Search as: Phrase #Search: working papers / articles #From the year 2005 to 2009 #JEL codes: (C14 OR C15 OR C23 OR C24 OR C32 OR C40 OR C41 OR C43 OR C44 OR C45 OR C46 OR C49 OR C51 OR C52 OR C53 OR C59) OR (G21 AND (G28 OR G33)) # Date is: Creation/revision of item # Sort by: Date modified	86 (Index: 1-84, 126, 127)
Total	**-**	**145**

3.2 Article classification

After eliminating unrelated papers (34 (23%)) and those appears in both database ones (9 (6%)), 103 of these 145 articles have been reviewed. Many criteria can be used to classify these papers. For example, we can divide them into empirical studies and theoretic studies; and according to the model inputs, there are accounting-data-based models, market-data-based models and macroeconomic-data-based models (Bonfim (2009)). Although various suggestions for classification are proposed, a consensus has emerged in recent 10 years: modern credit risk models can be generally classified into “structural models” and “reduced form models” (Saunders and Allen (2002)).

Structural models, also known as asset value models or option theoretic models, spring from Black and Scholes (1973) and Merton (1974)’s work in which default risk of debt is viewed as the European put options on the value of a firm’s assets and default happens if a firm’s assets value is lower than debt obligations at the time of maturity. The term “structural” comes from the fact that these models focus on the company’s structural characteristics such as the asset volatility or leverage. Relevant credit risk elements, such as default and losses given default, are functions of those variables.

On the other hand, unlike structural models, the models without examining underlying causalities of default are called reduced-form models. Reduced-form models argue that default time is a stopping time of some given hazard rate process and the payoff upon default is specified exogenously. An incomplete list of early studies by this approach contains Jarrow and Turnbull (1992, 1995), Lando (1994), Duffie and Singleton (1997), Jarrow et al. (1997) and Madan and Unal (1998).

We will explain them in detail in sections 5, including traditional credit risk models, which Altman and Saunders (1998) refers to as “credit scoring system models”, as well. However, the credit scoring system models can also be treated as a kind of reduced-form models. We name them "individual-level reduced-form models" to distinguish them from the above mentioned reduced-form models which focus on loan portfolio defaults. It should also be mentioned that the models developed by commercial companies, such as KMV model and CreditRisk+, are not covered in this classification. This is not only because of the clear distinction between them and academic models but also because of their translucency in techniques. However, we do not want to ignore the studies on these issues and will discuss them in a separated section latter. The classification of articles is illustrated in Table 3 and a complete listing of the reviewed papers is in the reference.

Table 3 Classification of Articles in terms of development directions

Model Framework	**Number (Proportions)**	**Most Recent Reference**
Structural models	12 (11%)	Shibata and Yamada (2009), Marcucci and Quagliariello (2009), Zambrano (2008) and so on;
Individual-level reduced-form models	9 (9%)	Das et al. (2009); Lin (2009), Agarwal and Taffler (2008), Hollo and Papp (2008) and so on;
Portfolio reduced-form models	49 (48%)	Bonfim (2009), Schmidt and Schmieder (2009), Witzany (2009), Feng et al. (2008), Frydman and Schuermann (2008), Feldhutter and Lando (2008), Kadam and Lenk (2008), Rösch and Scheule (2008), Varsanyi (2008) and so on;
Commercial models	2 (2%)	Gordy (2000), Crouhy et al. (2000);
Other topics	31 (30%)	Castren et al. (2009), Dunbar (2009), Chalupka and Kopecsni (2008), Fong and Wong (2008), Jankivuolle et al. (2008), Jakubik and Hermanek (2008) so on.
Total	**103 (100%)**	--

3.3 Development trend

Earlier works in credit risk modeling were characterized by a dominant focus on credit scoring and static assessment of default probabilities, (see Altman and Saunders (1998)). However, this situation does not hold in recent articles any longer. When we have a look at the list of the titles of recent finished articles, two salient changes are found. The focus of the studies on credit risk modeling moves from individual level to the loan-portfolio level and from static model to the dynamic model. That is because the internal needs of credit risk management as well as the availability of more and more historical default data.

4. Database used in model building and checking

In contrast to bonds, bank loans are usually not traded (the situation in United State is a little better) and data confidentiality may also be a problem. As a consequence, database used in modeling credit risk of loans are relatively limited, particularly in earlier studies.

Generally speaking, two kinds of databases are used in building and checking credit risk model. One is worldwide commercial databases from risk rating agencies, for example, Standard and Poor (S&P)'s CreditPro database and Moody's KMV Credit Monitor database. These databases have long time series (S&P's built from the year 1981 and Moody's from the early 1990s) and large number of observations, but the problem is that they are North American banking system and the loans for large companies dominated. The proportion of North American loans in CredirPro was 98% in 1980s, whereas, now it decreases to 63%. (Frydman and Schuermann (2008)). When the study aims to work on other countries, the available sample size becomes small.

Nowadays, it is still common for studies using this kind of databases and the proportion is up to 50%. For instance, in the 15 empirical articles published from 2005 to 2009, there are 7 studies using commercial databases. S&P's CreditPro are used by Feng et al. (2008), Frydman and Schuermann (2008), McNeil and Wendin (2008), Lucas and Klaassen (2006) and Hui et al. (2006). Schmidt and Schmiederb (2009) and Kadam and Lenk (2008) use Moody's database. Lin (2009) use BankScope database to target 37 listed banks in Taiwan over the time period of 2002-2004.

Another half of studies tend to use data sets provided by Central Credit Register, which is credit risk database managed by a country's central bank, or collected from commercial banks directly. These databases usually contain monthly information of loans granted to firms and households, including their current status, in order to improve their internal credit risk management. The availability of this kind of databases is increasing especially in Europe. Studies of them cover Czech, France, Germany, Italy, Portugal, Swiss, Sweden, as well as United Kingdom. Although most of these databases are built during the late 1990s or 2000s, which are much later than the S&P's CreditPro, within the recent five-year studies basing on them, 30% obtain panel data more than 10 years, 85% more than 5 years. This significant improvement in longitudinal data can still be foreseen in the following years. These data allow us to consider multiple business cycles into credit risk models, which lead to the developments in dynamic models.

