Modeling and forecasting China's GDP data with time series models

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Abstract

The gross domestic product (GDP), a basic measure of an economy's economic performance, is the market value of all final goods and services produced within the borders of a nation in a year. In this paper, the features of quarterly data of China's GDP obtained from aggregated annual data of the National Bureau of Statistics of China starting from 1962 to 2008 are studied. Testing for existing a data break in the data is carried out by Chow test. An evidence of a data break point is found between the 4th quarter of 1977 and the 1st quarter of 1978. To model the GDP, a class of ARIMA(autoregressive integrated moving average) models is built following Box-Jenkins method. Finally, we use the fitted ARIMA model to do an out-off-sample forecasting. Our forecasting value of the 1st quarterly of 2009 is 71054.8 hundred million Yuan, while the real value is 68745 hundred million Yuan which is published by National Bureau of Statistics of China.

Key words: ARIMA model, Box-Jenkins method, Chow test.

Table of Contents

1 Introduction

The gross domestic product (GDP), a basic measure of an economy's economic performance, is the market value of all final goods and services produced within the borders of a nation in a year. GDP can be defined in three ways, all of which are conceptually identical. First, it is equal to the total expenditures for all final goods and services produced within the country in a stipulated period of time (usually a 365-day year). Second, it is equal to the sum of the value added at every stage of production (the intermediate stages) by all the industries within a country, plus taxes less subsidies on products in the period. Third, it is equal to the sum of the income generated by production in the country in the period, that is compensation of employees, taxes on production and imports less subsidies, and gross operating surplus (or profits).

The economy of the People's Republic of China is the second largest in the world after that of the United States with a GDP of $7.8 trillion (2008) measured on a purchasing power parity (PPP) basis. China has been the fastest-growing major nation for the past quarter of a century with an average annual GDP growth rate above 10%.

China's importance in the world today is reflected through its role as the world's third largest economy nominally and a permanent member of the UN Security Council as well as being a member of several other multilateral organizations including the WTO, APEC, East Asia Summit, and Shanghai Cooperation Organization. Since the introduction of market-based economic reform in 1978, China has become one of the world's fastest growing economies and the world's second largest exporter and the third largest importer of goods. Rapid industrialization has reduced its poverty rate from 53% in 1981 to 10% in 2004.

In this paper, based on the time series data of China's GDP from 1962 to 2008, we consider the class of ARIMA (autoregressive integrated moving average) models. We aim at revealing the regularity of China's GDP growth, using the fitted ARIMA model to do an out-off-sample forecasting. On the other hand, Chow test is applied to test for the presence of a break point. Following from Chow test, there is a data break point between the 4th quarter of 1977 and the 1st quarter of 1978.

2 Models and methodology

At the very beginning, some models and statistical methodology will be introduced.

2.1 ARMA Models

ARMA models are used in time series analysis to describe stationary time series and to predict future values in this series. The ARMA model is a combination of an autoregressive (AR) model and a moving average (MA) model. The ARMA (p, q) model is as follow, where p is the order of the autoregressive part and q is the order of the moving average part (as defined below):

$$\text{ARMA (p, q)}: \; y_t = c + \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t \,, \;\; \varepsilon_t \sim \text{WN}\,(0, \sigma^2)$$

The ARMA model is combined with two parts: Autoregressive Model and Moving Average Model.

$$\text{AR(p)}: \; y_t = c + \sum_{i=1}^{p} \varphi_i y_{t-i} + \varepsilon_t \,, \;\; \varepsilon_t \sim \text{WN}\,(0, \sigma^2)$$

$$\text{MA(q)}: \; y_t = \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t \,, \;\; \varepsilon_t \sim \text{WN}\,(0, \sigma^2)$$

The error terms ε_t are generally assumed to be independent identically-distributed random variables.

2.2 ARIMA Models

In statistics and econometrics, and in particular in time series analysis, an autoregressive integrated moving average (ARIMA) model is a generalization of an autoregressive moving average (ARMA) model. In theory, the most general class of models for forecasting a time series which can be stationary by transformations such as differencing and logging. ARIMA models form an important part of the Box-Jenkins approach to time-series modeling. A non-seasonal ARIMA model is classified as an ARIMA (p, d, q) model, where:

p is the number of autoregressive terms

d is the number of non-seasonal differences

q is the number of moving average lags

$\{y_t\}$ is said to be ARIMA (p, d, q) if

$$(1-L)^d \phi^*(L) y_t = c + \theta(L)\varepsilon_t \text{, where:}$$

$\phi^*(L)$ is defined in $\phi(L) = (1-L)\phi^*(L)$, $\phi^*(z) \neq 0$ for all $|z| \leq 1$. And $\theta(L)$ is defined in $\theta(z) \neq 0$ for all $|z| \leq 1$.

