Testing For Common Stochastic Trends

An application to Sweden's macroeconomic data

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June, 2010

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Abstract

Economic time series display many distinctive stylized facts, called features of these time series. Stochastic trends are one kind of features. Common feature are features shared in common if each individual time series has a same feature and their linear combination of them does not have the feature. This article will focus on detecting common stochastic trends. So the article will describe the procedures of testing for non stationary time series which aims at the case about unit root and linear co-integration. Augmented Dickey-Fuller test and Residual-Based tests will be involved in it. The empirical application is based on real economic data, consumption and income data of Sweden. It is concluded that each individual series has a stochastic trend, and there exist a linear co-integration relation.

Key Words: Common feature stochastic trends Linear Co-integration Consumption function Augmented Dickey-Fuller test Residual-Based Tests

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1. Introduction

1.1 Background

Economic time series display many distinctive stylized facts, called features of these time series. Those features of time series exhibit serial correlation, heteroscedasticity, skewness, nonlinearity and various other features. Common feature are features shared in common if each individual time series has a same feature and their linear combination of them does not have the feature. A modern theme of econometric research, investigation of existing common features among a group of economic time series, is important for two reasons: one is of direct economic interest, and the other is for modeling a system of macroeconomics. A co-integrated economic system, for example, is a system with a common stochastic trend.

Making research on stochastic trend that is unit root process is very meaningful. Nelson and Plosser (1982) had been studied 13 macroeconomics variables as unit root process in 1929's Great Depression. And they considered that many economic series are better characterized by unit roots than by deterministic time trends. So we continue to research the feature of unit root process in common, which means a very important case in common feature research, co-integration system. Besides cointegration theory contribute much into constructing econometric model using nonstationary series. If several variables are cointegrated, they can merge into a stationary time series.

Since detecting common stochastic trend is also a kind of finding common features. In this article, we will also introduce the procedures testing for common features. Engle and Kozicki had proposed whether features detected in single data series are actually shared in common. In this article, the definition of common feature put forward by Engle and Kozicki is used. Engle and Kozicki, in the paper testing for common features, had discussed different situations on testing for different common features.

The data used in this article is the consumption and income quarterly data from 1993:1 to 2009:1 of Sweden. So the theory of consumption function is introduced in this article to construct model. The marginal propensity to consume (MPC) is assumed to be positive. Thus, as income increases, consumption increases. However, Keynes mentioned that the increases (for income and consumption) are not equal. According to him, "as income increases, consumption increases but not by as much as the increase in income". The MPC of USA is about 0.9, while the China's is about 0.3, so what is the situation of Sweden? We will give the answer of this question.

1.2 Aims and Structure

The main aims of this paper are to detect how to find common stochastic trends and whether the consumption and income time series are cointegration. If they are, we will construct a model for consumption and income, and make forecasting to check whether the model works well.

In this article, we will first describe the data we use, and then we will detect the procedures to test for common stochastic trends, and introduce the methodology we need including testing for unit root of individual series and residual based test for cointegration. After that, we enter into the empirical application, and apply the theory into this example and conclude the results.

2. Data description

The data used in this article is the consumption and income quarterly data from 1993:1 to 2009:1 of Sweden. And most economic and financial time series are better characterized by an exponential trend than a linear trend. So we take the natural log-transformation first. And because the log value is small, we deal with it by multiplying 100, and the initial value for 1993:1 was then subtracted. The purpose of subtracting the constant $\log(\text{consumption}_{1993:1})$ and $\log(\text{income}_{1993:1})$ from each observation is to normalize each series to be zero for 1993:1 so that the graph is easier to read.

$$\text{consumption} = 100 \times [\log(\text{consumption}) - \log(\text{consumption}_{1993:1})$$

The same as income series:

$$\text{income} = 100 \times [\log(\text{income}) - \log(\text{income}_{1993:1})$$

From the figure1, we can see that both of consumption and income series exist obvious upward trends. So next step we need to confirm which factor deterministic trend or unit root contribute to such trends. And from the graph, we can also judge them preliminarily that they are not stationary.