5. Risk measures

During the past 10 years, most approaches in credit risk modeling involve the estimation of three parameters: the probability of default (PD) on individual loans, the estimate of the loss given default (LGD) and the correlation across defaults and losses (Crouhy et al. (2000)). Actually, the first two are identified as two key risk parameters of the internal rating based (IRB) approach, which is central to Basel II. The IRB approach allows banks to compute the capital charges for each exposure from their own estimate of the PD and LGD. Although default and loss are universally acknowledged as critical terms of credit risk modeling, there are no standard definitions for them. We find their definitions and measurements to differ in the articles. To summarize and combine results from the various papers, it is necessary for us to have an overall understanding in the distinctions of the definitions being used.

5.1 Default

The default definition expressed by the Bank for International Settlements (BIS) is not a clear statement. According to it, default is the situation when the obligor is unlikely to pay its credit obligations or the obligor is past due more than 90 days on any material credit obligation. Many academic articles, such as Chalupka and Kopecsni (2008), Bonfim (2009) and Schmit (2004), follow the second part of this definition. However, some of the obligors may naturally happen to pay all their obligations back even after 90 days. Particularly in the case of retail clients, days overdue may just be a result of payment indiscipline, rather than a real lack of income to repay the loan. It means a problem of this definition that defaults do not necessarily imply losses.

Some other articles consider default to be when and only when obligor is "reorganization" bankrupt. This definition is based on United States Bankruptcy Code, which is often referred as "Chapter 11 Bankruptcy" (see Glossary). Some studies on European data also apply the similar definition, basing on their own bankruptcy law. Examples for this group are Grunert and Weber (2009) and Agarwal and Taffler (2008). In the option theoretic models, default probability is justified as the probability that obligor's asset value fall below the value of liabilities, that is, probability of bankruptcy. However, again, a firm can default on the debt obligations and still not declare bankruptcy. It depends on the negotiations with its creditors. Thus, for the question that which one is a better definition, there is still no clear universal answer in practice.

5.2 Losses given default

Another key issue in credit risk modeling is about the LGD or, equivalently, the recovery rate (RR). LGD is usually defined as the loss rate on a credit exposure if the counterparty defaults. It is in principle one minus RR, but also comprises the costs related to default of the debtor. However, because the costs is only a small part of losses, RR and LGD are always used the same conceptually in academic studies.

LGD of a portfolio can be restated as long-term average LGD on a portfolio level. Therefore, its value is determined by the measuring methods of LGD on individual level and the selection of average weights, which are summarized in Chalupka and Kopescsni (2008)'s work. It should be note that, the choice for individual LGD measurement and weights depends on data availability, and the all mentioned concepts have the indispensable importance in recovery estimation.

There are three classes of LGD for individual loan or instrument, referred to as market, workout and implied market LGD. The first one, market LGD, is estimated from market price of bonds or tradable loan when default events occur. But since after-default market is only available for the corporate bonds issued by large companies and bank loans are traditionally not tradable, this approach is highly limited in application. Therefore, most empirical articles for LGD of bank loans, such as Schmit (2004), Dermine and de Carvalho (2005) and Grunert and Weber (2009), suggest applying the second approach- workout LGD. It is calculated from the recovered part of the exposure arising in the long-running workout process, discounted to the default date. However, the disadvantage for this approach is that bankrupt settlements are common not only in cash but also with some assets without secondary market. It means that the cash flow generated from the workout process may not be properly estimated. Moreover, the work of selecting appropriate discount rate is difficult as well. The last method is called implied market LGD, which is estimated from market value of risky but non-defaulted bonds or bank loans by using theoretical asset pricing model. Thus, it is naturally applied in the articles related to structural or reduced-form models. It should be emphasized that there is a clear distinction between this approach and the other two: the first two kinds of LGD are hard to estimate before actual default happen or highly rely on historical default; while the estimation of implied market LGD does not rely on historical data so much and is suitable for the studies on the low-default loans or bonds, which do not have sufficient historical losses data.

In addition to selecting a specific individual LGD measurement, there are four approaches to calculate a portfolio average LGD. Table 4 is brought from Chalupka and Kopescsni (2008)'s work and shows these four approach. Where, $i=1,2,\ldots,m$ is the index of the default year, and $j=1,2,\ldots,n_i$ is the index of default observation in the i^{th} year.

Table 4 Different measurement of LGD on a portfolio level

	Default count averaging	Exposure weighted averaging
Default weighted averaging	Average LGD= $\dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n_i} LGD_{i,j}}{\sum_{i=1}^{m} n_i}$	Average LGD= $\dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n_i} EAD_{i,j} \times LGD_{i,j}}{\sum_{i=1}^{m}\sum_{j=1}^{n_i} EAD_{i,j}}$
Time weighted averaging	Average LGD= $\dfrac{\sum_{i=1}^{m} \dfrac{\sum_{j=1}^{n_i} LGD_{i,j}}{n_i}}{m}$	Average LGD= $\dfrac{\sum_{i=1}^{m} \dfrac{\sum_{j=1}^{n_i} EAD_{i,j} \times LGD_{i,j}}{\sum_{j=1}^{n_i} EAD_{i,j}}}{m}$

In practice, default weighted averaging is more frequently used than time weighted averaging. And default count averaging is usually recommended to be used in studies on non-retail segment. In contrast, retail portfolios and some small and medium-sized enterprises (SME) loans apply exposure weighted averaging.

6. Credit risk models

In the previous section, we summarized the outcome variables which are modeled by existing literature on credit risk. However, it seems more logical that the articles should be reviewed according to their modeling framework.

Three broad categories, being structural model, individual-level reduced-form model and portfolio reduced-form model, are introduced in this part. However, we want to point out that when we review the recent articles, one kind of cross model attracts our attentions. This kind of model assumes defaults to follow an intensity-based process, with latent variables that may not be fully observed because of imperfect accounting and market information (Allen et al. (2004)), and default happens when the latent variables fall behind a threshold value. Thus, this kind of model stems from both structural modeling and reduced-form modeling framework. Nevertheless, to review effectively, we put this kind of model in the sub-category of portfolio reduced-form models, titled and refer to it as "factor model".