When the process $\{y_t\}$ is stationary if and only if d=0, in which case it reduces to ARMA(p, q) process : $\phi(L)y_t = c + \theta(L)\varepsilon_t$, ε_t ~ WN $(0, \sigma^2)$

2.3 Chow test

Chow test is a statistical and econometric test of whether the coefficients in two linear regressions on different data sets are equal. The Chow test was invented by economist Gregory Chow. In econometrics, the Chow test is most commonly used in time series analysis to test for the presence of a structural break. Suppose that we model our data as:

$$y_t = a + bx_{1t} + cx_{2t} + \varepsilon$$

If we split our data into two groups, then we have

Model 1: $y_t = a_1 + b_1 x_{1t} + c_1 x_{2t} + \varepsilon$ and model 2: $y_t = a_2 + b_2 x_{1t} + c_2 x_{2t} + \varepsilon$

The null hypothesis of the Chow test asserts that $a_1 = a_2$, $b_1 = b_2$, and $c_1 = c_2$.

Let S_c be the sum of squared residuals from the combined data, S_1 be the sum of squares from the first group, and S_2 be the sum of squares from the second group. N_1 and N_2 are the number of observations in each group and k is the total number of parameters. Then the Chow test statistic is

$$\frac{(S_C - (S_1 + S_2))/(k)}{(S_1 + S_2)/(N_1 + N_2 - 2k)}$$

The test statistic follows the F distribution with k and $N_1 + N_2 - 2k$ degrees of freedom. The model in effect uses an F-test to determine whether a single regression is more efficient than two separate regressions involving splitting the data into two sub-samples. If the parameters in the above models are the same: $a_1 = a_2$, $b_1 = b_2$, and $c_1 = c_2$, then models1 and 2 can be expressed as a single model. That means there is no break point, so we don't need to split the data into two sub-samples. Furthermore, if observation is a large sample ($N_1 + N_2$ is large), the test statistics follows the χ^2 distribution with k degrees of freedom.

3 Modeling

3.1 Data description and modeling

The data employed in this paper is quarterly observations of China's GDP from 1962 to 2008. The data is obtained from National Bureau of Statistics of China. Firstly, the quarterly data of China's GDP from 1962 to 2008 are disposed and analyzed. We denote quarterly data of China's GDP by Pt. Figure1 presents the line graph of log(Pt), and $\log(P_t)$- $\log(P_{t-1})$.

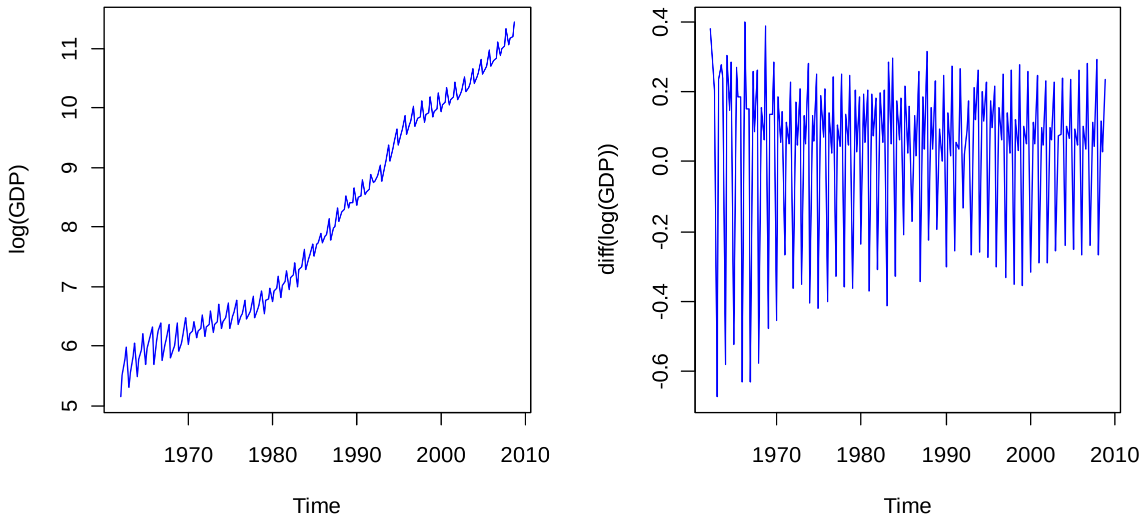


Figure1: the quarterly China's GDP Time series (1962:01-2008:04)

From the figure1, obviously, the left graph-log(GDP) is not stationary, according to the KPSS and ADF test. So we take first-order difference. We can see from the right graph, the time series seems stationary by taking first-order difference. We should ensure that the time series being analyzed is stationary before specifying a model. KPSS test is used for verifying whether or not the differenced series is stationary, and Augmented Dickey-Fuller test is used for verifying whether or not there is unit root. The p value of the KPSS test is greater than printed p-value (0.01), so it accepts the null hypothesis that x is level or trend stationary. This indicates that we may regard the differenced time series to be stationary. While the p value of ADF test is smaller than printed p-value, so it rejects the null hypothesis that x has a unit root. From above results, we find the differenced series is stationary and there is no unit root.

