Wind power production forecasting: Nonlinear approach

Author: Yang Lin
Supervisor: Changli He

School of Technology & Business Studies
Dalarna University
Abstract

This article considers the application of two nonlinear autoregressive models for wind power production forecasting. The first model is logistic smooth transition autoregressive (LSTAR) model and the second one is self-existing threshold autoregressive model. The article introduces the linearity tests against LSTAR model and SETAR model. Applying the Monte Carlo simulation method, theoretical limiting distributions and finite sample results is derived for the tests. Linearity tests are performed for wind power production series in Sweden and linearity are rejected for them. Therefore the empirical application to wind power production is modeled by LSTAR and SETAR models and forecast the production one month ahead. Comparing the forecasting results, LSTAR model overcomes SETAR model and Autoregressive model and thus can be applied in practical work.

Key words: Smooth Transition Autoregressive Model; Self-Existing Threshold Autoregressive Model; Linearity Test;
CONTENT

1. INTRODUCTION..............................................................................................................................................1
   1.1 BACKGROUND AND MOTIVATION................................................................................................................1
   1.2 LITERATURE REVIEW......................................................................................................................................2
   1.3 BRIEF INTRODUCTION ABOUT THIS ESSAY..................................................................................................5

2. DATA.....................................................................................................................................................................6

3. BASIC MODEL..................................................................................................................................................7
   3.1 LSTAR MODEL...............................................................................................................................................7
   3.2 SETAR MODEL...............................................................................................................................................7

4. TEST FOR NONLINEARITY............................................................................................................................8
   4.1 LINEARITY TEST THEORY..............................................................................................................................8
   4.2 ASYMPTOTIC DISTRIBUTION.........................................................................................................................9

5. MODEL SPECIFICATION ..........................................................................................................................10
   5.1 BUILDING LSTAR MODEL..........................................................................................................................10
   5.2 BUILDING SETAR MODEL..........................................................................................................................10

6. FORECASTING METHOD .........................................................................................................................13
   6.1 POINT FORECASTING OF LSTAR MODEL..................................................................................................13
   6.2 POINT FORECASTING OF SETAR MODEL..................................................................................................13
   6.3 COMPARING POINT FORECASTS BY ERROR MEASURES.........................................................................14

7. EMPIRICAL APPLICATION TO WIND POWER PRODUCTION DATA IN SWEDEN ......15
   7.1 AUTOREGRESSIVE MODEL SPECIFICATION ............................................................................................15
   7.3 LSTAR MODEL SPECIFICATION.............................................................................................................17
   7.4 SETAR MODEL SPECIFICATION.............................................................................................................17
   7.5 MODELS FORECASTING.............................................................................................................................19

8. CONCLUSION AND DISCUSSION .............................................................................................................20

APPENDICES .......................................................................................................................................................24
1. Introduction

These days, the wind energy becomes the most predominate renewable energy and wind power production has increasing influence on electricity price. The main application for wind power forecasting is to reduce the need for balancing energy and reserve power, which are needed to integrate wind power within the balancing of supply and demand in the electricity supply system. And the other application is to provide forecasts of wind power feed-in for grid operation and grid security evaluation. The target of our essay is to forecast the wind power production in median-term in Sweden.

1.1 Background and motivation

The wind power is gaining increasing importance throughout the world. Such result is based on the revolution of the energy consumption worldwide. As energy consumption, mainly from fossil fuels, increased more than tenfold over the 20th century, the finite and heavy pollution nature of fossil and nuclear fuel materials is concerned as dominantly energy issues. However, it is increasingly concerned that the harm of emissions, such as $\text{CO}_2$ in the atmosphere, and serious damage on ecological sustainability. Therefore, it is essential to exploit renewable energy supply and enhance the efficiency of generation and consumption of energy. With advantages such as pollution-free, sustainable and widely utilization, wind energy becomes the most fastest-growing renewable energy source these years. Taking climate change and energy security into account, the delegates of the Copenhagen Climate Summit supported the development of wind energy exploitation for electricity generation and that will be substantial tendency around the world in the foreseeable future.

Specifically, Sweden holds some of the best wind conditions in the world. For instance, the costs have decreased and the expansion has finally caught wind. At present, Sweden plays a leading role in the wind energy exploitation (see Figure 1). Government introduced a tradable green certificate support system, which gives
producers of renewable electricity (wind, small hydro, biomass based CHP) economic support for every MWh they produce. At the same time, a majority of Swedish prefer use wind energy in daily life and support building wind power farm in Sweden. Therefore, our research not only contributes to the wind power farms which benefits from the result, but also to the sustainable development of the society and nature.

Then, the question comes to does wind power production truly influences the electricity price? Does the research on wind power production forecasting can really help the enterprises determining the electricity price? The answer is definitely ‘yes’. Simply speaking, when the supply of wind power increases, at the fixed demand point, the spot price on the power market tends to decrease. The influence of wind power production has relation with the time of the day. That is to say, when there is plenty of wind power production at noon, during the peak power demand, most of the available storage energy will be consumption. Consequently, the price will be strongly affected by the amount of production. On the other hand, during the night time, the demand declines steeply and most power is produced on base load plants. And the impact of wind power production on the price has the slacking tendency (see Figure 2). Moreover, the production costs of non-renewable energy are consecutively increasing and cost associated with converting wind energy to electricity is not expected to rise. Based on above analysis, our research has empirical significance on forecasting the wind power production and gives accordingly suggestion on determining the electricity price.