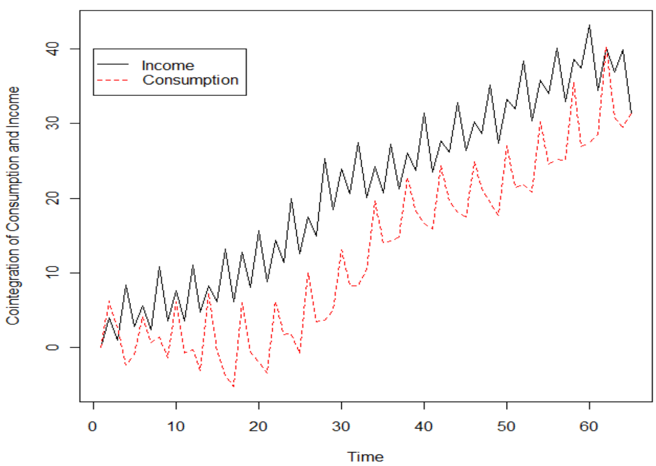

Figure 1 One hundred times of the log of consumption, income, subtract the log of First data of consumption and income in Sweden, quarterly, 1993:1-2009:4

Then ACF and PACF can be used to determine the lags, which is also helpful for us to choose the model.

From the ACF graphs of Figure2 and Figure3, we can see both of the consumption and income series are belong to ***$MA(\infty)$***, however, it is the different situation from Partial ACF. For consumption, when lag = 6, the PACF is fluctuating

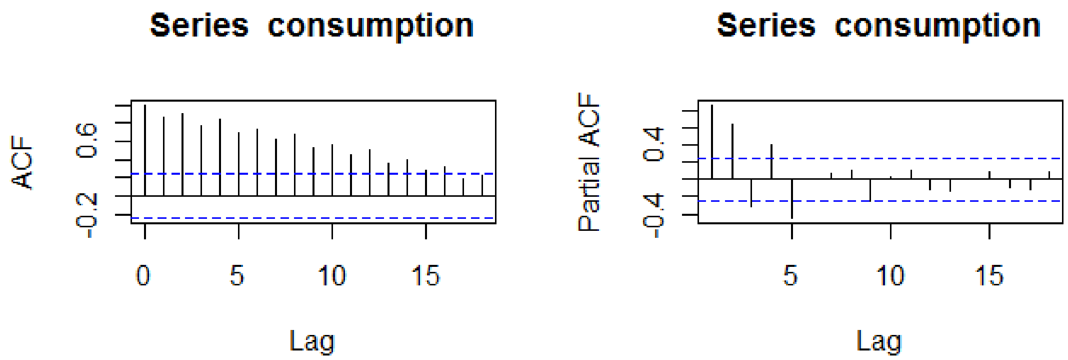

Figure 2 The ACF and PACF of consumption series

between $\pm 2\sqrt{T}$, so we preliminarily consider the consumption series is **AR(5)**.For income series, from PACF graph, it significant spikes at lag5 and it could be viewed as dying out after lag5. It implies that we should build a **AR(5)** model, while **$MA(\infty)$** from ACF plot. This information can provide the reference for selecting test method.

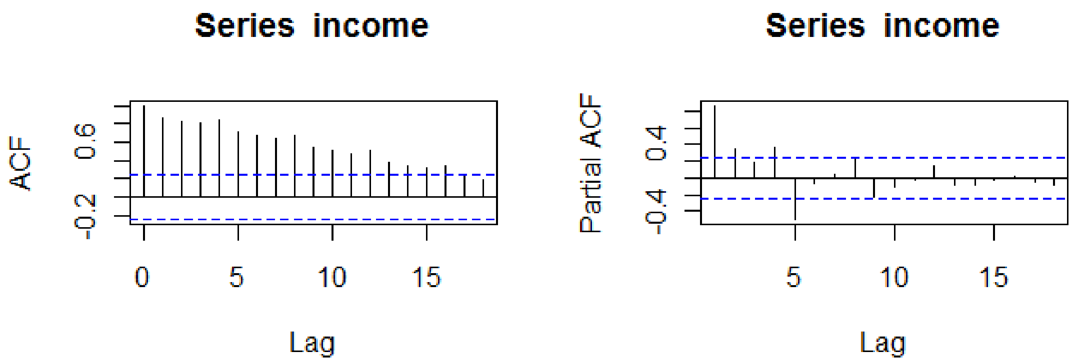

Figure 3 The ACF and PACF of income series

3. Methodology

3.1 Procedures of testing common stochastic trends

Stochastic trends are one kind of features. In order to test for common stochastic trends, we first need to know how to test for common feature.

From the Hamilton's (1994), we know that a feature that is present in each of a group of series is said to be common to those series if there exists a nonzero linear combination of the series that does not have the feature. We can also make this definition more specifically, that is a feature will be said in common if a linear combination of the series fails to have the feature even though each of the series individually has the same feature.