And then we plot the graphs for sample autocorrelations function and sample partial autocorrelations function. These two plots are useful in determining the p autoregressive terms and the q lagged error terms. Figure 2 following:

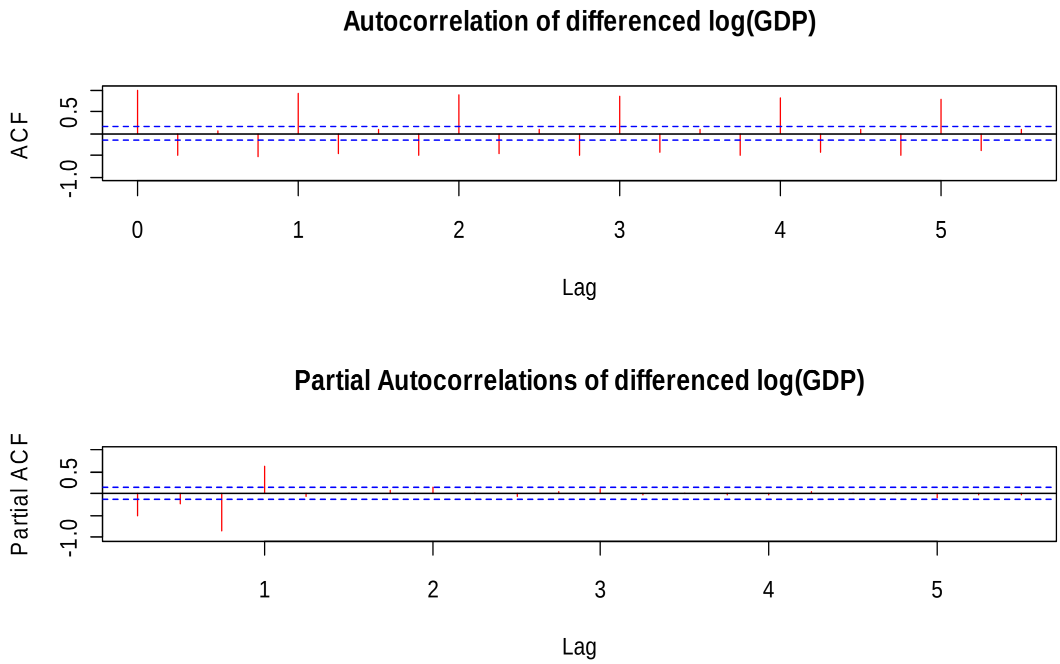


Figure 2: the quarterly China's GDP differenced series (1962:01-2008:04)

Figure 2 consists of plots of the ACF and the PACF for the quarterly China's GDP differenced series from 1962 to 2008, 95% confidence brands are plotted on the both panels. The ACF and the PACF of the differenced values obtained by using the transformation $x_t = y_t - y_{t-4}$.We apply the Box-Jenkins approach to choose the value p and q by ACF and PACF plot. From PACF plot, it significant spikes at lag4 and it could be viewed as dying out after lag4. It implies that we should build a AR(4) model, while MA(∞) from ACF plot. We compare the AIC of all the possible models and find out a model to fit the data better than others, which is the one has the lowest AIC value. The final model is ARIMA(4,1,0), we denote y_t= diff(log(GDP)), then $\{y_t\}$

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varphi_3 y_{t-3} + \varphi_4 y_{t-4} + \varepsilon_t, \quad \varepsilon_t \sim \text{WN}(0, \sigma^2)$$

3.2 Estimation

The final model ARIMA(4,1,0) is estimated by Maximum likelihood estimation(MLE) including estimation of the parameters: $\varphi_1, \varphi_2, \varphi_3, \varphi_4$. The estimated model is

$$\hat{y}_t = -0.0753\ y_{t-1} - 0.0774\ y_{t-2} - 0.0882\ y_{t-3} + 0.9020\ y_{t-4} + \varepsilon_t$$

s.e. (0.0318) (0.0300) (0.0321) (0.0304)

$\hat{\sigma}^2 = 0.003436$: log likelihood = 259.75, and AIC = -509.5

3.3 Ljung and Box

Ljung and Box statistic is computed as the weighted sum of squares of a sequence of autocorrelations, which is used to determine whether a time series consists simply of random values (white noise). Residual Sum of Squares (RSS) is the sum of squares of residuals. It is a measure of the discrepancy between the data and an estimation model. RSS $= \sum_{i=1}^{n} \hat{\varepsilon}_i^{\,2}$

