Modeling the Dynamic Conditional Correlation between Hong Kong and Tokyo Stock Markets with Multivariate GARCH models

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Abstract

The raw data is the daily return denoted by $R_t$ of the two stock markets Hong Kong and Tokyo. They are collected to get the residuals. The model we used to fit the data in our paper is the bivariate DCC-GARCH model.

For the $p$-th order vector autoregressive model, we choose the value of $p$ equal to one by using some model selection criteria: AIC, HQ and SC. Then we got the estimations of the DCC-GARCH(1,1) and give out the dynamics conditional correlation between HS and NK and the conditional standard deviations which is seen as the volatilities of the stock price.

At last we come to the point that between the two stock markets there is a dynamic conditional correlation which depends on the time change and also the volatilities of each stock market is a time-varying volatility.

Key words: Volatilities, Vector Autoregressive model (VAR), GARCH model, Dynamic Conditional Correlation (DCC).
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1. Introduction

1.1 Literature review

After the Generalized Autoregressive Conditional Heteroscedasticity model (GARCH model) was first introduced by Bollerslev (1986), it was concerned more and more by the application of those who use it as one of the powerful tools to analysis the financial return time series data. The financial return series with high frequency data display stylized facts such as volatility clustering, fat-tailness, high kurtosis and skewness. Modeling volatility in asset returns with such stylized facts is considered as a measure of risk, and investors want a premium for investing in risky assets.

Many researchers took quite a lot of in-depth studies and developed the univariate GARCH models and multivariate GARCH models. For instance, the Exponential GARCH model (EGARCH model) was pointed out by Nelson (1991). He pointed out that the volatilities aroused by negative news are larger than that by same level positive news. That is to say it is an asymmetry phenomenon. In order to solve this problem, he introduced a parameter “g” in the conditional variance part. Then it can reflect different volatilities when the random error takes negative or positive values.

Also the Threshold GARCH (TGARCH model) was mentioned by Zakoian (1990) and then by Glosten, Jaganathan and Runkle (1993) is an asymmetry GARCH models. \[ \sigma_t^2 = \omega + \alpha \cdot \mu_{t-1}^2 + \gamma \cdot \mu_{t-1}^2 \cdot d_{t-1} + \beta \cdot \sigma_{t-1}^2 \] in the function above, \( d_{t-1} \) is a dummy variable: when \( \mu_{t-1} < 0 \), \( d_{t-1} = 1 \); otherwise, \( d_{t-1} = 0 \). Therefore, in this model, good news (\( \mu_{t-1} > 0 \)) has an \( \alpha \) impact effect on conditional variance. And bad news (\( \mu_{t-1} < 0 \)) has an \( \alpha + \gamma \) impact effect. It is very famous because of the name “impact curve”.

All the achievements above are the basis for the multivariate GARCH models like Constant Conditional Correlation (CCC) GARCH model by Bollerslev (1990) and is later extended by Jeantheau (1998). Then Engle (2000) introduced a Dynamic Conditional Correlation (DCC) GARCH model which the conditional correlation is not a constant
term any more. And his main finding in his paper is that: The bivariate version of GARCH model provides a very good approximation to a variety of time varying correlation processes. The comparison of the DCC-GARCH model with simple multivariate GARCH and several other estimators shows that the DCC is often the most accurate. This is true whether the criterion is mean absolute error, diagnostic tests or tests based on value at risk calculations.

So in our study, the authors mainly used the bivariate DCC GARCH to model the time varying volatility (conditional heteroscedasticity) in stock indexes of two main financial centers Hong Kong and Tokyo comparatively instead of individually.

1.2 Aim and outline of the paper

The aim of the paper is: Trying to find the internal links between the financial return and the past errors and the relations between the two stock indexes using the DCC GARCH model.

You will find mainly six parts in this paper. First section is the introduction. Then the main part of the paper will be consisted by five parts: the author will present the data collected to be modeled which is from the two Asian Stock markets first. Secondly we will show the details of basic models and set ups used in the paper. The following section will be the analyzing process and going to give the estimation results. Last part comes to the conclusions.

2. Data Description

The samples analyzed in this research include Hong Kong stock market represented by the HangSeng (HS) index and Japan stock market represented by Tokyo Nikkei 225 (NK) index.