Therefore, when we test for a specific common feature of a group of data, we always set null hypothesis as "the feature is common", while the alternative hypothesis is that the feature is not common. So first we would detect the feature of series individually. Owning the same feature is the premise of testing for common feature.

Now that we make sense how to detect the common features, we will apply this procedure into testing for common stochastic trends. As we known, almost all time series data cannot reach stationary. So if we study the real economic data, we must analyze the properties of nonstationary data. And usually the real economic data show some trend, which should be incorporated in any forecast of the series. There are two most popular approaches mentioned in Hamilton's (1994) to describe such trends to the nonstationary data. The first approach includes a deterministic time trend, and the other one includes a unit root process. It is important to know that whether the data generated by a deterministic time trend or a unit root. Because unit root process is one kind of stochastic trend. And there are lots of differences between these two situations, such as forecasts, forecast errors, dynamic multiplier and the transformation that the data required to generate a stationary time series. The transformation of deterministic time trend should remove the time trend term, while for unit root process, should difference the data. The following definitions are about deterministic time trend and unit root process.

Definition 1: Trend-stationary process includes a deterministic time trend. The equation below shows the form of trend-stationary.

$$y_t = \alpha + \delta t + \psi(L)\varepsilon_t \qquad [2.1]$$

If [2.1] subtract the trend $\alpha + \delta t$, the result is a stationary process.

Definition 2: A unit root process,

$$(1 - L)y_t = \delta + \psi(L)\varepsilon_t \qquad [2.2]$$

where $\psi(1) \neq 0$. Another expression that is sometimes used is that the process [2.2] is *integrated* of order 1.This is indicated as $y_t \sim \boldsymbol{I}(\mathbf{1})$.

Since a co-integrated economic system is a system with a common stochastic trend, it is crucial to understand cointegration.

Definition 3: The components of the vector y_t are said to be co-integrated

of order d, b, denoted $y_t \sim$ *CI (d, b)*, if,

a) all components of y_t are *I (d)*;

b) there exists a vector $\alpha(\neq 0)$ so that $z_t = \alpha' y_t \sim I(d - b)$; b > 0. The vector α is called the co-integrating vector.

This definition was given by Engle and Granger's Nobel paper in 1987. In Hamilton's (1994), the description of cointegration is that an $(n \times 1)$ vector time series $\boldsymbol{y_t}$ is said to cointegrated if each of the series taken individually is $\boldsymbol{I}(\mathbf{1})$, that is nonstationary with a unit root, while some linear combination of the series $\boldsymbol{\alpha}'\boldsymbol{y_t}$ is stationary ,or $\boldsymbol{I}(\mathbf{0})$, for some nonzero $(n \times 1)$ vector $\boldsymbol{\alpha}$.

Thus, in this article, we will focus on testing whether the individual series is $\boldsymbol{I}(\mathbf{1})$. If all of them are $\boldsymbol{I}(\mathbf{1})$, we will test whether the linear combination of the series $\boldsymbol{\alpha}'\boldsymbol{y_t}$ is $\boldsymbol{I}(\mathbf{0})$. If the linear combination is $\boldsymbol{I}(\mathbf{0})$, we can say that the series are cointegrated. That is we find the common stochastic trend.

3.2 Methods

3.2.1 Processes with unit roots

In this part, I would like to use Dickey-Fuller and Augmented Dickey-Fuller tests as the main tests to judge whether the data is a unit root process. For Dickey-Fuller test, it suit for the situation AR(1), while the Augmented Dickey-Fuller test suit for AR(p). Besides, the maintained assumption for Dickey-Fuller test has been that the disturbance term in the regression is i.i.d. There is no strong reason to expect this for either of these time series. While Augmented Dickey-Fuller test can be used to test for unit roots in serially correlated processes. So which test will be used depends on the data situation.

1. Dickey-Fuller test

Dickey-Fuller test of constant term without time trend In this case, we could consider the OLS estimation of ρ based on an AR(1) model,

$$\mathrm{y_t} = \alpha + \rho y_{t-1} + u_t, u_t \sim N\left(0,\ \sigma^2\right) \quad [2.3]$$

We set the null hypothesis is $\rho = 1$ and $\alpha = 0$. To the statistics of constant term and lag term, they have different convergent rate to the asymptotic distribution. We should introduce a scaling matrix to amend the rate of convergence, when we calculate the asymptotic distribution. The scaling matrix should be,

$$\Gamma_t \equiv \begin{pmatrix} \sqrt{T} & 0 \\ 0 & T \end{pmatrix}$$

The calculation of ρ statistic should be,

$$\rho_T = T(\widehat{\rho_T} - 1) \quad [2.4]$$

$$t_T = \frac{\widehat{\rho}_{T-1}}{\widehat{\sigma}_{\widehat{\rho_T}}} \quad [2.5]$$