Ho: No residual autocorrelation at lag 1 to m

$$LM(m) = n(n+2)\sum_{k=1}^{m}(n-k)^{-1} r_k^2(\hat{\varepsilon}) \xrightarrow{d} \chi^2(m-n-p), \quad \text{where } r_k(\hat{\varepsilon}) = \frac{\sum_{t=k+1}^{n} \hat{\varepsilon}_t \hat{\varepsilon}_{t-k}}{\sum_{t=k+1}^{n} \hat{\varepsilon}_t^{\,2}}$$

Where n is the sample size, k is lag of autocorrelation, and m is the number of lags being tested. Thus, for ARIMA(4,1,0) model, we can calculate the X-squared = 0.5434, p-value = 0.461 > 0.05. It means we can accept null hypothesis that the residuals are random, and they are independent and identically distributed. Figure3 shows the residuals of the model.

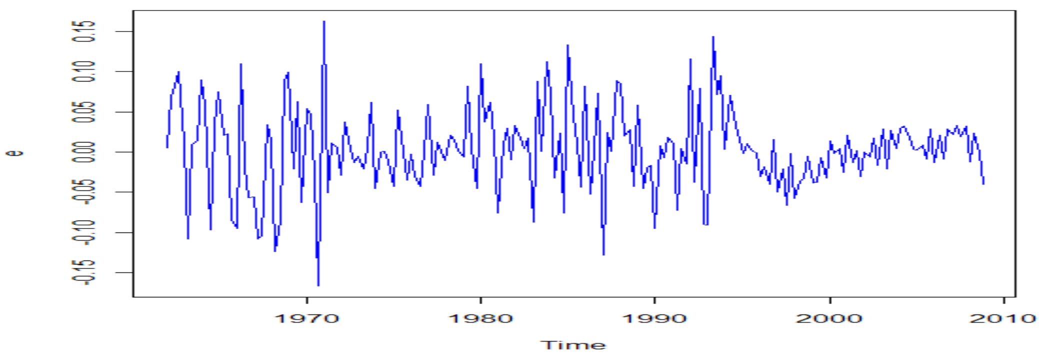


Figure3: the residuals of the first-order difference quarterly China's GDP series

3.4 Data break

Chow test is a statistical and econometric test of whether the coefficients in two linear regressions on different data sets are equal. In econometrics, the Chow test is most commonly used in time series analysis to test for the presence of a structural break. A series of data can often contain a structural break, due to a change in policy or sudden shock to the economy. Looking back in economy of the People's Republic of China history, we can know China's influence in the world economy was minimal until the late 1980s. At that time, economic reforms began after 1978, which generated significant and steady growth in investment, consumption and standards of living.

From the figure1, it shows clearly there is a break point on the 1st quarter of 1978, so we suppose it is a structural break. In order to know whether or not point 1978(01) is a structural break, we use Chow test to test it.

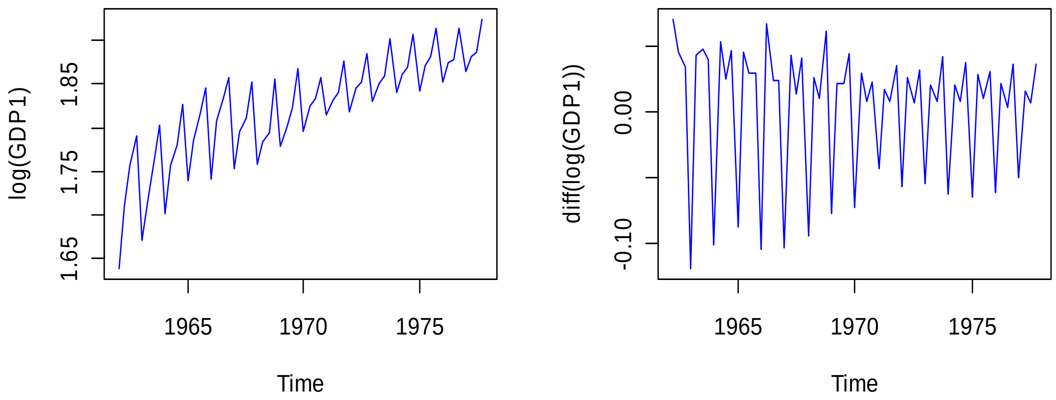
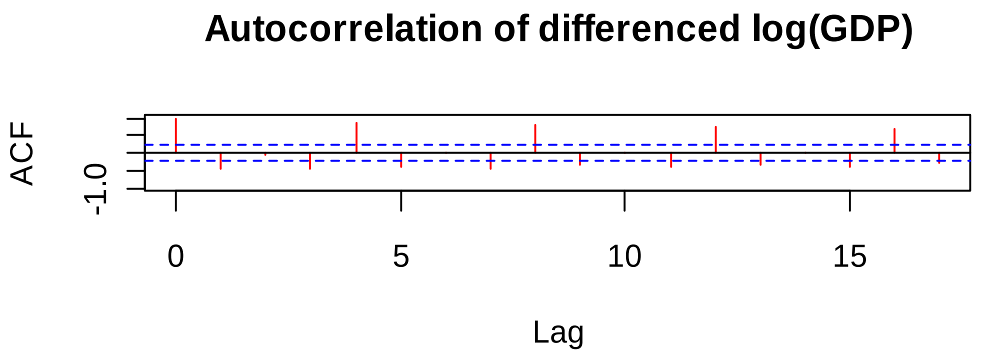