In order to get the synchronism of the data, we only collect the raw data of the days when the two markets are both business days. All the stock prices are the closing value of each day over the period January 5, 1998 to April 30, 2010 for a total
of 2930 observations. The raw data are available at Yahoo Finance website\(^1\).

Let \( Y_t \) denote the closing price at time \( t \). To achieve stationarity, we transform the nominal stock price data by taking the first-difference of the logarithm for each stock price series and multiplied by 100. \( R_t \) denotes the daily returns.

\[
R_t = 100 \times \log(Y_t / Y_{t-1}) = 100 \times \left[ \log(Y_t) - \log(Y_{t-1}) \right]
\]  

(1)

Figure 1 is the trend charts of the two stock prices indices in the sample period and Figure 2 is the trend charts of the two stock price index returns.

Figure 1 shows that the two stock price indices present the same trend direction during the entire sample period. To visualize the returns for these two markets, we depict the series in Figure 2. The plots show that the volatilities of these two stock market returns not only have a volatility clustering phenomenon during the selected sample period, but have certain relevance on their return volatility processes. That is, when the fluctuation of the Japan stock price index grew larger, the volatility of Hong

\(^1\)http://finance.yahoo.com/q/hp?s=^N225+Historical+Prices
http://finance.yahoo.com/q/hp?s=^HSI+Historical+Prices
Kong market return also became larger. This is the main motive for discussing the relationships of stock price returns between Hong Kong and Japan stock market.

Table 1 shows descriptive statistics for each stock price returns, the average return and the risk of the Hong Kong stock price index are higher than that of Japan stock price index. As with most financial time series, both of the stock prices exhibit leptokurtosis, a coefficient of kurtosis significantly in excess of the normal distribution's reference value of the two, and the stock price return series also show significant skewness. This means that the data have a heavy tail distribution. For the two markets, big shocks of either sign are more likely to be present and that the stock return series may not be normally distributed.

We also can recognize from Table 1 that the difference between the standard statistics and the robust ones are large, meaning that the conventional measures are affected by a small number of outliers.
Table 1  Summary statistics for the two stock price returns

<table>
<thead>
<tr>
<th></th>
<th>HS</th>
<th>NK</th>
</tr>
</thead>
<tbody>
<tr>
<td>nobs</td>
<td>2930.000</td>
<td>2930.000</td>
</tr>
<tr>
<td>mean</td>
<td>0.024</td>
<td>-0.010</td>
</tr>
<tr>
<td>maximum</td>
<td>13.407</td>
<td>13.235</td>
</tr>
<tr>
<td>minimum</td>
<td>-13.582</td>
<td>-12.111</td>
</tr>
<tr>
<td>s.d.</td>
<td>1.850</td>
<td>1.635</td>
</tr>
<tr>
<td>std.sk</td>
<td>0.249</td>
<td>-0.204</td>
</tr>
<tr>
<td>rob.sk</td>
<td>0.006</td>
<td>0.016</td>
</tr>
<tr>
<td>std.kr</td>
<td>6.852</td>
<td>5.492</td>
</tr>
<tr>
<td>rob.kr</td>
<td>0.350</td>
<td>0.202</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>5762.25</td>
<td>3702.702</td>
</tr>
<tr>
<td>p-value</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 2 illustrates the correlations of stock returns between Hong Kong market and Japan market. The coefficient is positive.

Table 2  Constant Correlation Estimates

<table>
<thead>
<tr>
<th></th>
<th>HS</th>
<th>NK</th>
</tr>
</thead>
<tbody>
<tr>
<td>HS</td>
<td>1.0000</td>
<td>0.54768</td>
</tr>
<tr>
<td>NK</td>
<td>0.54768</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

3. Models

3.1 Multivariate GARCH Model

In financial applications, extending from univariate to multivariate modeling opens the door to better decision tools in various areas, such as asset pricing models and portfolio selection. The univariate GARCH model has been extended to an n-variate model requires allowing the conditional variance-covariance matrix of the n-dimensional zero mean random variables $\epsilon_t$ to be depend on elements of the information set.