Dickey-Fuller test in Case 4 For Dickey-Fuller test in Case 4, the joint null hypothesis is $\rho = 1$ and $\delta = 0$, and the regression model is,

$$y_t = \alpha + \rho y_{t-1} + \delta t + u_t, u_t \sim N\left(0，\sigma^2\right) \qquad [2.6]$$

The calculation of ρ statistic should be

$$\rho_T = T(\widehat{\rho_T} - 1) \qquad [2.7]$$

$$t_T = \frac{\widehat{\rho}_T - 1}{\widehat{\sigma}_{\widehat{\rho_T}}} \qquad [2.8]$$

2.Augmented Dickey-Fuller test

An alternative representation of an AR(p) process

Suppose the data generated from,

$$\left(1 - \Phi_1 L - \Phi_2 L^2 - \cdots - \Phi_p L^p\right) y_t = \varepsilon_t \qquad [2.9]$$

Define that,

$$\rho \equiv \Phi_1 + \Phi_2 + \cdots + \Phi_p$$

$$\zeta_j \equiv -\left[\Phi_{j+1} + \Phi_{j+2} + \cdots + \Phi_{j+p}\right] \ for\ j = 1,2, \ldots, p-1$$

Thus,

$$(1 - \rho L) - \left(\zeta_1 L + \zeta_2 L^2 + \cdots + \zeta_{p-1} L^{p-1}\right)(1 - L) \qquad [2.10]$$

And then, the equation [2.11] can be written equivalently as,

$$y_t = \rho y_{t-1} + \zeta_1 \vartriangle y_{t-1} + \zeta_2 \vartriangle y_{t-2} + \cdots + \zeta_{p-1} \vartriangle y_{t-p+1} + \varepsilon_t \qquad [2.11]$$

Augmented Dickey-Fuller test in Case 2

The regression model should be,

$$y_t = \alpha + \rho y_{t-1} + \zeta_1 \vartriangle y_{t-1} + \zeta_2 \vartriangle y_{t-2} + \cdots + \zeta_{p-1} \vartriangle y_{t-p+1} + \varepsilon_t \qquad [2.12]$$

The Null hypothesis is ρ = 1 and $\alpha=\alpha_0$, and the alternative hypothesis of ρ is $|\rho| < 1$, which means that y_t is stationary.

The calculation of ρ statistic should be,

$$\rho_T = T\frac{\lambda}{\sigma}(\widehat{\rho_T} - 1) \qquad [2.13]$$

$$t_T = \frac{\hat{\rho}_T - 1}{\hat{\sigma}_{\widehat{\rho_T}}} \qquad [2.14]$$

Augmented Dickey-Fuller test in Case 4

The regression model should be,

$$y_t = \alpha + \rho y_{t-1} + \zeta_1 \vartriangle y_{t-1} + \zeta_2 \vartriangle y_{t-2} + \cdots + \zeta_{p-1} \vartriangle y_{t-p+1} + \delta t + \varepsilon_t \qquad [2.15]$$

The Null hypothesis is $\rho = 1$ and $\alpha = \alpha_0$ and $\delta = 0$, and the alternative hypothesis

of ρ is $|\rho| < 1$, which means that y_t is stationary.

The calculation of ρ statistic should be,

$$\rho_T = T\frac{\lambda}{\sigma}(\widehat{\rho_T} - 1) \qquad [2.16]$$

$$t_T = \frac{\hat{\rho}_T - 1}{\hat{\sigma}_{\widehat{\rho_T}}} \qquad [2.17]$$

3.2.2 Testing for linear co-integration system

In the linear co-integration test, we can set that,

H_0: There is no co-integration relation in the system.

H_1: There exists co-integration relation in the system.

If we can prove there exists a linear combination that satisfies that $z_t = \alpha' y_t \sim I(d-b)$ is an ***I (0)*** process in a critical region, $P_{H_0}(z_t$ is stationary$) \leq 5\%$, then we will reject the null hypothesis, and accept the alternative hypothesis. That means there exits linear co-integration relation in the system. Then it is time to estimate the co-integrating vector, we suppose that $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \dots, \alpha_n)'$. Then we normalize the parameters as $\alpha_1 = 1$, and $\boldsymbol{\alpha} = (1, -\gamma_2, \dots, -\gamma_n)'$.That is,