Figure4: the quarterly China's GDP Time series (1962:01-1977:04)

From the figure4, obviously, the left graph-log(GDP) is not stationary, according to the KPSS and ADF test. We can see from the right graph in figure4, the time series seems stationary by taking first-order difference. We use KPSS test and Augmented Dickey-Fuller test to verify whether or not the differenced series is stationary, and whether or not there is unit root. The p-value (0.1) of the KPSS test is greater than printed p-value, so it accepts the null hypothesis that x is level or trend stationary. It indicates that we may regard the differenced time series to be stationary. The p-value (0.01) of ADF test is smaller than printed p-value, so it rejects the null hypothesis that x has a unit root. From above results, we find the differenced series is stationary and there is no unit root.



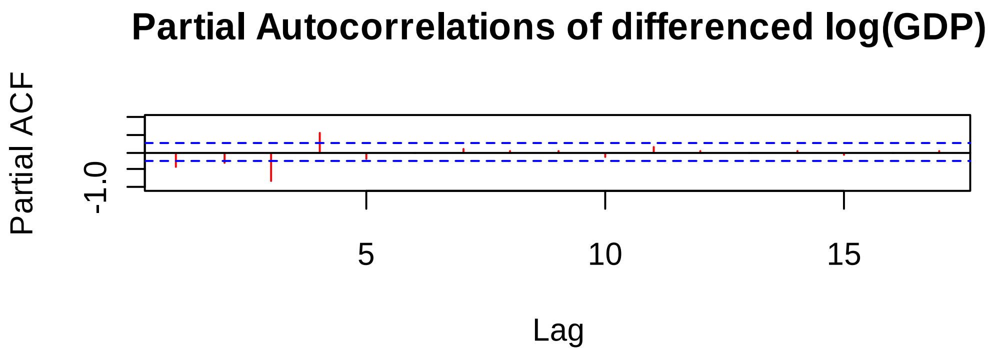


Figure5: the quarterly China's GDP Time series (1962:01-1977:04)

From figure5, it consists of plots of the ACF and the PACF for the quarterly China's GDP differenced series from 1962 to 1977, 95% confidence brands are plotted on the both panels. We apply the Box-Jenkins approach to choose the value p and q by ACF and PACF plot. From PACF plot, it spikes at lag5 and it could be viewed as dying out after lag5. It implies build a AR(5) model, while MA(∞) from ACF plot. We compare the AIC of all the possible models and find out a model to fit the data better than others, which is the one has the lowest AIC value. The final model is AR(5), we denote y_t = diff(log(GDP)), then { y_t }

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varphi_3 y_{t-3} + \varphi_4 y_{t-4} + \varphi_5 y_{t-5} + \varepsilon_t \text{, where } \varepsilon_t \sim \text{WN } (0, \sigma^2)$$

The estimated model is

$$\hat{y}_t = -0.2041 y_{t-1} - 0.3825 y_{t-2} - 0.4335 y_{t-3} + 0.5139 y_{t-4} - 0.1848 y_{t-5} + \varepsilon_t$$

s.e. (0.0919) (0.1201) (0.1223) (0.1268) (0.0867)

Through Ljung-Box test, the X-squared = 2.2308, p-value = 0.1353 > 0.05. It means we can accept null hypothesis that the residuals are random, and they are independent and identically distributed.

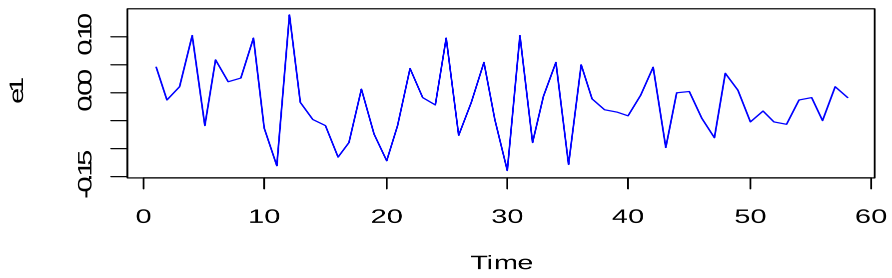


Figure6: the residuals of the first-order difference quarterly China's GDP Time series (1962:01-1977:04)

Figure6 shows the residuals of the model1.