Suppose that we have n assets in a portfolio and the return vector is a column vector $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t}, \cdots, \epsilon_{nt})'$ with assumptions $\epsilon_t|\mathcal{F}_{t-1} \sim D(0, H_t)$. That is, $E[\epsilon_t|\mathcal{F}_{t-1}] = 0$ and $E[\epsilon_t' \epsilon_t|\mathcal{F}_{t-1}] = D_tP_tD_t = H_t$. 


Nakatani and Teräsvirta (2008) define the family of CC-GARCH models as follows. Consider the multivariate GARCH process:

\[ y_t = \mu_t + \varepsilon_t \]  
\[ \varepsilon_t = D_t z_t \]

Notation:

\( y_t \): \( n \times 1 \) vector of log returns of \( n \) assets at time \( t \).
\( \mu_t \): \( n \times 1 \) vector of the expected value of the conditional \( y_t \). \( \mu_t = E(y_t | \mathcal{F}_{t-1}) \).
\( \varepsilon_t \): \( n \times 1 \) vector of mean-corrected returns of \( n \) assets at time \( t \).
\( D_t = \text{diag} \{ \sqrt{h_{1t}}, \ldots, \sqrt{h_{nt}} \} \), and \( \sqrt{h_{it}} \) is the conditional standard deviation of \( \varepsilon_{it} \).

The conditional variance \( h_{it} \) follows a univariate GARCH process:

\[ h_{it} = \alpha_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{i,t-j}^2 + \sum_{j=1}^{p} \beta_j h_{i,t-j}. \]

\( z_t \): \( n \times 1 \) vector of iid errors with \( E[z_t | \mathcal{F}_{t-1}] = 0 \) and \( E[z_t' z_t | \mathcal{F}_{t-1}] = P_t = [\rho_{ij,t}] \),

where \( \mathcal{F}_{t-1} \) is the information set up to and including time \( t-1 \).

\( H_t \): time-varying conditional covariance matrix of the process \( \varepsilon_t \). \( [H_t]_{ij} = h_{it} h_{jt} \rho_{ij,t} \) \( i \neq j \), where \( 1 \leq i, j \leq n \).

\( P_t \): time-varying positive definite conditional correlation matrix of the process \( \varepsilon_t \).

\[ \rho_{ij,t} = E[z_{it} z_{jt} | \mathcal{F}_{t-1}] = \frac{E[z_{it} z_{jt} | \mathcal{F}_{t-1}]}{\sqrt{E[z_{it}^2 | \mathcal{F}_{t-1}] E[z_{jt}^2 | \mathcal{F}_{t-1}]}} = \frac{E[\varepsilon_{it} \varepsilon_{jt} | \mathcal{F}_{t-1}]}{\sqrt{E[\varepsilon_{it}^2 | \mathcal{F}_{t-1}] E[\varepsilon_{jt}^2 | \mathcal{F}_{t-1}]}} = \text{Corr}[\varepsilon_{it}, \varepsilon_{jt} | \mathcal{F}_{t-1}] \]

3.2 DCC-GARCH Model

The simplest way of modeling the correlation structure is to assume a constant relationship among variables in the model. The CCC-GARCH model is proposed by Bollerslev (1990) and is later extended by Jeantheau (1998). In the CCC-GARCH model, the conditional correlation matrix is constant over time, that is,

\[ P_t = P \]  

(4)
However, the assumption of Bollerslev’s (1990) model that the conditional correlations are constant over time may seem too restrictive in practice, and for this reason many authors have proposed models of time-varying conditional correlations. Thus, Engle (2002) and Tse and Tsui (2002) propose a generalization of Bollerslev’s (1990) constant conditional correlation model by making the conditional correlation matrix time-dependent. This type of model is called a dynamic conditional correlation (DCC) model and it is one of the most popular CCC-GARCH models with time-varying conditional correlations.

Engle (2002) applies GARCH-type dynamics in modeling the conditional correlations. Its correlation structure is defined as follows:

\[ P_t = (Q_t \odot I_N)^{-1/2} Q_t \odot I_N^{-1/2} \]  
\[ Q_t = (1 - \alpha - \beta)Q + \alpha z_{t-1}'z_{t-1} + \beta Q_{t-1} \]

where

\[ \alpha + \beta < 1 \text{ and } \alpha > 0, \beta > 0 \]

Notation:

- \( I_N = E[z_t'z_t] \)
- \( \odot \): the Hadmard or element wise product of the two conformable matrices.
- \( Q \): a sample covariance matrix of \( z_t \).