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_n \end{bmatrix} = \begin{bmatrix} 1 \\ -\gamma_2 \\ -\gamma_3 \\ \cdot \\ \cdot \\ \cdot \\ -\gamma_n \end{bmatrix}$$

Besides, for $\boldsymbol{z_t} = \boldsymbol{\alpha}'\boldsymbol{y_t} = T^{-1}\sum_{t=1}^{T} z_t^2 = T^{-1}\sum_{t=1}^{T}(\alpha' y_t)^2$, therefore, the objective is to choose $(\gamma_2, \gamma_3 \dots, \gamma_n)$ so as to minimize,

$$T^{-1}\sum_{t=1}^{T}(\alpha' y_t)^2 = T^{-1}\sum_{t=1}^{T}(y_{1t} - \gamma_2 y_{2t} - \dots - \gamma_n y_{nt})^2 \qquad [2.19]$$

This minimization is obtained by an OLS regression of the first element of y_t on all of the others:

$$y_{1t} = \gamma_2 y_{2t} + \gamma_3 y_{3t} + \dots + \gamma_n y_{nt} + u_t \qquad [2.20]$$

When a constant term is included in [2.22], as in

$$y_{1t} = \alpha + \gamma_2 y_{2t} + \gamma_3 y_{3t} + \dots + \gamma_n y_{nt} + u_t \qquad [2.21]$$

which can also be presented as follows:

$$y_{1t} = \alpha + \boldsymbol{\gamma}' \boldsymbol{y_{2t}} + u_t \qquad [2.22]$$

Residual-Based Tests for Cointegration

Now we need use Residual-Based Tests to test whether the linear combination can achieve ***I(0)*** . ***case 1*** :

Estimated cointegrating regression:

$$y_{1t} = \boldsymbol{\gamma}' \boldsymbol{y_{2t}} + u_t \qquad [2.23]$$

where $\hat{u}_t = y_{1t} - \widehat{\gamma'}_T y_{2t}$. Assuming

$$\hat{u}_t = \rho \hat{u}_{t-1} + e_t, \; t = 2,3,\dots,T \qquad [2.24]$$

True model:

$$\Delta\, y_t = \sum_{s=0}^{\infty} \Psi_s\, \varepsilon_{t-s} \qquad [2.25]$$

And from [2.23], we can obtain,

$$\hat{\rho}_T = \left(\sum_{t=2}^{T} \widehat{u^2}_{t-1}\right)^{-1} \left(\sum_{t=2}^{T} \hat{u}_t \hat{u}_{t-1}\right)$$

case 2 :

Estimated cointegrating regression:

$$y_{1t} = \alpha + \boldsymbol{\gamma}' \boldsymbol{y_{2t}} + u_t$$

Where $\hat{u}_{\mathrm{t}} = y_{1t} - \hat{\alpha}_{\mathrm{T}} - \widehat{\gamma'}_{\mathrm{T}} y_{2t}$. Assuming

$$\hat{u}_t = \rho \hat{u}_{t-1} + e_t, \qquad t = 2,3, \dots, T$$

True model:

$$\Delta\, y_t = \sum_{s=0}^{\infty} \Psi_s\, \varepsilon_{t-s}$$

And from [2.23], we can obtain,

$$\hat{\rho}_T = \left(\sum_{t=2}^{T} \widehat{u^2}_{t-1}\right)^{-1} \left(\sum_{t=2}^{T} \hat{u}_t \hat{u}_{t-1}\right)$$

case 3 :

Estimated cointegrating regression:

$$y_{1t} = \alpha + \boldsymbol{\gamma}' \boldsymbol{y_{2t}} + u_t$$

Where $\hat{u}_{\mathrm{t}} = y_{1t} - \hat{\alpha}_{\mathrm{T}} - \widehat{\gamma'}_{\mathrm{T}} y_{2t}$. Assuming

$$\hat{u}_t = \rho \hat{u}_{t-1} + e_t, \qquad t = 2,3, \dots, T$$

True model:

$$\triangle y_t = \delta + \sum_{s=0}^{\infty} \Psi_s \, \varepsilon_{t-s} \qquad [2.26]$$

And from [2.23], we can obtain,

$$\hat{\rho}_T = \left(\sum_{t=2}^{T} \widehat{u^2}_{t-1} \right)^{-1} \left(\sum_{t=2}^{T} \hat{u}_t \hat{u}_{t-1} \right)$$

Note that the different cases of 1and 2 refer to whether a constant term is included in the cointegrating regression [2.22] and not to whether a constant term is included in the form of [2.24]. In each case, the autoregression for the residuals is estimated in the form of [2.24] with no constant term.