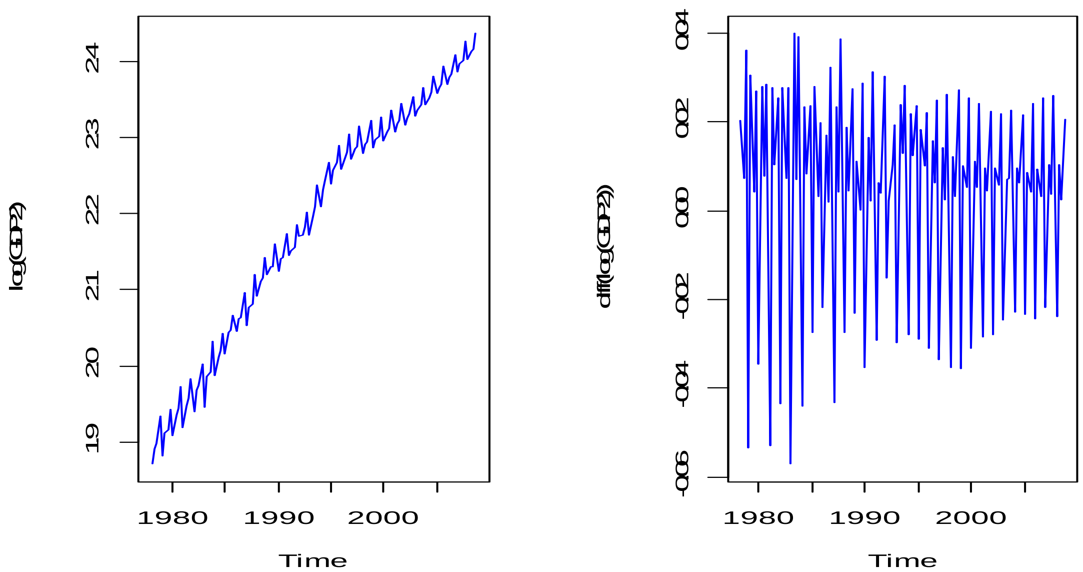


Figure7: the quarterly China's GDP Time series (1978:01-2008:04)

From figure7, log(GDP) is not stationary, but by taking first-order difference, the time series is stationary. We use KPSS test and Augmented Dickey-Fuller test to verify whether or not the differenced series is stationary, and whether or not there is unit root.

The p-value (0.1) of the KPSS test is greater than printed p-value, so it accepts the null hypothesis that x is level or trend stationary. It indicates that we may regard the seasonal differenced time series to be stationary. The p-value (0.032) of ADF test is smaller than printed p-value, so it rejects the null hypothesis that x has a unit root. From above results, we find the differenced series is stationary and there is no unit root.

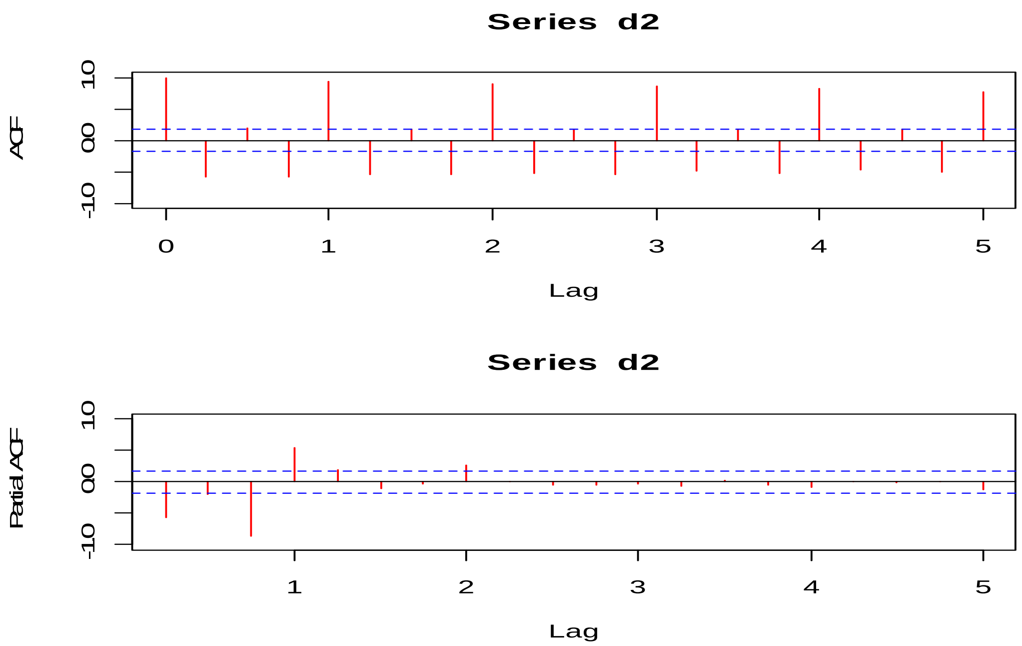


Figure8: the quarterly China's GDP Time series (1978:01-2008:04)