6.1 BSM framework structural models

As mentioned before, the idea of this kind of model is proposed by Merton (1974). Merton (1974) derived the value of option for a defaultable company. In the classical Black-Scholes-Merton (BSM) model of company debt and equity value, it is assumed that there is a latent firm asset value *A* determined by the firm's future cash flows, where *A* follows Brownian motion. Its value at time t, given by A_t, satisfies

$$\frac{dA_t}{A_t} = r_A(t)dt + \sigma_A(t)dz_t$$

where $r_A(t)$ and $\sigma_A(t)$ denote asset return rate ($r_A(t)$ can be view contains there components: risk-free interest rate r, asset risk premium λ and asset payment ratio δ) and volatility of asset value, z_t follows the standard Wiener process and dz_t is standard normally distributed. In Merton (1974)'s work, $r_A(t)$ and $\sigma_A(t)$ are constants and non-stochastic. And, he assumes the firm's capital structure just relate to two things: pure equity (that means preference stocks are not considered) and a single zero-coupon debt maturing at time *T*, of face value *B*. The default event only occurs when the asset value at maturity is less than B. Upon the random occurrence of default, the stock price of the defaulting company is assumed to go to zero. Thus, we have following payment equations

$$\begin{cases} \text{Receives of debt holders} = \min(A_T, B) \\ \text{Receives of equity holders} = \max(A_T - B, 0) \end{cases}$$

Thus, debt holder can be considered as a seller of European put option, equity holder can be considered as a buyer of European call option in Merton's (1974) work and asset value *A* can be considered as the price of foundation security. By assuming there are no dividends, we can use standard Black-Sholes option-pricing equation to get a relation between the equity market value E_t and A_t and the bond market value Y_t and A_t. In general form, that is,

$$\begin{cases} \overline{E}_t = f_1(A_t, \overline{B}, \overline{r}, \sigma_A(t), \overline{T}) \\ Y_t = f_2(A_t, \overline{B}, \overline{r}, \sigma_A(t), \overline{T}) \end{cases}$$

where *r* means short-term risk-free interest rate and the meaning of other variables are as mentioned before. Variables which have a bar above them are observable and exogenous. However, because, according to Modigliani-Miller Theorem I (1958), it hold that $E_t + Y_t = A_t$ in any capital structure, any one of the above two equations can be derived from the other. It means that we actually do not need to calculate the second one. As Delianedis and Geske (1998) discussed, there should be linkage between observable stock volatility σ_t and unobservable asset volatility $\sigma_A(t)$, so if we specify the form of this linkage $\overline{\sigma}_E(t) = g(\sigma_A(t))$ (e.g. many articles directly use $\sigma_E(t)$ to substitute $\sigma_A(t)$), Y_t and A_t can be known.

The biggest disadvantage of Merton (1974)'s model is that there are too many simplified assumptions for its derivation. This restricts the empirical applied value of the model. Thus, its subsequent researches mainly focus on relaxing these assumptions. The most worthy to be elaborated is maturity. We find three kinds of extending methods for it in our review. Geske (1977) extends the original single debt maturity assumption to various debt maturities by using compound option modeling. In Leland and Toft (1996)'s work, firms allow to continuously issue debts of a constant but infinite time to maturity. Duffie (2005) mentions that, comparing with Merton (1974)'s assumption that the default occurs only at the maturity date, another group of structural models is developed by Black and Cox (1976) and often referred to as "first-passage-time model". In this class of models, default event can happen not only at the debt's maturity, but also can be prior to that date, as long as the firm's asset value falls to the "pre-specified barrier" (that is, default trigger value). Thus, the model not only allows valuation of debt with an infinite maturity, but, more importantly, allows for the default to arrive during the entire life-time of the reference debt or entity.

Staying with this first-passage-time idea, other parameters that Merton assumes constant also are extended to be dynamic. First, Longstaff and Schwartz (1995) treat the short-term risk-free interest rate as a stochastic process which converges to long-term risk-free interest rate and is negatively correlated to asset value process, so that the effect of monetary policy to macro economy are considered.

Additionally, the default barrier B_t is also treated dynamically in various papers. Briys and Varenne (1997) assume that the change of B_t follows the change of risk-free interest rate r_t . Hui et al. (2003) argue that default barrier should decrease when time goes, because they observe that there is high default risk at time close to maturity. Collin-Dufresne and Goldstein (2001), based on their observation that firms tend to issue more debt when their asset value increases, propose that the default trigger value B_t, which is considered as a fixed face value of debt in Merton model, should be a process converging to a fraction of asset value A_t. Actually, this model implied a widespread strategy that firms tend to maintain a constant leverage ratio. Hui et al. (2006) develop this stationary-leverage-ratio model to "incorporate a time-depending target leverage ratio". They argue that firm's leverage ratio varies across time, because of the movement of initial short-term ratio to long-term target ratio as stated in Collin-Dufresne and Goldstein (2001). Their model assumes default occurs when a firm's leverage ratio increases above a pre-specific default trigger value and the dynamic of interest rate follows the set of Longstaff and Schwartz (1995). By doing so, the model captures the characters of the term structure of PD, which was mentioned in Hui et al. (2003).

What's more, Huang and Huang (2003)'s model postulates the asset risk premiums λ_t is a stochastic process and there is negative correlation between it and the unexpected shock to the return of asset value. They consider the empirical finding that risk premiums of a security tend to move reversely against the returns of stock index in it.

There is another classification of the family of structural models. The models can be divided into exogenous default group and endogenous default group (Tarashev (2005)). The distinction between this two groups is somewhat related to the two definitions of default in the last section. All the above mentioned works belong to the former group, in which default is defined as when the asset value fall below a trigger value. While the endogenous default models allow obligors choose the time of default strategically, in which default also depends on negotiation. For example, the latter group of models contains Anderson et al. (1996). Anderson et al. (1996) allow firm to renegotiate the terms of debt contract. When default trigger value is touched, a firm can either bankrupt or give a new but higher interest rate debt contract to debt holder. An empirical comparison of these two groups of model can be found in Tarashev (2005).

The last noteworthy BSM structural model is proposed by Shibata and Yamada (2009). They develop BSM structural model to model bank's recovery process for a firm in danger of bankruptcy. When obligor bankrupts, the bank's choice whether the firm should be run or be liquidated affects the losses of the loan. Shibata and Yamada (2009) assume this decision is made at continuous time t after the bankruptcy. Using this option approach and incorporating the application of game theory, the paper shows the properties of the bank's collecting process.

6.2 Individual-level reduced-form models

The reduced-form models are an approach to credit-risk modeling that contrasts sharply with the "structural credit models". All the models not structural model belong to this class. The individual-level reduced form model is commonly called as credit scoring system or credit scoring model. As we will show in following, now just a small part of articles (less than 9%) fully study on it. So we do not want to mention them too much, but just present some important developments.