1.2 literature review

In the retrospect of wind power production forecasting history, two principal directions of researches on forecasting are statistics and meteorology fields and the history can be divided into three eras: before 1990s, during 1990s and after 2000. Here, the brief review of the development of wind power forecasting is demonstrated by the time-line.
At end of the 1970s, the importance of the forecasting wind power was realized by the electricity plants, and wind power production data was treated as time series data. Later, Notis(1987) developed a semi-objective method, which refines the output from weather services for model output statistics. Then, Geerts compared ARMA models with Kalman filter and suggested to employ other relative variables to improve the forecasting accuracy. At the end of 1990s, Bosasnyi models Kalman filter to forecast with one minute time scale and eventually he applied the forecasting for furling operation of machines.

During the 1990s, the boost of wind power production capacity motivated hundreds of wind power plants and relative scientists in both statistics and meteorology fields into the wind power forecasting research. Watson et al. employed numerical weather prediction (NWP) with model output statistics (MOS) to forecast wind speed and direction with horizon of up to 18h and an hourly time-step. Applied in the UK grid system case, they concluded that the NWP-MOS forecasting performed significantly better than that by the persistence. Then, Jensen et al. introduced the wind power prediction tool (WPPT), with a half hourly time-step and a forecast horizon of up to 36 hours. The breakthrough of their research was an upscaling method to estimate the total production in the ELSAM service area based on seven plants. And they also presented the structure of the software, the operational experiences and meteorological method to exogenous variables.

At the end of 1990s, Akylas et al. obtained slightly overcome persistence. They proved three classes’ models against the persistence: one is multi-variable regression which employed from meteorological masts, considering speed, temperature, pressure and pressure tendency; another is regression of meteorological forecasts over the time series from meteorological masts and the third one is corrections of meteorological forecasts for microscale effects applying wind field simulation.
When time goes to 21th century, it is prominent breakthrough that the development of tools for on-line operation and the integration between the mathematical and physical research methods (Costa et al., 2003). Before that, Sfetsos compared linear models with non-linear models (feed forward neural networks, Elman recurrent network and neural logic network) to forecast wind speed one hour ahead. His conclusion was all the non-linear models presented comparable RMSE and overcame the linear models, especially the neural logic network which was slightly superior to the others. Lange and Wald presented that the uncertainty on wind power forecast is related with wind speed and the whole weather situation.

In these years, Pinson and Kariniotakis presented a methodology system to estimate the risk of wind power forecasting, that is to say, they adjusted the confidence band of forecasting. Also, they presented the spread of NWPs from the idea of meteo-risk index and concluded that the NWP spatial resolution was the major importance in complex terrain. However, the previous work inspired Costa (2003) research group to introduce integration with both statistical and meteorological models. This mechanism of integration based on a tracking of the intersection point between the error curves of the models. Eventually, they extended the proposed meteorological models with a dynamic downscaling through mesoscale models.

Generally the idea based on using the meteorology method to forecast wind power production, has been applied for years. In calculating the wind power production it is necessary to construct some model to define the relation between meteorological elements about wind power and the production. Since the geometric conditions and production capacity of different electricity plants vary, it is hard to build a universe model presenting such varied relations. Therefore, this article focuses on the application of forecasting method and considers the production data as time series data in order to make the forecasting method clear and widely utilized.
1.3 Brief introduction about this essay

1.3.1 General problem and purpose

According to the review, the main challenges of wind power production forecasting can be summarized as following: the magnitude of fluctuations in power output; the uncertainties in forecasting climate change and meteorological disasters; the diversity characteristics of wind power in different areas and the criterion of evaluation the forecasting methods and results. In order to overcome these challenges, I investigate such characteristics of production daily data in Swedish and try some nonlinear models for forecasting.

1.3.2 Method

The LSTAR model describes a situation where the contraction and expansion phases of an economy may have rather different dynamics, and a transition from one to the other regimes may be smooth (More discussion about LSTAR model see Teräsvirta (1992)). And the SETAR model described a situation where the time series separately follow different autoregressive models based on logit transition function. That is to say, the features of the data changes on varied regimes. Following the previous researches, the nonlinear wind power production is proved as a fact and the characteristic is similar as the LSTAR and SETAR model described. This inspires me starting the research on whether the LSTAR model and SETAR model overcome linear model in wind power production forecasting.

1.3.3 Outline

The rest of the article is constructed as follows. In section 2 the data set is described by graphics and data transition is presented. In section 3, the linear against nonlinear test is introduced to determine whether the data set is nonlinear. Meanwhile, a theoretical asymptotic of the test is presented. In section 4 I introduced regime-switching models——the LSTAR and SETAR model. Section 5 provides empirical application on both modeling and forecasting results for production and compares the results. Section 6 concludes and makes suggestion for future work.
2. Data

The original data, provided by Svensk Energi’s database (the organization represents companies involved in the production, distribution and trading of electricity in Sweden), is hourly observations of Sweden wind power production from January 1st 2008 to September 30th 2009 (15336 observations). Figure 3 illustrates hourly changes of wind power production during the one and half year. In order to forecast wind power production in median-term, I sum up the original hourly data to daily data which contains 639 observations of Sweden wind power production.

In this article, the simple time series specifications without exogenous variables are comprised models. The calibration was performed in R software (nonlinear test, LSTAR model, forecasting error estimate) and S-plus (SETAR model and relative test). In addition, the data is summarized by the descriptive statistics method and four different time series $Y_t$, $\log(Y_t)$, $Y_t - Y_{t-1}$, $100 \times (\log(Y_t) - \log(Y_{t-1}))$ are calculated. The result is given by Table 1 and Figure 4. According to this result, the significant difference exists in the wind power production series. The reason must be that when lakes and rivers freeze in winter, the capacity of production electricity is decreased and the wind power production accordingly increases. Based on the summary, the logarithmic transformation is applied to forecasting because the data attains a more stable variance, compare to the others (see Table 1).