Figure8 consists of plots of the ACF and the PACF for the quarterly China's GDP differenced series from 1978 to 2008, 95% confidence brands are plotted on the both panels. We apply the Box-Jenkins approach to choose the value p and q by ACF and PACF plot. From PACF plot, it spikes at lag4 and it could be viewed as dying out after lag4. It implies build a AR(4) model, while MA(∞) from ACF plot. We compare the AIC of all the possible models and find out a model to fit the data better than others, which is the one has the lowest AIC value. The final model is ARIMA(4,1,0), we denote y_t = diff(log(GDP)), then $\{ y_t \}$

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varphi_3 y_{t-3} + \varphi_4 y_{t-4} + \varepsilon_t, \quad \varepsilon_t \sim \text{WN}\,(0, \sigma^2)$$

The estimated model is

$$\hat{y}_t = -0.0594\, y_{t-1} - 0.0397 y_{t-2} - 0.0474 y_{t-3} + 0.9178 y_{t-4} + \varepsilon_t$$

s.e. (0.0333) (0.0311) (0.0337) (0.0314)

$\hat{\sigma}^2 = 0.002800$: log likelihood = 182.28, AIC = -354.56

Through Ljung-Box test, the X-squared = 0.2258, p-value = 0.613 > 0.05. It means we can accept null hypothesis that the residuals are random, and they are independent and identically distributed. Figure9 shows the residuals of the model.

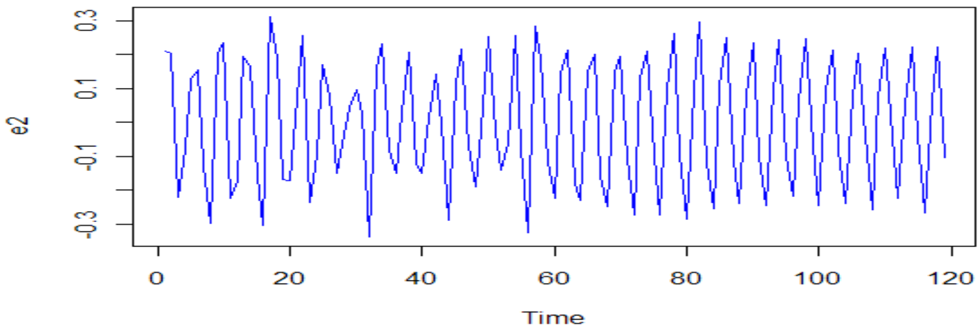


Figure9: the residuals of the first-order difference quarterly China's GDP time series (1978:01-2008:04)

Model1 (period from 1962:01 to 1977:04), and the time series process $\{y_t\}$ at a point (T_0) and before it

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varphi_3 y_{t-3} + \varphi_4 y_{t-4} + \varphi_5 y_{t-5} + \varepsilon_t \quad (t< T_0) \qquad (1)$$

Model2 (period from 1978:01 to 2008:04), ARIMA(4,1,0), let the model after the point (T_0)

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varphi_3 y_{t-3} + \varphi_4 y_{t-4} + \varepsilon_t \quad (T_0<t< T) \qquad (2)$$

Model, which is the whole model for the whole period from 1962:01 to 2008:04

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varphi_3 y_{t-3} + \varphi_4 y_{t-4} + \varepsilon_t \quad (t< T) \qquad (3)$$

The null hypothesis of the Chow test asserts that a structural change doesn't exist.

H_0: no data break point

H_1: there is a data break point

Let S_c be the sum of squared residuals from the combined data, S_1 be the sum of squares from the first group, and S_2 be the sum of squares from the second group. N_1 and N_2 are the number of observations in each group and k is the total number of parameters. Then the Chow test statistic is

$$\frac{(S_C - (S_1 + S_2))/(k)}{(S_1 + S_2)/(N_1 + N_2 - 2k)}$$

First, run the regression using all the data, before and after the structural break, and collect Sc. Second, run two separate regressions on the data before and after the structural break, collecting S_1 and S_2. Third, using these three values, calculate the test statistic from the above formula. And then, find the critical values in the F-test tables, in this case it has F (k, $N_1 + N_2 - 2k$) degrees of freedom. In this paper, T_0=64, the main test at the 5% level are summarized in Table1.

Table1: the results of Chow test about data break

Series	Number of observation(T)	k	F-value	reject or accept H_0
P_t	176	5	4.159448	reject H_0

From table1, F=4.16, while critical value F (5,176) < 3. That means F=4.16 > F (5,176), the result rejects the null hypothesis, so T_0 is structural break. The parameters in the model1 and model2 are not the same, so two separate models fit the data best and more efficient than a single model.