The credit scoring model is proposed by Altman (1968). By identifying accounting variables that have statistical explanatory power in differentiating defaulting firms from non-defaulting firms, this approach uses linear or binomial (such as logit or probit) models to regress the defaults. And once the coefficients of model are estimated, loan applicants are assigned a Z-score to classify they are good or bad.

This individual-level model got significant development in the decades after its proposal. Earlier results of this issue are discussed in some overview studies at the end of 1990s comprehensively. Altman and Saunders (1998) mentioned the widespread use of credit scoring models as well as the model developments. Altman and Narayanan (1997) surveyed the historical explanatory variables in credit scoring models throughout the world and they found most studies suggest use financial ratios which measure profitability, leverage and liquidity, such as Earnings Before Interest and Tax (EBIT)/sales, market value equity/debt, working capital/debt and so on, in their models. However, the consensus about specific choice of these variables is not made.

Whereas in recent 10 years, studies concentrating on credit scoring models are not so much. In our article pool, just 4 papers base on it. Jacobson and Roszbach (2003) builds on bivariate probit model and "proposes a method to calculate portfolio credit risk" without bias. Lin (2005) proposes a new approach by three kinds of two-stage hybrid models of logistic regression-artificial neural network (ANN). Altman (2005) specifies a scoring system, namely Emerging Market Score Model, for Emerging Corporate Bonds, which is not related to out topic much. The last one is Luppi et al. (2007)'s work. They apply logit model to Italian non-profit SMEs and find that traditional accounting based credit scoring model shows less explanatory power in non-profit firms than that in for-profit firms.

One of the main criticisms about credit scoring models is that because their predominant explanatory variables are based on accounting data, these models may fail to pick up fast-moving changes in borrower conditions. Some studies, of which an incomplete list is given by Agarwal and Taffler (2008), test this argument and successfully show the market-based model such as structural models are better at forecasting distress than credit scoring models such as Altman's Z-score, although it should be noted that two recent papers show a reversed result. Agarwal and Taffler (2008) using data of UK, show in term of predictive accuracy, there are few differences between the two specified BSM structural model and the Z-score model. Moreover, when considering differential misclassification costs and loan pricing considerations, Z-score model has greater bank profitability. Das et al. (2009) also find these two kinds of models perform comparably. However, it is a fact that more interests have been brought to dynamic portfolio model from this purely static and individual-level credit scoring model.

6.3 Portfolio reduced-form models

Similar to the situation of structural models that structural models become popular after their introduction by commercial firms such as KMV in 1990s, PD calculated by portfolio reduced-form models have been growing rapidly in popularity since the early 2000s (Das et al. (2009)). Actually, in recent contributions, there are around 50% papers based on this kind of model.

These models were originally introduced by Jarrow and Turnbull (1992) and widely mentioned by latter studies. Jarrow and Turnbull (1992)'s idea behind these models is highly associated with the concept "risk neutral", which in finance means a common technique to figure out the probability of a future cash flow and then to discount this cash flow at the risk-free interest rate. It is valid because Arrow (1953) proves that, in no arbitrage market, it is irrelevant to assets price that whether probabilities assigned to the future cash flows are the real world probabilities or imaged. Thus, we can use risk neutral probabilities to get a simplified calculation of asset price. Based on that, Jarrow and Turnbull (1992) decompose the credit risk premium (can also be called credit spread) to two components, $PD \times LGD$, and the core problem of credit risk modeling becomes to model the distributions of PD and LGD.

Although structural models are very attractive because of their fine theoretical bases, in empirical studies, reduced-form models are reported to performance better to capture the properties of firms' credit risk. Generally speaking, we can consider each of the reduced-form models in three aspects: default time is unpredictable and driven by a default intensity which is a function of latent variables; default in their framework is exogenous and directly specified, for instance, as Poisson or "jump" process with default intensities; RR or LGD can be specifically assumed.

6.3.1 Poisson / Cox process model

This framework is referred to by Gaspar and Slinko (2005) as "doubly stochastic marked point process". In fact, these two names have the same connotation. Cox process is also known as "doubly stochastic Poisson process", and "marked point process", which is more commonly called as counting process, is a generalization of Poisson process. About 8% articles are based on Poisson/ Cox process model.

The simplest model of reduced-form consideration is proposed by Jarrow and Turnbull (1995). In that work, the default process is modeled as a Poisson process $N(t)$ with constant intensity λ , default time τ is exponentially distributed as a consequence, and they also assume that RR is a fixed value. However, it is a somewhat strong assumption that the intensity λ is constant over time and across the loan clusters (e.g. across different credit ratings or industries). The same situation lies in its assumption of RR and in reality, it is hard to consider it as an independent constant. Thus, the earlier works about reduced-form model put a lot of concerns to modify these two assumptions.

Two main methods are employed to extend the assumption of $\lambda(t)$. One is as what is done by Madan and Unal (1998). They assign the intensity $\lambda(t)$ to be a function of the excess return on the issuer's equity. Similar idea lies in Duration models, which are mentioned in Carling et al. (2007). Because the aim is to make the intensity change over time and differ across the loans' properties, in practice, it is a natural idea that allows default intensities to depend on some observable variables which can affect PD. These variables can be accounting variables such EBIT/Asset, market variables such as market equity price, macroeconomic variables such as GDP index, and other variables such as duration of loan. Carling et al. (2007)'s model considered all these kinds of variables, by assuming a linear relation is held between the selected variables and the log value of intensities. And they found that accounting variables and macroeconomic variables are most powerful to explain the credit risk.

The other kind of approach is to modify constant intensity to a stochastic process, which is proposed by Lando (1998) and commonly mentioned as Cox process model. The mathematic form is described here as an example. In this model, default time τ are treated as the first jump time of a Cox process $N(t)$. Thus, default time τ can be written as the infimum of the following set.

$$\tau = \inf\left\{t \in R^+ \middle| N(t) > 0\right\}$$

$N(t)$ denotes the count of default events. Cox process is defined as a Poisson process with time-dependent and stochastic intensity $\lambda(t)$.

$$\lambda(t) = \mu_\lambda(t)dt + \sigma_\lambda(t)dW_t$$

Above equation shows, in Lando (1998)'s case, $\lambda(t)$ are assumed to be Brown motion. $\mu_\lambda(t)$ and $\sigma_\lambda(t)$ are mean and volatility of the intensity; W_t is standard Wiener process and $dWt \sim N(0,1)$. Alternative distributions are assumed by other articles as well. Gaspar and Sliko (2005) propose a concrete model where both PD and LGD are dependent on market index, which is log-normally distributed, and therefore the correlation between PD and LGD can readily be computed.