<table>
<thead>
<tr>
<th>Time Series</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>SEMean</th>
<th>Sk</th>
<th>Ku</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_t$</td>
<td>252.29</td>
<td>173701.4</td>
<td>56083.91</td>
<td>1663.62</td>
<td>0.77</td>
<td>-0.35</td>
</tr>
<tr>
<td>$\log(Y_t)$</td>
<td>5.53</td>
<td>12.07</td>
<td>10.53</td>
<td>0.04</td>
<td>-1.29</td>
<td>2.32</td>
</tr>
<tr>
<td>$Y_t - Y_{t-1}$</td>
<td>-38388.89</td>
<td>43628.94</td>
<td>328.24</td>
<td>423.91</td>
<td>-1.82</td>
<td>0.88</td>
</tr>
<tr>
<td>$100(\log(Y_t) - \log(Y_{t-1}))$</td>
<td>-314.65</td>
<td>336.76</td>
<td>0.31</td>
<td>2.39</td>
<td>-0.41</td>
<td>9.97</td>
</tr>
</tbody>
</table>

The whole data set, from the period January 1st 2008 to September 30th 2009 is used solely for linear against nonlinear test. And the data, from the period January 1st 2008 to August 31th 2009, is used for the nonlinear modeling. When forecasting the wind
power production thirty days ahead, the data, from the period October 1st 2009 to October 30th 2009, is used for out-of-sample forecasting evaluation.

3. Basic Model

3.1 LSTAR model

Consider the following LSTAR model introduced by Terväsvirta (1994)

\[ y_t = (\pi_1 + \pi_1' w_t) + (\pi_2 + \pi_2' w_t)G(z_t; \gamma, c_0) + u_t \]  \hspace{1cm} (1)

where \( u_t \sim \text{iid}(0, \sigma_u^2) \)

\[ w_t = (y_{t-1}, y_{t-2}, \ldots, y_{t-p}), \pi_j = (\pi_{j1}, \ldots, \pi_{jp}), j = 1, 2 \]

\[ G(z_t; \gamma, c) = \frac{1}{1 + \exp(-\gamma(y_{t-1} - c))}, \gamma > 0 \]  \hspace{1cm} (2)

and \( G(z_t; \gamma, c) \) is a transition function. The main property of LSTAR model is “Smooth transition” between two regimes instead of a sudden jump from one regime to the other. The transition function \( G(z_t; \gamma, c) \) is a monotonically increasing function of \( z_t \), \( z_t \) is the transition variable and here defined as \( y_{t-1} \) and \( c \) is the threshold value determining where the transition occurs. The slope parameter \( \gamma \) represents the speed of the transition from 0 to 1. Note that when \( \gamma \to \infty \) in (2), if \( y_t \leq c, G(z_t; \gamma, c) = 1 \) if \( y_t > c, G(z_t; \gamma, c) = 0 \).

So the equation (1) becomes a TAR(p) model. And When \( \gamma \to 0 \), equation (1) becomes an AR(p) model.

3.2 SETAR model

Consider a two-regime continuous time, self-exciting threshold autoregressive model of

\[ y_t = X_t \Phi^{(1)} \left( 1 - I(y_{t-d} > \gamma_1) \right) + X_t \Phi^{(12)} \left( 1 - I(y_{t-d} \leq \gamma_1) \right) + \varepsilon_t \]

where \( I(A) \) is the indicators function that is equal to 1 if \( A \) is logical true and equals to 0 otherwise, \( \varepsilon_t \sim \text{iid}(0, \sigma^2) \)

and there is only one non-trivial threshold \( \gamma_1 \).
The concept of SETAR model is first introduced by Tong(1983,1990) and it is easy to understand but rich enough to generate complex nonlinear dynamics. The main idea of SETAR model is: supposed under some regime, the series follows autoregressive model and it switches between two regimes depending on the value taken by a lag of \( y_t \), say, \( y_{t-d} \), so that \( d \) is the length of the delay, and here I denote \( d=1 \).

4. Test for nonlinearity

4.1 Linearity test theory

The test linearity against nonlinearity described by Terväsvirta (1994) is used here. Following the suggestion of Davies (1977), Terväsvirta derived the test statistics when the unidentified values fixed. Since the first order expansion would lead a low power if the transition took place only in the intercept. Thus I expand the transition function into a third order Taylor series around \( \gamma = 0 \), merging terms and estimating the parameters again. The model term is constructed as follows:

\[
y_t = \left( (\pi_{10} + \pi_1'w_t) + (\pi_{20} + \pi_2'w_t) \right) \left[ \frac{1}{4}r(z_t - c) + \frac{1}{48}r^3(z_t - c)^3 \right] + \eta_t = c^* + \pi_1'w_t + \pi_2'w_t z_t + \eta_t
\]

Test linear against non-linear hypothesis are:

\[
H_{20} : \pi_1' = \pi_2' = 0 \iff \gamma = 0
\]

\[
H_{21} : \pi_1', \pi_2' \text{ not be zero at the same time}
\]

In order to calculate the value of F statistics, I constructed:

Restricted model:

\[
y_t = \pi_{10} + \pi_1'w_t^*
\]

Unrestricted regression:

\[
y_t = \pi_{10} + \pi_1'w_t^* + \pi_2'w_t^*z_t
\]

Then, the statistics is

\[
F = \frac{(SSR_0 - SSR_1)/m}{SSR_0/(T-K)}
\]

Where \( SSR_0 \) is the sum of the residuals squares of the restricted model. \( SSR_1 \) is the sum of the squared residual squares of the unrestricted model, \( m \) is the number of
restricted conditions, T is the sample size, and k is the number of parameters in the unrestricted model.