3.5 Forecasting

Forecasting is the process of estimation in unknown situations, which is commonly used in discussion of time-series data. We suppose a variable Y_{t+1} based on a set of variables X_t observed at data t. For intuitive notion, short-term forecasting should be more reliable than long-term forecasting. Using the model2 obtained above, we forecast 2009 China's GDP, which is outside the region of observation of the independent variable t, with the statistical software R. Estimated ARIMA(4,1,0):

$$\hat{y}_t = -0.0594\ y_{t-1} - 0.0397\ y_{t-2} - 0.0474\ y_{t-3} + 0.9178\ y_{t-4} + \varepsilon_t,\ \ \varepsilon_t \sim \mathrm{WN}\,(0,\sigma^2)$$

The point predicts, standard errors and 95% forecasting interval as following:

Table 2

	2009:01	2009:02	2009:03	2009:04
Pred (μ_0)	11.17121	11.28240	11.30123	11.52326
Se (σ)	0.05291867	0.07264969	0.08699945	0.09822259

The confidence interval is $e^{\mu_0 \pm z_{0.975} \cdot \sigma}$. We can obtain the forecasts and the prediction intervals for 1st quarter to 4th quarter of 2009 China's GDP. The observed 2009 China's GDP, forecasting and prediction intervals are shown in table3.

Table 3

	GDP(forecasted) (hundred million Yuan)	GDP(observed) (hundred million Yuan)	confidence interval (hundred million Yuan)
2009:01	71054.8	68745.0	[64054.40, 78820.27]
2009:02	79411.86	----------	[68872.50, 91564.03]
2009:03	80920.91	----------	[68234.92, 95965.43]
2009:04	101038.8	----------	[83345.26, 122488.50]

We find the forecasting GDP value of the 1st quarter of 2009 is 71054.8 hundred million Yuan, comparing with the observed value:68745 hundred million Yuan which is published by National Bureau of Statistics of China, the forecasted value is very closed to the observed value. And the observed value falls in the 95% confidence interval [64054.40, 78820.27], so it is a successful forecasting.

4 Conclusions

In this paper, we use statistical methods to collect, analyze, explain, and present the time series data of China's GDP from 1962 to 2008. Comparing with other models, ARIMA (4, 1, 0) model has been selected as the final model. On the other hand, Chow test is applied to test for the presence of a break point. Following from Chow test, an evidence of a data break point is found between the 4th quarter of 1977 and the 1st quarter of 1978. We also provide method for prediction and forecasting based on data, which is applicable and useful to government and business.

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Appendix I: R codes

```
rm(list = ls())
library(tseries)
gdp <- read.table("C:/007.txt",header=T)
y <- ts(gdp$GDP,start=c(1962,1),end=c(2008,4),frequency=4)
y <- log(y)
plot.ts(y)
kpss.test(diff(y))
adf.test(diff(y))
par(mfrow = c(2,1))
acf(diff(y))
pacf(diff(y))

##################################################################
##model
model <- arima(y,c(4,1,0))
e <- model$residuals
Box.test(e)
S <- sum(e^2)
plot.ts(e)

##seperate
y1 <- y[1:64]
y2 <- y[65:length(y)]
kpss.test(diff(y2))
adf.test(diff(y2))

##model1
par(mfrow = c(2,1))
op=par(mfrow = c(2,1))
acf(diff(y1),main="Autocorrelation of differenced log(GDP)",ylim=c(-1,1),
col="red")
pacf(diff(y1),main="Partial Autocorrelations of differenced log(GDP)", ylim=c(-1,1),
col="red")
model1 <- ar(diff(y1))
e1 <- model1$resid
e1=e1[6:63] #NA
Box.test(e1)
S1 <- sum(e1^2)

##model2
par(mfrow = c(2,1))
acf(diff(y2), ylim=c(-1,1), col="red")
```

```
pacf(diff(y2), ylim=c(-1,1), col="red")
model2=arima(y2,c(4,1,0))
e2 <- model2$resid
e2=e2[6:length(e2)]
Box.test(e2)
S2 <- sum(e2^2)

##Chow test
k <- 5
N1 <- length(diff(y1))
N2 <- length(diff(y2))
chowtest <- (S-S1-S2)*(N1+N2-2*k)/k/(S1+S2)
chowtest

######################################################################
predict<-predict(model2,n.ahead=4)
pd <- predict$pred
pe <- predict$se
exp(pd[1]) #a2009.1

q<-qnorm(0.975)
lb<-ub<-numeric(0)
for (i in 1:4){
lb[i]<-exp(pd-q*pe)[i]
ub[i]<-exp(pd+q*pe)[i]
}
lb
ub
```