6.3.2 Markov chain model

To the authors' knowledge, this kind of model is first mentioned in Jarrow et al. (1997). The Markov chain model considers default event as an absorbing state and default time as the first time when a continuous Markov Chain hits this absorbing state. In Jarrow et al. (1997)'s model, it assume fixed probabilities for credit quality changes, which is estimated from historical credit transition matrices, and a fixed RR in the event of default. These time-homogenous discrete-time Markov chain models are widely used (Bangia et al. (2002)). Several developments of Markov chain model are made. There are11 recent papers from our selection in this group.

First, similar to Poisson process model, there are also modifications for the homogeneous assumption. However, this modification is focusing on rating transition probabilities, rather than the intensity in above section.

One of the key models here is ordered probit model. Nickell et al. (2000) and Feng et al. (2008) fit ordered probit model to rating transition. The rating transition probabilities are viewed as functions of latent variables. However, the former work assumes latent variables derived by observable factors such as industry, residence of the obligor and variables related to business cycle, while Feng et al. (2008) introduce unobservable factors and argue recent literature on credit risk shows preference for the use of unobservable factors (related details are shown in the factor model section latter). In addition, the ordered probit model can also be applied in sovereign credit migration estimating, as Kalotychou and Fuertes (2006) do. They also do a comparison between homogeneous and heterogeneous estimators. Gagliardini and Gourieroux (2005) apply ordered probit model for another aim: to estimated migration correlations. They also point out that the traditional cross-sectional estimated migration correlations are inefficient.

Monteiro et al. (2006) suggest using "finite non-homogenous continuous-time semi-Markov process" to model time-dependent matrices. As the definition, semi-Markov process is a Markov chain with a random transformation of the time scale. Monteiro et al. (2006) show that the nonparametric estimators of the hazard rate functions can be used for consistently estimating these time-dependent transition matrices.

Second, Jarrow et al. (1997)'s discrete-time model is commonly extended to continuous-time model. Many papers aim to check Jarrow et al. (1997)'s argument that the results based on discrete-time Markov Chain processes can be improved if we adopt the continues-time ones. Many articles are involved in this topic, such as the contributions made by Monteiro et al. (2006), Fuertes and Kalotychou (2006), Frydman and Schuermann (2008), Kadam and Lenk (2008). Lucas and Klaassen (2006) apply both discrete-time and continuous-time Markov chain model in empirical studies.

At last, although all the above models use Markov Chain process as a core in their framework, their emphases are not on Markovian behaviors but on the non-Markovian behaviors, such as heterogeneous and time-varying rating transition probabilities due to industry class and macroeconomic variables. The literatures which are really studying on Markovian behavior in credit risk are just the recent papers. Thus, we divided Markov Chain articles into non-Markovian behavior group and Markovian behavior group. For the latter, there are studies in following.

Hidden Markov Models (HMM) is a statistical model in which the system being modeled is assumed to be a Markov process with unobserved state. It is used to forecast quantiles of default rates in credit risk modeling. Banachewicz and Lucas (2007) do a further study on this area, and test the sensitivities of the forecasted quantiles if the underlying HMM is mis-specified.

Frydman and Schuermann (2008) and Kadam and Lenk (2008) apply Markov mixture model to their analysis. In their work, the original Markov chain model is extended to a mixture of two Markov chains, where the mixing is on the speed of movement among credit ratings. The main difference of these two works is that the estimation of the former one is based on maximum likelihood method while the latter uses Bayesian estimation.

6.3.3 Factor model

Although the intuitions of structural model and reduced-form model appear significantly different, clear distinction does not exist under some model frame work. Actually, there is no pure "non-structural" model, and as McFadden (1974) stated, even the simplest logit model bears a structural instruction in it. This fact is more obvious in factor models. Gordy (2000) pointed out structural models (CreditMatric model in his work) can map to reduced-form models (CreditRisk+ model in his work) in some degree under factor models framework. Regardless of the different distributions and functional forms these two kinds of models assume, both of them actually use the similar correlation structures that the correlation between defaults is totally driven by some specific common risk factors, which can be called as "systematic factors".

Partly because of IRB approach in Basel II, factor models are most widespread in current literature. About 25% papers in literature pool follow this framework. In these models, the default event of firm *i* in period *t* is modeled as a random variable Y_{it} so that

$$Y_{it} = \begin{cases} 1 & \text{if firm } i \text{ defaults in } t \\ 0 & \text{otherwise} \end{cases}$$

And hazard rate is defined as

$$\lambda_{it} = \Pr(Y_{it} = 1)$$

One of the key characters of factor models is that they model hazard rate through one or a set of latent variables, which follows the structure model's idea: the obligor will default if latent variables fall below a given threshold C_{it}. Usually the latent variable is the firm's returns rate R_{it}., however, some papers use firm's asset value A_{it} as latent variable (Kupiec (2007a)). In the common cases,

$$\lambda_{it} = \Pr(Y_{it} = 1) = \Pr(R_{it} \le C_{it})$$

Factor models often consider two vectors of explanatory variables for latent variable. The first one ($\boldsymbol{X}_{it}$) is a set of macro-economical variables, such as GDP growth, interest rate, money supply growth, inflation rate, stock index and firm's industry as well. This vector intends to explain systematic risk, which leads the correlations of default events. The second vector is a set of firm-specific variables ($\boldsymbol{Z}_{it}$,) which account for individual risk. This vector may include contemporaneous and lagged variables regarding several dimensions of the firm's property, such as age, size, asset growth, profitability, leverage and liquidity.

Some models, such as Pederzoli and Torricelli (2005) and Borio et al. (2001), considered these variables simultaneously. These models are called multi-factor models. These models assume that, set ε_{it} as error term

$$R_{it} = \Gamma X_{it} + \Delta Z_{it} + \varepsilon_{it}$$

Thus,

$$\lambda_{it} = \Pr\left(\Gamma X_{it} + \Delta Z_{it} + \varepsilon_{it} \le C_{it} \middle| X_{it}, Z_{it}\right) = F(C_{it} - \Gamma X_{it} + \Delta Z_{it})$$

where F(.) is cumulative distribution function of the error term ε_{it}.