### 4.2 Asymptotic distribution

In order to find the asymptotic and finite sample critical values, we let T=1,000,000 to simulate a Brownian motion \( \dot{W}(\gamma) \) on [0,1] and the replications are set to 100,000 times. The finite critical values for the same test are obtained by simulation data from the model \( y_t = y_{t-1} + \mu_t \) where \( \mu_t \sim \text{n.i.d}(0,1) \) with desired sample size.

Then, with auxiliary regression and the OLS estimation to parameters, the value of F statistics can be calculate as,

\[
\begin{align*}
    b_T &= (X'X)^{-1}y_{2t}, X = (h_{2t}, h_{3t}) \\
    s^2 &= (y_{3t} - Xb_T)'(y_{3t} - Xb_T)/T - K \\
    F &= (Rb_T - r)'(s^2R(X'X)^{-1}R')^{-1}(Rb_T - r)/m
\end{align*}
\]

Where R is a diagonal matrix and r is a vector fixed by the null hypothesis. Repeat the above process 10,000 times to simulate OLS F statistic’s asymptotic distribution.

Finite sample critical values are reported in Table 2. The simulation result of F statistics is also presented in Figure 5 with sample size 100, 300, 500 and 1000 separately.

Table 2: Asymptotic and finite sample critical values

<table>
<thead>
<tr>
<th>Sample Size (T)</th>
<th>.99</th>
<th>.975</th>
<th>.95</th>
<th>.90</th>
<th>.05</th>
<th>.025</th>
<th>.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.07</td>
<td>1.65</td>
<td>2.16</td>
<td>2.68</td>
</tr>
<tr>
<td>200</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.07</td>
<td>1.59</td>
<td>2.06</td>
<td>2.56</td>
</tr>
<tr>
<td>300</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.07</td>
<td>1.57</td>
<td>2.03</td>
<td>2.53</td>
</tr>
<tr>
<td>500</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.07</td>
<td>1.56</td>
<td>2.04</td>
<td>2.51</td>
</tr>
<tr>
<td>600</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.07</td>
<td>1.55</td>
<td>2.02</td>
<td>2.50</td>
</tr>
<tr>
<td>1000</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.07</td>
<td>1.54</td>
<td>2.02</td>
<td>2.48</td>
</tr>
</tbody>
</table>
5. Model Specification

5.1 Building LSTAR model

After confirming the existence of the LSTAR type nonlinearity in a time series, I proceed to build a LSTAR model. This estimation involves choosing the transition variable and the delay parameter. Once the AR order and the transition variable have been chosen, LSTAR models can be estimated by nonlinear least squares (NLS):

\[
\Theta = \arg\min_{\gamma, c} \sum_{t} \hat{\varepsilon}_t^2
\]

Where

\[
\hat{\varepsilon}_t = y_t - \hat{\mathbf{w}}_t \hat{\Phi}
\]

\[
\hat{\mathbf{w}}_t = \begin{bmatrix}
W_t (1 - G(z_t; \gamma, c)) \\
W_t G(z_t; \gamma, c)
\end{bmatrix}
\]

\[
\hat{\Phi} = \begin{bmatrix}
\hat{\phi}_{(1)}^{(1)} \\
\hat{\phi}_{(2)}^{(2)}
\end{bmatrix} = \left[ \sum_t (\hat{\mathbf{w}}_t' \hat{\mathbf{w}}_t) \right]^{-1} \left[ \sum_t (\hat{\mathbf{w}}_t' y_t) \right]
\]

Note that the minimization of the NLS objective function is only performed over \( \gamma \) and \( c \) because \( \hat{\phi}_{(1)}^{(1)} \) and \( \hat{\phi}_{(2)}^{(2)} \) can be estimated by least squares once \( \gamma \) and \( c \) are known. Under the additional assumption that the errors are normally distributed, NLS is equivalent to maximum likelihood estimation.

5.2 Building SETAR model

I proposed the Hansen's Approach to test the threshold nonlinearity values and estimate the SETAR model by Hansen (1997). In following procedure, the threshold can be estimated together with other model parameter and valid confidence intervals can be constructed for the estimated thresholds. The limitation for Hansen’s approach is that only applied in two-regime SETAR model and only one threshold can be estimated.

5.2.1 The choice of the delay parameters

Firstly, the delay parameter and the thresholds is identified by applying least square to estimate the unknown parameters in (2) with given a start value of \( d \) and thresholds.
value. When the number of observations in each regime is enough, the least square is calculates as:

$$\hat{\psi}(\gamma) = \left( \sum_{t=1}^{n} x_t(\gamma)x_t(\gamma)^{'} \right)^{-1} \left( \sum_{t=1}^{n} x_t(\gamma)y_t \right)$$

In the given AR (p) model, Tsay proposed the delay parameter d should satisfies that

$$d = \arg \max_{\nu \in S} F(p, \nu)$$

where $F(p, \nu)$ is the F statistic of the auxiliary regression with AR(p) and the delay parameter is $\nu$, and $S$ is a set of $d$ to consider.