After [2.21] is estimated by OLS, we will construct one of the standard unit root tests on the estimated residuals, such as the augmented Dickey-Fuller t test or the Phillips Z_ρ or Z_t test. Although these test statistics are constructed in the same way as when they are applied to an individual series y_t, when the tests are applied to the residuals from spurious regression, the critical values that are used to interpret the test statistics are different from those employed in single series.

Let s_T^2 be the OLS estimate of the variance of e_t for the regression of [2.26]:

$$s_T^2 = (T-2)^{-1} \sum_{t=2}^{T} (\hat{u}_t - \hat{\rho}_T \hat{u}_{t-1})^2 \qquad [2.27]$$

And let $\hat{\sigma}_{\hat{\rho}_T}$ be the standard error of $\hat{\rho}_T$ as calculated by the usual OLS formula:

$$\hat{\sigma}_{\hat{\rho}_T} = s_T^2 \div \left\{ \sum_{t=2}^{T} \hat{u}_{t-1}^2 \right\} \qquad [2.28]$$

Finally, let $\hat{c}_{j,T}$ be the jth sample autocovariance of the estimated residuals associated with [2.26]:

$$\hat{c}_{j,T} = (T-1)^{-1} \sum_{t=j+2}^{T} \hat{e}_t \hat{e}_{t-1} \quad for\, j = 0,1,2,\dots,T-2 \qquad [2.29]$$

For $\hat{e}_t \equiv \hat{u}_t - \hat{\rho}_T \hat{u}_{t-1}$; and let the square of $\hat{\lambda}_T$ be given by

$$\hat{\lambda}_T^2 = \hat{c}_{0,T} + 2 * \sum_{j=1}^{q} [1 - j/(q+1)] \hat{c}_{j,T}, \qquad [2.30]$$

Where q is the number of autocovariances to be used, Phillips's Z_ρ statistic (1987) can be calculated as:

$$Z_{\rho,T} = (T-1)(\hat{\rho}_T - 1) - \left(\frac{1}{2}\right) * \left\{(T-1)^{-1} * \hat{\sigma}_{\hat{\rho}_T}^2 \div s_T^2\right\} * \left\{\hat{\lambda}_T^2 - \hat{c}_{0,T}\right\} \qquad [2.31]$$

If the vector y_t is not cointegrated, then [2.24] will be spurious regression and $\hat{\rho}_T$ should be near 1. On the other hand, if we find that $\hat{\rho}_T$ is well below 1, that is, if calculation of [2.31] yields a negative number that is sufficiently large in absolute value, then the null hypothesis that [2.22] is a spurious regression should be rejected, and we would conclude that the variables are cointegrated.

Similarly, Phillips's Z_t statistic associated with the residual autoregression [2.24] would be

$$Z_{t,T} = \left(\hat{c}_{0,T}/\hat{\lambda}_T^2\right)^{1/2} t_T - \left(\frac{1}{2}\right) * \left\{(T-1)^{-1} * \hat{\sigma}_{\hat{\rho}_T}^2 \div s_T^2\right\} * \frac{\left\{\hat{\lambda}_T^2 - \hat{c}_{0,T}\right\}}{\hat{\lambda}_T} \qquad [2.32]$$

For t_T the usual OLS t statistic for testing the hypothesis $\rho = 1$:

$$t_T = \frac{\hat{\rho}_T - 1}{\hat{\sigma}_{\widehat{\rho_T}}}$$

4 Empirical Application to Sweden's macroeconomic data

Based on the data description we mentioned before, and the methodology we introduced, we first test whether each of the elements of y_t is individually ***I(1)***. This can be done using Dickey-Fuller or Augmented Dickey-Fuller test. Assuming that the null hypothesis of a unit root in each series individually is accepted, we next construct the scalar $\boldsymbol{z_t} = \boldsymbol{\alpha' y_t}$. Notice that if $\boldsymbol{\alpha}$ is truly a cointegrating vector, then $\boldsymbol{\alpha' y_t}$ will be **I(0)**. If $\boldsymbol{\alpha}$ is not a cointegrating vector, then $\boldsymbol{\alpha' y_t}$ will be **I(1)**. Thus, a test of the null hypothesis that $\boldsymbol{z_t}$ is **I(1)** is equivalent to a test of the null hypothesis that $\boldsymbol{y_t}$ is not cointegrated. If the null hypothesis is rejected, we could conclude that $\boldsymbol{y_t}$ is cointegrated with cointegrating vector $\boldsymbol{\alpha}$**.**