However, it is more popular just using one systematic random factor to model credit risk, and considering individual risk as a non-deterministic random variable. In these models, defaults are assumed to be driven by the single systematic factor, rather than by a multitude of correlated factors. These one-factor models, which are also called single-factor models, are mentioned in works of Altman et al. (2004), Diesch and Petey (2002, 2004), Repullo and Suarez (2004), Ebnöther and Vanini (2007) and Witzany (2009) and so on. They follow the IRB approach which utilizes a one factor model to calibrate risk weights. Additionally, these models always base on the assumption that the economic conditions which cause defaults to rise might also cause LGD to increase. They model both PD and LGD dependent on the state of the systematic factor. The intuition behind factor model is relatively simple: if a borrower defaults on a loan, a bank's recovery may depend on the value of the loan collateral. The value of the collateral, like the value of other assets, depends on economic conditions.

The simplest version of the singled-factor model is probably the model proposed by the Tasche (2004), which assumes both systematic risk factor X_{it} and individual error term ε_{it} follow the standard normal distributions, and the sum of their risk weight equals to 1. Conditional PD can be calculated by

$$\lambda_{it} = \Pr\left(\omega X_{it} + (1-\omega)\varepsilon_{it} \le C_{it} \middle| X_{it}\right) = \Phi(\frac{C_{it} - \omega X_{it}}{1-\omega})$$

And the unconditional PD, that is, long term PD can be calculated by

$$\overline{\lambda_{it}} = \Pr\left(\omega X_{it} + (1-\omega)\varepsilon_{it} \le C_{it}\right) = \Phi(C_{it})$$

It should be pointed out that the square of risk weight of systematic risk factor ω^2 is also the correlation coefficient between the defaults event.

Both in multi-factor model and single-factor model, they always assume the distribution of the error term follows normal distribution, as the above mentioned papers. But there are alternative assumptions of the factors' distributions. Gordy(2000) and Dietsch and Petey (2002) show a factor model where the factors are gamma distributed with mean 1 and variance σ^2.

Ebnöther and Vanini (2007) extent the standard single factor model to multi-period framework. That is, latent variables are a set of return rates in different time period.

6.3.4 Mortality analysis

Mortality analysis of loan can also be viewed as a type of reduced-form models, because it is based on the survival time of loan as well. Altman and Suggitt (2000) applied this actuarial method to study mortality rates of obligation. Although before them there were some prior works in the area of credit risk based on mortality analysis, most works concentrate on corporate bonds while Altman and Suggitt (2000)'s study focuses on US large bank loan. They find that loans show higher default rates than bonds for the first two years after issuance. This approach was followed by studies on recovery rates, on the probability of default over time for different credit ratings, on recovery rates based on market prices at the time of default, on estimates of rating transition matrices and on the degree of correlation between default frequencies and recovery rates.

The instances in our article pool contain Nickell et al. (2000), Dermine and de Carvalho (2006) and so on. Nickell et al. (2000) study on the issue that choosing an appropriate survival probability for representative banks over a specific horizon. Dermine and de Carvalho (2006) is applied morality analysis to recovery rate. They find that beta distribution does not capture the bimodality of data.

7. Commercial models

Both Crouhy et al. (2000) and Gordy (2000) compare different commercial models, such as the KMV model, CreditRisk+ model and CreditPortfolioView model in their articles.

Unlike CreditMetrics, KMV does not use Moody's or S&P's statistical data to assign a probability of default which only depends on the rating of the obligor. Instead, KMV derives the actual probability of default and the Expected Default Frequency (EDF), for each obligor based on a Merton (1974)'s type model, of the firm.

CreditRisk+ applies an actuarial framework for the derivation of the loss distribution of a loan portfolio. Only default risk is modeled, not downgrade risk. Contrary to KMV, default risk is not related to the capital structure of the firm.

CreditPortfolioView is a multi-factor model which is used to simulate the joint conditional distribution of default and migration probabilities for various rating groups in different industries, It is based on the observation that default probabilities, as well as credit migration probabilities, are linked to the economy.

There are only 3 articles that mentioned the commercial models and all of them from the ELIN database in the relative early period. It might be due to the fact that less recent developments in these models, or, because of the business confidentiality, their technical improvements are not open to the pubic.

8. Performance tests of credit risk

Because there are statistical uncertainties in realized default rates, it is important to develop mechanisms to show how well the credit assessment source estimates the losses. Although only 7 papers specifically focus on this issue, most papers more or less are considering it.

8.1 Statistical tests in credit risk modeling

These evaluating tasks are generally done by using statistical tests to check the significance of the deviation between the realized and in-sample predicted PD (or default frequency). Coppens et al. (2007) have summarized these statistical tests for this purpose. There are extensive articles applying Wald/normal test to test the realized default rates, based on the model assumption that realized default frequency follows a binomial distribution, which could be approximated by a normal distribution. However, Wald test is only suited to testing single default rate, sometimes we need to test several default rates simultaneously, to allow for variation in PDs within the same credit rating or loan cluster, and to take into account default correlations. For these problems, other statistical tests are introduced into this area. Hosmer-Lemeshow test is applied to analyze deviations between predicted probabilities of default and realized default rates of all rating grades or loan clusters, by using the sum of the squared differences of predicted and observed numbers of default, weighted by the inverse of the theoretical variances of the number of defaults as statistic. Spiegelhalter test, which focus on the mean square error (MSE), is used when the probability of default is assumed to vary for different obligors within the rating grade or loan cluster. Tasche (2003) summaries two statistic that considered the correlation between defaults, namely “granularity adjustment approach” and “moment matching” approach, under internal rating based model.

In the context of evaluating VaR models, the test are against losses directly rather than PD. Lopez and Saidenberg (2000) discuss statistical tools used in this issue, for example, they refer likelihood ratio (LR) statistic in binomial method to evaluating the forecasted critical value of losses.

8.2 Test Strategies

There are two strategies to implement above tests. One is back test, the other is stress test. Back test (or backtesting) evaluates the model’s expected outcomes based on historical data. Whereas, stress test (or stress-testing) is examine the model’s expected outcomes under extreme conditions.

In recent articles, only Coppens et al. (2007) refer to back test and propose a simple mechanism to check the performance of credit rating system estimate the probability of default by applying traffic light approach, which is a simplified back test incorporated the above mentioned statistical tests, in their framework.