5.2.2 Identify the threshold value

There are two graphical tools for identifying the threshold values:

a. the scatter plot of standardized predictive residuals $\hat{e}_{x_1}$ from the arranged autoregressive versus the order threshold variable;

b. the scatter plot of the t-statistics of the recursive least squares (RLS) estimates from the arranged autoregressive versus the order threshold variable.

Both plots may exhibit structural breaks at the ordered threshold values.

5.2.3 Testing SETAR models by Hansen’s sup-LR test

In order to investigate the null hypothesis of no threshold nonlinearity is rejected or not, the likelihood ratio test assuming normally distributed errors can be calculated as:

$$F(\gamma_1) = \frac{RSS_0 - RSS_1}{\hat{\sigma}_1^2(\gamma_1)} = n'\frac{\hat{\sigma}_0^2 - \hat{\sigma}_1^2(\gamma_1)}{\hat{\sigma}_1^2(\gamma_1)}$$

Where $RSS_0$ is the residual sum of squares from SETAR(1)

$RSS_1$ is the residual sum of squares from SETAR(2) given the threshold $\gamma_1$

$\hat{\sigma}_0^2$ is the residual variance of SETAR(1) and $\hat{\sigma}_1^2$ is the residual variance of SETAR(2)

The above test is the standard F test since (2) is a linear regression. However, based on the threshold $\gamma_1$ is usually unknown, Hansen (1997) computed the following sup-LR test:

$$F_s = \sup_{\gamma_1 \in \gamma_d} F(\gamma_1),$$
by searching over all the possible values of the threshold variable \( y_{t-d} \). In practice, to ensure each regime has a non-trivial proportion of observations, a certain percentage of \( Y_d \) at both ends are usually trimmed and not used. In this article, the assumption is ten percent of \( Y_d \) satisfies this condition.

The sup-LR test has near-optimal power as long as the error term is identical independent distributed (iid). If the error is not iid, the F test needs to be replaced by Lagrange multiplier test. Hansen shows that the asymptotic distribution may be approximated by a bootstrap procedure in general and gave the analytic form of the asymptotic distribution for testing against SETAR(2) models.

In practice, to test the threshold nonlinear in daily production, I proposed the same AR(2) specification and choose the threshold variable to be \( z_t = y_{t-1} \) as in Tsay’s F test. Note that the optional argument is used to trim 10% observations at both ends of \( Y_d \) and bootstrap is used to set the number of bootstrap simulations for computing the p-value of the test. If the null hypothesis of no threshold nonlinearity is strongly rejected, the SETAR model can be constructed then.

5.2.4 Sequential estimation of SETAR models

After confirming the existence of threshold nonlinearity, Hansen (1997) proposed to estimate the threshold value \( \gamma_1 \) together with \( \phi \) using the least square method

\[
\hat{\gamma}_1 = \arg\min_{\gamma_1 \in Y_d} \hat{\sigma}^2 (\gamma_1, d)
\]

where \( \hat{\sigma}^2 (\gamma_1, d) \) is the residual variance of the LS estimate \( \hat{\gamma}_1 \) and the delay parameter \( d \)

Note that for the asymptotic inference on SETAR models to work correctly, each regime must have a non-trivial proportion of observations in the limit. Therefore, just as in computing Hansen’s sup-LR test, a certain percentage of \( Y_d \) at both ends are usually trimmed and not applied when searching for the value of \( \gamma_1 \).
6. Forecasting method

6.1 Point forecasting of LSTAR model

The forecasts from nonlinear models have to be generated numerically as discussed in Granger and Terväsvirta. Let

\[ y_t = f(y_{t-1}, \ldots, y_{t-p}; r) + \epsilon_t \]

Be a nonlinear model with additive error term, where \( r \) is the parameter vector and \( \epsilon_t \sim \text{iid}(0, \sigma^2) \). The one-step ahead point forecasting for \( y_{t-1} \) equals

\[ \hat{y}_{t+1|t} = f(y_{t}, \ldots, y_{t-p+1}; \hat{\theta}_t) \]

Where \( \hat{\theta}_t \) indicates that parameter estimates are obtained using observations up to time period \( t \). Then, I proposed the bootstrap approach (see Lundbergh and Terväsvirta 2002). In particular, I simulate \( N \) paths for the forecasting time series and here \( N=500 \). Then I obtained the \( h \)-period ahead point forecast as the average of these paths. For instance, the two step ahead point forecast is computed as

\[ \hat{y}_{t+2|t} = \frac{1}{N} \sum_{i=1}^{N} \hat{y}_{t+2|t}(i) = \frac{1}{N} \sum_{i=1}^{N} f(\hat{y}_{t+1|t} + \epsilon_t, y_t, \ldots, y_{t-p+2}; \hat{\theta}_t) \]

6.2 Point forecasting of SETAR model

For a SETAR model, the forecast task is to generate forecast of future values of the time series that is of interest. Forecasting from SETAR models can be easily computed using Monte Carlo simulations as long as the threshold value is known. A number of sampling schemes (see, e.g., West and McCracken, 1998, pp.818-9) could be adopted: we use a ‘recursive’ scheme. As the forecast regime moves forward out the sample, the model order is re-specified and the parameters is re-estimated in each period. Thus for an observation vector of length \( T \), the model is first specified and estimated on the data up to period \( T_0 \) \((T_0 < T)\) and a forecast (point, interval and density) of \( T_0 \) is made. The analysis is confined to one-step ahead forecasts.

\[ \hat{y}_{t+1|t} = X_t \Phi^{(1)} \left( 1 - I(y_{t-d} > \gamma_1) \right) + X_t \Phi^{(12)} \left( 1 - I(y_{t-d} \leq \gamma_1) \right) + \epsilon_t \]
\[ \hat{y}_{t+2} = (\phi_0 + \phi_1 \hat{y}_{t+1} + \cdots + \phi_p y_{t-p+1}) \left( 1 - I(y_{t-d} > \gamma_1) \right) + (\phi_0 + \phi_1 \hat{y}_{t+1} + \cdots + \phi_p y_{t-p+1}) \left( 1 - I(\hat{y}_{t+1} \leq \gamma_1) \right) + \epsilon_{t+2} \]

Then, the model is re-specified and re-estimated on data up to and including \( T_0 \), and forecasts of \( T_0 \) are made. This continues up to a forecast of \( T \) made from models specified and estimated on data up to \( T \). Thus we generate a sample of \( T_0 \), \( T_0 \) one-step forecasts.