4.1 Testing for individual series

First, we will try Augmented Dickey-Fuller case 2. And we define AR(5), and the estimated regression is as follows,

$$y_t = \alpha + \rho y_{t-1} + \zeta_1 \vartriangle y_{t-1} + \zeta_2 \vartriangle y_{t-2} + \zeta_2 \vartriangle y_{t-3} \zeta_2 \vartriangle y_{t-4} + \varepsilon_t$$

True process is the same specification as estimated regression with $\alpha = 0$ and $\rho = 1$. When we make the test for consumption series, we find that the coefficient of $\vartriangle y_{t-2}$, $\vartriangle y_{t-3}$,$\vartriangle y_{t-4}$ is not significant. So we try it again with *p=2*. So the estimated regression changed into

$$y_t = \alpha + \rho y_{t-1} + \zeta_1 \vartriangle y_{t-1} + \varepsilon_t$$

The model we estimated

$$y_t = 48.7142 + 0.9619 y_{t-1} - 0.8682 \vartriangle y_{t-1} + \varepsilon_t$$

Se. (0.04328) (0.03218) (0.06585)

For this time, all of the coefficients are significant. And AIC is smallest. Then we calculate the ρ statistic and t statistic. The result is as follows.

Table 1 The comparison of consumption between ρ statistic and t statistic

T=65	Consumption	Critical value (5%)
ADF rho statistic	-1.264360	-13.7
ADF t statistic	-1.184017	-2.89,

We can see from the table that both of rho statistic and t statistic is larger than critical value. So it is obvious that we cannot reject the null hypothesis of $\rho = 1$ for any the consumption series, which means that the data owns unit root processes.

Then we begin to study income series. We also start with AR(5), but some coefficients are not significant. Then we try AR(4), AR(3). AR(3) show good result. So we set the following model:

$$x_t = \alpha + \rho x_{t-1} + \zeta_1 \vartriangle x_{t-1} + \zeta_2 \vartriangle x_{t-2} + \varepsilon_t$$

The estimated model is:

$$x_t = 21.37330 + 0.98365 x_{t-1} - 0.59028 \vartriangle x_{t-1} - 0.32618 \vartriangle x_{t-2} + \varepsilon_t$$

Se. (0.03218) (0.05607) (0.13083) (0.12509)

When we make the test, we will find all of the coefficients are significant. Then we calculate the ρ statistic and t statistic. The result is as follows.

Table 2 The values of ρ statistic and t statistic

T=65	Income	Critical value (5%)
ADF rho statistic	-0.528934	-13.7
ADF t statistic	-0.2916164	-2.89

We can see from the table that both of rho statistic and t statistic is larger than critical value. So it is obvious that we cannot reject the null hypothesis of $\rho = 1$ for any the consumption series, which also means that the data owns unit root processes. Each original series of the data is actually an ***I (1)*** process. Three individual series are ***I(1)*** processes, and they all own the stochastic process.

4.2 Testing for linear co-integration

By the test analysis above, it is obvious that the individual series of data own the property of unit root process. Now, we are more interested in knowing whether these individual data own the common feature of these two properties. The theory of consumption function can be used to support the next test.

According to the theory of consumption function, and in order to estimate the co-integrating vector, we can regress

$$C_t = \gamma I_t + u_t$$

Where the variables are,

C_t= consumption(Sweden krone);

I_t= income(Sweden krone);

Then we can obtain,

$$C_t = 9.4086 + 0.9308 I_t$$

Se. (0.9748) (0.0562)

Then we can get,

$$u_t = C_t - c - \gamma I_t$$

Next step is to test whether u_t is stationary. We regress u_t as:

$$\hat{u}_t = \delta \hat{u}_{t-1} + \hat{e}_t$$

We can obtain:

$$\hat{u}_t = 0.3521 \hat{u}_{t-1} + \hat{e}_t$$

Se. (0.1166)

$$s_T^2 = (T-2)^{-1} \sum_{t=2}^{T} (\hat{u}_t - \hat{\rho}_T \hat{u}_{t-1})^2 = 23.96963$$

$$\hat{c}_0 = 23.57767$$

$$\hat{c}_j = (T-1)^{-1} \sum_{t=j+2}^{T} \hat{e}_t \hat{e}_{t-1}$$

$$\hat{\lambda}_T^2 = \hat{c}_{0,T} + 2 * \sum_{j=1}^{q} [1 - j/(q+1)] \hat{c}_{j,T} = 22.60392$$

where q is equal to 4. Phillips's Z_ρ statistic (1987) can be calculated as:

$$Z_{\rho,T} = (T-1)(\hat{\rho}_T - 1) - \left(\frac{1}{2}\right) * \left\{(T-1)^{-1} * \hat{\sigma}_{\hat{\rho}_T}^2 \div s_T^2\right\} * \left\{\hat{\lambda}_T^2 - \hat{c}_{0,T}\right\}$$
$$= -39.68657$$