In contrast, an amount of studies on stress test have emerged after Basel II requires banks to conduct systematic stress tests on their potential future minimum capital. Stress test is an assessment to determine if the largest U.S. financial organizations have sufficient capital buffers to withstand the recession and the financial market turmoil. Two macroeconomic scenarios are applied, one based on baseline conditions and the other with a more pessimistic expectation. The recent stress tests are mostly integrated with macroeconomic credit risk models.

Cihak et al. (2007) provide a brief overview of stress tests applied by the Czech National Bank. They put their emphasis in introducing a model-based macro stress test and derive scenarios according to the forecast. Both Jokivuolle et al. (2008) and Valentinyi-Endresz and Vásáry (2008) adopt Wilson (1997a, 1997b)'s macro model and generate scenarios though Monte Carlo simulation. Fong and Wong (2008) extend macro stress testing under mixture vector autoregressive (MVAR) model, which assume either default rate or macroeconomic variable is a mixture normal distribution, and apply Monte Carlo simulations as well.

8.3 Evaluation of Bank's Internal Rating

Recent interests of internal rating are mainly from the suggestion of internal rating based (IRB) approach by Basel II and focus on the following two aspects.

One is about how to design specific banks' internal ratings systems suited to Basel II. For example, Crouhy et al. (2001) suggest how an internal rating system could be organized according to their analysis of standardized external rating system, such as Moody's and S&P's. Fernandes (2005) shows the probability to "build a relatively simple but powerful and intuitive rating system for privately-held corporate firms".

The other one is about how the implementation of internal credit rating by banks will lead to differences in minimum capital requirements. Jacobson et al. (2006) compare the loss distributions computed by two of the largest Swedish banks with equally regulatory internal ratings and find that their results are widely different in many cases. They point out these differences may be due to the design of a rating system and the ways in which they are implemented. Van Roy (2005) considered another possible reason for these differences associated with which external rating agency the bank selects. Because implementing the internal rating by banks generally calibrate their assessments to existing external ratings, differences of opinion among external raters may also cause differences of opinion among internal ratings systems and thus lead to different results in internal rating based approach.

9. Studies on SME

Before 2000, few studies have been devoted to modeling credit risk in small and medium enterprises (SMEs) or retail section. The conclusions from the credit risk models were mostly facing to wholesale commercial loans at that stage. This situation is partly because of the data availability, which we discuss in Section 4, and it has been changed in recent studies. 5% of all articles concentrate on SME loans study and more empirical study is based on dataset of SME loans. This is not only because of the quickly development of SME loan business, but also the suggestion in Basel II.

Historically, discriminant analysis and logistic regression have been the most widely used methods for constructing scoring systems for SME. Several studies have proposed to use some new models. Bharath and Shumway (2004) employ a time-dependent proportional-hazards model to evaluate the predictive value of the Merton structural model. Dietsch and Petey (2002, 2004), basing on large samples of French and German SMEs, suggest to apply a one-factor ordered probit model to catch properties of further losses in SME portfolio. They show the PD of SME is positively correlated rather than negatively to firm's asset value. What's more, both of these two studies mention correlations are weak between the SMEs' asset prices, especially for the SMEs with small size. Their result suggests that the properties of SME loans differ according to company scale, and there should be size-specified models even if inside SME.

10. Conclusion

During the past decade, two remarkable things happened in the field of credit risk modeling. One is the proposal of Basel II, whose influence have been considered by many researchers, including Repullo and Suarez (2004), Sironi and Zazzara (2003), Riportella et al. (2008) and so on. The other is the increasing availability of longitude data in this decade provides convenience to do dynamic study on credit risk. These two changes brought important consequences for banks, bank supervisors as well as for the direction of academic works. In this paper, we review the recent development in credit risk modeling, and the main conclusions are in the following.

1. It can be noted that current focuses on credit risk modeling have shifted from static individual loan models to dynamic portfolio models. Credit scoring system model is no longer the dominant model in this field. Studies make more efforts to consider the correlations between default and business cycle, and between defaults in portfolios. As a result, macro-economic variables are added to different modeling frameworks, which reduce the importance of accounting variables.

2. Earlier studies only model the PD, while this situation has been reversed in term recent increasing works on LGD and RR. This is partly because the assumption in traditional analysis that LGD is a constant is widely doubted based on recent

empirical results. Nowadays, more and more works are dedicated to modeling the distribution of LGD and PD simultaneously and the correlation between them.

3. Factor models have increased significantly in the number of applied studies. As a suggestion of internal rating based (IRB) approach in Basel II, factor models, especially, single factor model begin to appear frequently in academic works and step by step become the largest part of studies on credit risk modeling.

4. Another observation is that there is increasing concern on modeling the credit risk of SMEs. The accepted distinction between the properties of retail and cooperate loans lead to the development of sector-specific models.

5. As we proved in the methodology part, JEL codes have become the standard to classify researches in credit risk modeling.

6. A few JEL codes, namely (C14 OR C15 OR C23 OR C24 OR C32 OR C4 OR C5) OR (G21 AND (G28 OR G33)), have historically been used by researchers to labeled their contribution on credit risk modeling. We suggest use these JEL codes to label the coming studies on credit risk modeling so that the future literature reviews will be easier

7. No consensus on definitions of the three key parameters, being PD, LGD and correlation has emerged. However, we note that there are some conclusions made on how to choose the definitions of these parameters, according to data availability, property of loan and its study aim. In this sense, we suggest further empirical studies to show how they use the definitions of PD and LGD

GLOSSARY

Source: 1. *Saunders, A.., Allen, L., 2002. Credit risk measurement: New Approaches to Value at Risk and Other Paradigms, second edition John Wiley & Sons, Inc., New York.;*
2. Wikipedia. http://en.wikipedia.org .

Basel Accord, BCBS, BIS Basel Accord refers to the banking supervision Accords (recommendations on banking laws and regulations), Basel I and Basel II issued by the Basel Committee on Banking Supervision (BCBS). They are called the Basel Accords as the BCBS maintains its secretariat at the Bank of International Settlements (BIS) in Basel, Switzerland and the committee normally meets there. BCBS is sometimes referred to as the BIS Committee after its meeting location. However, the BIS and the Basel Committee remain two distinct entities.

Black-Scholes- Merton (BSM) model BSM model refers to a model proposed by Robert C. Merton in 1974 for assessing the credit risk of a company by characterizing the company's equity as a call option on its assets with a strike price equal to the face value of the debt. The company defaults if the value of its assets is less than the promised debt repayment at time T. This European call option can be pricing by Black-Scholes model, which is first articulated by Fischer Black and Myron Scholes in 1973.