### 6.3 Comparing point forecasts by error measures

In this part, different evaluation measures are utilized to access the forecasting performance of the three models. First I calculate the root-mean-squared forecast error (RMSFE). The formula of RMSFE is as follows:

\[
\text{RMSFE}(L) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (g(x_i) - \hat{g}(x_i))^2}
\]

Also, the mean absolute percentage error (MAPE) is measure of accuracy in a fitted time series model and it usually expresses accuracy as a percentage:

\[
\text{MAPE} = \frac{1}{n} \sum_{t=1}^{n} \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right|
\]

where \( Y_t \) is the actual value and \( \hat{Y}_t \) is the forecast value.

In the empirical application, the ratio of the RMSFE for a given forecast horizon, \( h=20 \) days, relative to the RMSFE of the linear AR(p) model with \( p \) selected by AIC, which is as benchmark and the ranks of the three forecasting models.
7. Empirical application to wind power production data in Sweden

7.1 Autoregressive model specification

7.1.1 Testing for stationary

The KPSS test is presented to investigate whether the logarithmic transformation data set is stationary. More accurately, the kind of stationary should be non-trend stationary during the test. The reasonable lag value is determined based on the above autoregressive model with lag equals to 1, the result of KPSS test is as follows:

<table>
<thead>
<tr>
<th>T=639</th>
<th>Yt</th>
<th>Log(Yt)</th>
<th>Diff(Yt)</th>
<th>100*(diff(log(Yt)))</th>
<th>Critical (5%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.367</td>
<td>0.1967</td>
<td>0.0137</td>
<td>0.0113</td>
<td>.463</td>
</tr>
</tbody>
</table>

According to the above result, the value of KPSS statistics are all smaller than the critical value under 5% significance. As the asymptotic distribution is left-tailed, so the null hypothesis is not rejected and the conclusion is the data is stationary. Thus, I can start to the modeling part.

7.1.2 Autoregressive model specification

According to Figure 6, the autoregressive function (ACF) decays very solely and remains significant even after 20 lags, while the partial autoregressive function (PACF) function is significant for the first four lags. This suggests that an autoregressive process with lag length ranging from 1 to 4 is estimated and the AIC statistics is summarized as follows:

<table>
<thead>
<tr>
<th>Lag</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-695.81</td>
</tr>
<tr>
<td>2</td>
<td>-694.14</td>
</tr>
<tr>
<td>3</td>
<td>-694.83</td>
</tr>
<tr>
<td>4</td>
<td>-692.97</td>
</tr>
</tbody>
</table>

Table 3 is clearly reported the results for the data: the first line refers to the lag length of autoregressive model and the second one to the AIC corresponding to that lag.
Since the value of AIC is slightly smaller than other models, we choose lag equals to one and take evaluation of AR (1). Applying the maximum likelihood estimation method and the data taking the logarithmic transformation:

\[ y_t = 1.6627 (0.0004) + 0.8424 y_{t-1} (0.001^*) + \mu_t \]

Where the \( y_t \) is the logarithmic transformation of wind power production daily data and \( \mu_t \) is assumed to be a sequence of identical independent random variables distributed according to Normality distribution. All estimated parameters are statistically significant at 5% significance level. Then we will check whether the autoregressive model is suitable for the logarithmic transformation data set.

### 7.2 Test for linearity

To test the linearity, we calculate the nonlinear F statistics we have derived in subsection applied into the wind power production data. The nonlinear Dickey-fuller F statistics is calculate for both the four time series and is listed in Table 5.

<table>
<thead>
<tr>
<th>Time series</th>
<th>F statistics</th>
<th>Critical (5%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_t )</td>
<td>2.7</td>
<td>2.04</td>
</tr>
<tr>
<td>( \log(y_t) )</td>
<td>134.24</td>
<td>2.04</td>
</tr>
<tr>
<td>( y_t - y_{t-1} )</td>
<td>3.968</td>
<td>2.04</td>
</tr>
<tr>
<td>( 100 \times (\log(y_t) - \log(y_{t-1})) )</td>
<td>31.553</td>
<td>2.04</td>
</tr>
</tbody>
</table>

Table 5 is clearly reported the results for the data: the first column is four time series, the second column refers to F statistics in each series and the third one to critical value corresponding to that series. The computer program give the critical value \( P(X > 2.04) = 1 - \alpha \) where \( \alpha = .05 \) (see Table 1). According to the critical value, the calculating results implies that in four cases we reject the null hypothesis of linearity, which means the wind power production data in Sweden can be specified as nonlinearity. And the data set with logarithmic transformation will be utilized in the following steps.
7.3 LSTAR model specification

7.3.1 LSTAR model estimation

The final model is estimated by nonlinear least squares including estimation of the parameter \( \gamma \) and \( c \). Terväsvirta (1994) proposed that transition \( G(z_t; \gamma, c) \) should be standardized to make \( \gamma \) scale-free. This implies dividing the exponent in \( G(z_t; \gamma, c) \) by the standard deviation of \( z_t = y_{t-1} \). Grid search method is used here to give final value of \( \gamma \) and \( c \) in both high and low regimes according to AIC (see Figure 7).