Similarly, Phillips's Z_t statistic associated with the residual autoregression [2.24] would be

$$Z_{t,T} = \left(\hat{c}_{0,T}/\hat{\lambda}_T^2\right)^{1/2} t_T - \left(\frac{1}{2}\right) * \left\{(T-1)^{-1} * \hat{\sigma}_{\hat{\rho}_T}^2 \div s_T^2\right\} * \frac{\left\{\hat{\lambda}_T^2 - \hat{c}_{0,T}\right\}}{\hat{\lambda}_T} = -5.521377$$

We set the null hypothesis,

H_0: There is no co-integration relation in the system.

H_1: There exists co-integration relation in the system.

Then will use one of residual-based test that is Phillips's test to make the following test.

Given the evidence of nonzero drift in the explanatory variables, this is to be compared with the case 3 section of Table B.8. For (n-1)=2, the critical value for Z_ρ is -21.5. Since -39.68657 < -21.5, the null hypothesis of no cointegration is rejected. Similarly, the Phillips-Ouliaris Z_t statistic is -5.521377**.** Comparing this with the case 3 section of Table B.9, we see that -5.521377 < -3.24, so that the null hypothesis of no cointegration is also rejected by the test. Thus consumption and income appear to be cointegrated.

Table 3 The value of Phillips-Ouliaris Z_ρ and Z_t statistics

T=65	Compute Value	Critical Value(5%)
Z_ρ Statistic	-39.68657	-21.5
Z_t Statistic	-5.521377	-3.42

4.3 Forecasting

Forecasting is the process of estimation in unknown situations, which is commonly used in discussion of time-series data. We suppose a variable C_{t+1} based on a set of variables I_t observed at date *t*. We use the model:

$$C_t = 9.4086 + 0.9308 I_t$$

to make the forecasting. We leave the real data from 2006:4 to 2009:1, and make the forcasting for this period, and then to compare the predict value with the real data. The confidence interval is $e^{\mu_0 + z_{0.975}\sigma}$. We can obtain the forecasts and the prediction intervals from 2006:4 to 2009:1 of Sweden's Consumption. The observed value, forecasting and prediction intervals are shown in Table 4.

Table 4

Time	Consumption(Predicted)	Consumption (Observed)	Confidence Interval
2006，4	330400.3	331005	[329670.9 ,336531.3]
2007，1	307537.8	308180	[301709.2 ,313479.0]
2007，2	320540.8	326034	[313249.6, 328001.7]
2007，3	324500.7	322367	[310692.1 ,324746.0]
2007，4	339079.5	341456	[329444.0 ,348996.9]
2008，1	307537.8	312675	[306589.0 ,319564.8]
2008，2	324500.7	331211	[316730.2 ,332461.8]
2008，3	314574	320616	[307977.4 ,321311.9]
2008，4	326473.5	330312	[318459.5 ,334689.2]
2009，1	308088.9	303215	[302202.4 ,314090.0]

From the table, we can see that all the predict value falls in the 95% confidence interval. Compared with the observed value, the predict value is very closed to the real data. So this model is effective and forecasting also works well.

5 Conclusion and Discussion

5.1 Conclusion

This article has detailed and presented the procedures to test for common stochastic trends, including testing for unit root process, and testing for cointegration by residual based test. And also with the help of cointegration, we exclude the spurious regression phenomenon to construct the proper model to the real application. Take the consumption and income data of Sweden for empirical example, both of the time series are ***I(1)*** and they are cointegrated. We construct the model for the data and make the forecasting, and the results show that the model works well. Besides, we answer the question put forward in the introduction, which is how large the MPC is of Sweden. Through the estimating, we know it is about 0.9308. That means when Sweden people earn 1 Kr, they will spend 0.93 Kr, and keep 0.07Kr.

5.2 Discussion

Testing for common stochastic trends is the only tiny part of testing for common features, so there are still many features can be detected. But different feature need different test methods. With the time and ability limit, I can only focus on this part. In future, other features can be research on. And in my paper, only linear situation is mentioned, however, non-linear situation is also worth to study. Even for cointegration, there are many representations to construct model, such as famous error-correction method. And we can use more method to construct model and to choose the best one to apply.

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