Book Value of Liability The value of a liability according to its balance sheet account balance.

Chapter 11 Bankruptcy A chapter of the United States Bankruptcy Code, which permits reorganization under the bankruptcy laws of the United States. In the case of a corporation, reorganization occurs under the existing management.

Collateralized loan obligations (CLOs) A form of securitization where payments from multiple middle sized and large business loans are pooled together and passed on to different classes of owners in various tranches. CLO is a special purpose vehicle (SPV).

Conditional Value at Risk (CVaR) Also known as "Mean Excess Loss", "Mean Shortfall" and "Tail VaR".A risk assessment technique often used to reduce the probability a portfolio will incur large losses. This is performed by assessing the likelihood (at a specific confidence level) that a specific loss will exceed the value at risk. Mathematically speaking, CVaR is derived by taking a weighted average between the value at risk and losses exceeding the value at risk.

Credit cycle A cycle involving the access to credit by borrowers. Credit cycles first go through periods in which funds are easy to borrow; these periods are characterized by lower interest rates, lowered lending requirements and an increase in the amount of available credit. These periods are followed by a contraction in the availability of funds. During the contraction period, interest rates climb and lending rules become more strict, meaning that less people can borrow. The contraction period continues until risks are reduced for the lending institutions, at which point the cycle starts again.

Credit Default Insurance The use of a financial agreement - usually a credit derivative such as a Credit Default Swap, total return swap, or credit linked note - to mitigate the risk of loss from default by a borrower or bond issuer.

Credit rating An assessment of the credit worthiness of individuals and corporations. It is based upon the history of borrowing and repayment, as well as the availability of assets and extent of liabilities.

Credit spread The spread between Treasury securities and non-Treasury securities that are identical in all respects except for quality rating.

Default Premium The additional amount a borrower must pay to compensate the lender for assuming default risk.

Default probability (PD) The degree of likelihood that the borrower of a loan or debt will not be able to make the necessary scheduled repayments. Should the borrower be unable to pay, they are then said to be in default of the debt, at which point the lenders of the debt have legal avenues to attempt obtaining at least partial repayment. Generally speaking, the higher the default probability a lender estimates a borrower to have, the higher the interest rate the lender will charge the borrower (as compensation for bearing higher default risk).

Default risk The risk that companies or individuals will be unable to pay the contractual interest or principal on their debt obligations.

Expected Default Frequency (EDF) Usually the default probability calculated for a one year horizon is called as Expected Default Probability. It is one of the common identifier used along with credit ratings of the client.

Exposure at default (EAD) A total value that a bank is exposed to at the time of default. Each underlying exposure that a bank has is given an EAD value and is identified within the bank's internal system. Using the internal ratings board (IRB) approach, financial institutions will often use their own risk management default models to calculate their respective EAD systems.

Loss given default(LGD) Equal to one minus the recovery rate (RR) in the event of default. The amount of funds that is lost by a bank or other financial institution when a borrower defaults on a loan. Academics suggest that there are several methods for calculating the loss given default, but the most frequently used method compares actual total losses to the total potential exposure at the time of default. Of course, most banks don't simply calculate the LGD for one loan. Instead, they review their entire portfolio and determine LGD based on cumulative losses and exposure.

Off-balance sheep financing A form of financing in which large capital expenditures are kept off of a company's balance sheet through various classification methods. Companies will often use off-balance-sheet financing to keep their debt to equity (D/E) and leverage ratios low, especially if the inclusion of a large expenditure would break negative debt covenants.

Option A financial derivative that represents a contract sold by one party (option writer) to another party (option holder). The contract offers the buyer the right, but not the obligation, to buy (call) or sell (put) a security or other financial asset at an agreed-upon price (the strike price) during a certain period of time or on a specific date (exercise date).

Recovery rate(RR) See the term **Loss given default**.

Risk based capital (RBC) requirement A stated requirement of liquid reserves placed upon banks and institutions that deal in risky ventures.

Risk-adjusted return A concept that refines an investment's return by measuring how much risk is involved in producing that return, which is generally expressed as a number or rating. Risk-adjusted returns are applied to individual securities and investment funds and portfolios. There are five principal risk measures: alpha, beta, r-squared, standard deviation and the Sharpe ratio. Each risk measure is unique in how it measures risk. When comparing two or more potential investments, an investor should always compare the same risk measures to each different investment in order to get a relative performance perspective.

Subprime loan A type of loan that is offered at a rate above prime to individuals who do not qualify for prime rate loans. Quite often, subprime borrowers are often turned away from traditional lenders because of their low credit ratings or other factors that suggest that they have a reasonable chance of defaulting on the debt repayment.

Value-at-Risk (VaR) VAR is a technique used to estimate the probability of portfolio losses based on the statistical analysis of historical price trends and volatilities.

Working capital Also known as "net working capital", or the "working capital ratio". A measure of both a company's efficiency and its short-term financial health. The working capital ratio is calculated as: Working Capital= Current Asset- Current Liability.

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APPENDIX

Appendix 1 Ordered logit regression results

polr(formula = pf ~ NoJel, data = jel.t)

Coefficients:

	Value	Std. Error	t value
NoJel	-1.0638	0.42893	-2.4802

Intercepts:

	Value	Std. Error	t value
Not interested|Little interested	-0.8956	0.3067	-2.9197
Little interested|Interested	0.4280	0.2948	1.4518
Interested|Very interested	0.7618	0.3068	2.4827

Residual Deviance: 192.4157

AIC: 200.4157

polr (formula = Preference ~ Va1 + Va2, data = subset (jel.t, NoJel ==0))

Coefficients:

	Value	Std. Error	t value
Va1	3.5372	0.9462	3.7384
Va2	1.8148	0.8319	2.1815

Intercepts:

	Value	Std. Error	t value
Not interested |Of little interest	-0.2152	0.3816	-0.5640
Of little interest |Interested	1.8234	0.5098	3.5766
Interested |Very interested	2.2259	0.5680	3.9192

Residual Deviance: 86.60887

AIC: 96.60887

		Forecasting Preference			
Real Preference		Not interested	Of little interested	Interested	Very interested
Not interested	Mean:	0.3630	0.3883	0.0518	0.1969
Very interested	Mean:	0.0722	0.1961	0.0598	0.6720