Here I presented the results of the logarithms transformation of daily wind power production is \( p = 2 \), \( z_t = y_{t-1} \) and the model should be:

\[
y_t = \left( 9.4122 - 0.1031y_{t-1} \right) + \left( -10.1001 + 1.6581y_{t-1} - 0.4985y_{t-2} \right) \times \left[ 1 + \exp\{10.04 \times (y_{t-1} - 9.0727)\} \right]^{-1} + \tilde{u}_t
\]

\( SSE = 142.3451, AIC = -879.402, BIC = -848.5193 \)

\( sk = -2.2987, kurtosis = 11.843, LJB = 0.7914(0.6732) \)

Where figures in parentheses below is the parameter estimates which denote for the p-value of the test. SSE is the estimated standard deviation of residuals; LJB test does not reject normality for residuals. The estimated value for the threshold value \( c \), \( c = 9.0727 \) shows the intermediate point between the wind power production increasing and decreasing. The estimate of \( \hat{\gamma} = 10.04 \), suggests the regime transition speed from one regime to the other and this result can be checked in Figure 4.

7.4 SETAR model specification

7.4.1 The estimation of delay parameter and threshold

Based on Figure 8, the both estimates are significant with t-statistics greater than 2 in absolute values in most cases. In addition, the trend in the t-statistics seems to have two breaks which occurs in the threshold variable is around 10.4. This suggests a SETAR(2) model with non-trivial threshold values: \( \gamma_1 = 10.4 \). For more exact study, the sup-LR nonlinearity test is presented in the next step.
7.4.2 The threshold nonlinearity test

In practical, we use Hansen sup-LR Nonlinearity test method by S+FinMetrics function in S-plus to test for the threshold nonlinearity and the null hypothesis is no threshold with the specified threshold variable under Maintained Assumption of Homoskedastic Errors.

Table 6: Nonlinearity Test: Threshold Nonlinearity

<table>
<thead>
<tr>
<th>Number of Bootstrap Replications</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trimming percentage</td>
<td>10%</td>
</tr>
<tr>
<td>Threshold Estimate</td>
<td>10.4121</td>
</tr>
<tr>
<td>F-test for no threshold</td>
<td>49.0198</td>
</tr>
<tr>
<td>Bootstrap P-Value</td>
<td>0</td>
</tr>
</tbody>
</table>

Note that the optional argument is used trim 10% observations at both ends of $Y_d$ and the number of bootstrap simulations is for computing the p-value of the test. The null hypothesis of no threshold nonlinearity is strongly rejected. The threshold value is determined as 10.4121 which is utilized in SETAR modeling.

7.4.3 SETAR model estimation

After choosing the delay parameter $d=0$ and the threshold equals to 10.4, the estimated model of the first difference of daily wind power is:

$$y_t = \begin{cases} 
0.4833y_{t-1} - 0.0288y_{t-2} + 4.9453 & y_{t-1} < z_t, \text{containing 35.32\% points} \\
0.14123y_{t-1} - 0.1306y_{t-2} - 0.6356 & y_{t-1} \geq z_t, \text{containing 64.68\% points}
\end{cases}$$

threshold: $z_t = y_{t-1}$, threshold = 10.4 AIC value = $-788$

residuals variance = .2861 MAPE = 3.237%

Note that AR coefficients for the first regime are estimated to be (0.48, 0.03) which appear to be significant, while the AR coefficients for the second regimes are estimated to be 0.41 and the constant term is not significant. In the SETAR model, the regime switching happens in the threshold value equals to 10.4 point. The logarithmic wind power production is in increasing tendency in both the high and low regimes. However, the increasing speed of low regime is less than high regime.
7.5 Models Forecasting

After building models, the next step is to forecasting wind power production for one month ahead. Before coming to out-sample point forecasting, I compare the three models by AIC, BIC and Mean Absolute Percentage Error. And the residuals of the three models are plotted in Figure 9-11.

Table 7 AIC value of models

<table>
<thead>
<tr>
<th>Model name</th>
<th>AR(2)</th>
<th>SETAR(2)</th>
<th>LSTAR(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIC</td>
<td>-695.8096</td>
<td>-787.7544</td>
<td>-917.7604</td>
</tr>
<tr>
<td>BIC</td>
<td>-686.8898</td>
<td>-760.995</td>
<td>-886.541</td>
</tr>
<tr>
<td>MAPE</td>
<td>3.61%</td>
<td>3.237%</td>
<td>2.871%</td>
</tr>
</tbody>
</table>

Based on Table 7, the LSTAR(2) overcomes the other two model in all the three statistics. The graphic is drawn to illustrate the forecasting result (see Figure 12, and Figure 13). According to Figure 12 and Table6, the LSTAR model is preferred to applied into the practice forecasting work due to its outperformance. When comparing with the residuals for these three models, the same conclusion is drawn.

Table 8 Forecasting result for 20 days ahead.

<table>
<thead>
<tr>
<th>Forecast Step</th>
<th>AR(1)</th>
<th>SETAR(2)</th>
<th>LSTAR(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.424339</td>
<td>8.962763</td>
<td>8.962842</td>
</tr>
<tr>
<td>3</td>
<td>9.742371</td>
<td>9.579083</td>
<td>9.316035</td>
</tr>
<tr>
<td>4</td>
<td>9.864751</td>
<td>8.365076</td>
<td>8.000203</td>
</tr>
<tr>
<td>5</td>
<td>9.967781</td>
<td>9.130477</td>
<td>8.73596</td>
</tr>
<tr>
<td>6</td>
<td>10.05452</td>
<td>12.5668</td>
<td>12.4011</td>
</tr>
<tr>
<td>7</td>
<td>10.12754</td>
<td>8.51814</td>
<td>8.468463</td>
</tr>
<tr>
<td>8</td>
<td>10.18902</td>
<td>8.916796</td>
<td>8.766144</td>
</tr>
<tr>
<td>9</td>
<td>10.24078</td>
<td>8.155572</td>
<td>7.906198</td>
</tr>
<tr>
<td>10</td>
<td>10.28435</td>
<td>10.51435</td>
<td>10.1645</td>
</tr>
<tr>
<td>11</td>
<td>10.32103</td>
<td>8.725795</td>
<td>8.315746</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>12</td>
<td>10.35191</td>
<td>9.570414</td>
<td>9.384557</td>
</tr>
<tr>
<td>13</td>
<td>10.37791</td>
<td>9.630541</td>
<td>9.574433</td>
</tr>
<tr>
<td>14</td>
<td>10.3998</td>
<td>9.425905</td>
<td>9.269183</td>
</tr>
<tr>
<td>15</td>
<td>10.41823</td>
<td>9.824574</td>
<td>9.568981</td>
</tr>
<tr>
<td>16</td>
<td>10.43374</td>
<td>8.756572</td>
<td>8.400545</td>
</tr>
<tr>
<td>17</td>
<td>10.44681</td>
<td>9.7702</td>
<td>9.36551</td>
</tr>
<tr>
<td>18</td>
<td>10.4578</td>
<td>8.626796</td>
<td>8.449886</td>
</tr>
<tr>
<td>19</td>
<td>10.46706</td>
<td>8.075284</td>
<td>8.022386</td>
</tr>
<tr>
<td>20</td>
<td>10.47485</td>
<td>9.27446</td>
<td>9.120675</td>
</tr>
</tbody>
</table>

In order to illustrate the forecast result, Figure 13 is plotted and the brief conclusion can be drawn that the LSTAR model forecasts more accurate than the SETAR model. There is five points cross the true value in the LSTAR forecast result, while the SETAR model holds only three points cross the true value line and the autoregressive model holds three points too. Then the forecasting evaluation is presented in Table 9.

**Table 9** Forecasting result evaluation for 30 days ahead

<table>
<thead>
<tr>
<th></th>
<th>AR(1)</th>
<th>SETAR(2)</th>
<th>LSTAR(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSFE</td>
<td>1.78214</td>
<td>1.72447</td>
<td>1.55810</td>
</tr>
<tr>
<td>MAPFE</td>
<td>0.19686</td>
<td>0.16248</td>
<td>0.14406</td>
</tr>
</tbody>
</table>

In Table 9, the LSTAR model outperformed both SETAR model and Autoregressive model based on RMSFE and MAPFE. That is to say, while nonlinear model outperformed linear model in the model evaluation section, the forecasting result is on the same line that LSTAR model can be used in the empirical forecasting work. The logarithmic data is calculate to the original form and the plot is drawn to investigate that the wind power production experiences the fluctuation and then on the rise tendency in the following 20 days.

**8. Conclusion**

In this paper, the feature of the daily wind power production of Sweden is tested from the period of Jan. 1st, 2008 to May 31st, 2009. I propose LSTAR model that
accommodates a nonlinear change in dynamics with transition function containing first ordered delay parameter of $z_t = y_{t-1}$ and also propose a SETAR model that represents a nonlinear change in dynamics with transition function divided into two regimes. The change of wind power production exhibits non-linearity features such as regime switching. An important advantage of our test is that they are computationally easy to carry out and in order to find the asymptotic distribution for the test; we generate theoretical critical value by Mont Carlo method which is derived in the test literature.

While nonlinear model outperformed linear model in the model evaluation section, the forecasting result is on the same conclusion that LSTAR model, which has smaller forecasting error, can be used in the empirical forecasting work.
REFERENCES


Appendices

Figure 1

![Bar chart showing the growth rate in 2009 and 2008 for different countries.](chart1)

- Turkey: 1.39
- China: 1.13
- Sweden: 0.48
- Poland: 0.41
- Canada: 0.40
- USA: 0.39
- France: 0.33
- Italy: 0.30
- United Kingdom: 0.28

Figure 2

![Diagram showing the relationship between price and energy demand/supply.](chart2)

- Price A (low wind)
- Price B (high wind)
- Demand: Night, Day, Peak
- Supply: Gas turbines, Condensing plants, CHP plants, Wind and nuclear

MWh
Figure 3 The plot of five time series

Figure 4 The plot of five time series
Figure 5 Simulation results of F statistics in sample size equals to 100, 300, 500, 1000.

Figure 6 Autocorrelation and Partial autocorrelation of logarithmic production.
Figure 7 The scatter plot of the t-statistics for choosing threshold value

Figure 8 grid search for threshold in LSTAR model specification
Figure 9 AR(1) model residuals

Figure 10 LSTAR(2) model residuals
Figure 11 SETAR model residuals

Figure 12 The forecasting result of the AR(1), SETAR(2) and LSTAR(2) model
Figure 13 The forecasting result of true value in the AR(1), SETAR(2) and LSTAR(2) model