In this formulation, the correlation process is driven by two parameters, \( \alpha \) and \( \beta \). This is one of the advantages of the DCC-GARCH model in the sense that the number of parameters to be estimated for conditional correlations does not depend on the number of variables in the model. With this property, one can alleviate the computational burden and yet obtain large-dimensional correlations. But the simple structure of the DCC-GARCH parameterizations may be seen as a weakness because all the correlation processes are assumed to have the same dynamic behavior.

3.3 Model Building

In this paper, we use the bivariate DCC-GARCH model to discuss the relationships between Japan stock market and Hong Kong stock market, and impact on returns of
the two markets. The simplest but often very useful DCC-GARCH process is of course
the DCC-GARCH (1, 1) process. The constructions of the model are as follows:

\[
\text{RHS}_t = \phi_0 + \sum_{k=1}^p \phi_{1k} \text{RHS}_{t-k} + \sum_{k=1}^p \phi_{2k} \text{RNK}_{t-k} + \epsilon_{1,t} \tag{7}
\]

\[
\text{RNK}_t = \varphi_0 + \sum_{k=1}^p \varphi_{1k} \text{RHS}_{t-k} + \sum_{k=1}^p \varphi_{2k} \text{RNK}_{t-k} + \epsilon_{2,t} \tag{8}
\]

\[
h_{11,t} = \alpha_{10} + \alpha_{11} \epsilon_{1,t-1}^2 + \beta_{11} h_{11,t-1} \tag{9}
\]

\[
h_{22,t} = \alpha_{20} + \alpha_{21} \epsilon_{2,t-1}^2 + \beta_{21} h_{22,t-1} \tag{10}
\]

\[
P_t = [Q_t \otimes I_N]^{-1/2} Q_t [Q_t \otimes I_N]^{-1/2} \tag{11}
\]

\[
Q_t = (1 - \alpha - \beta) Q + \alpha z_{t-1} z_{t-1}' + \beta Q_{t-1} \tag{12}
\]

### 4. Analysis and Results

#### 4.1 Estimation of DCC-GARCH Model

In this part, we describe how the parameters of a DCC-GARCH model may be
determined. The likelihood function for \( \epsilon_t = D_t z_t \) is

\[
L(\theta) = \prod_{t=1}^N \frac{1}{\sqrt{2\pi|H_t|}} \exp \left\{ -\frac{1}{2} \epsilon_t' H_t^{-1} \epsilon_t \right\} \tag{13}
\]

The log-likelihood function of the DCC-GARCH model at time \( t \) is in general
given by

\[
\ell(\theta) = \ln[L(\theta)] = -\frac{N}{2} \ln(2\pi) - \frac{1}{2} \ln(|H_t|) - \frac{1}{2} \epsilon_t' H_t^{-1} \epsilon_t \tag{14}
\]

\[
= -\frac{N}{2} \ln(2\pi) - \frac{1}{2} \ln(|D_t P_t D_t'|) - \frac{1}{2} \epsilon_t' D_t^{-1} P_t^{-1} D_t^{-1} \epsilon_t
\]

The log-likelihood function can be decomposed into two parts, namely the
volatility component and the correlation component. The volatility component at
time \( t \) is given by

\[
\ell_{\nu,t}(\omega) = -\frac{N}{2} \ln(2\pi) - \frac{1}{2} \ln(|V_t|) - \frac{1}{2} \epsilon_t' V_t^{-1} \epsilon_t \tag{15}
\]

where \( V_t = D_t^2 \), and the correlation component at time \( t \) is

\[
\ell_{c,t}(\omega, \varphi) = -\frac{1}{2} \ln(|P_t|) - \frac{1}{2} z_t' P_t^{-1} z_t + \frac{1}{2} z_t' z_t \tag{16}
\]

By applying this decomposition, the estimation of a DCC-GARCH model can be
carried out in two steps. First, maximize (15) with respect to $\omega$ which denotes the parameters in the volatility component. Second, maximize (16) with respect to $\varphi$ which denotes the parameters in the correlation component, given the estimates from the preceding step. It is worth mentioning that the constraints on $\alpha$ and $\beta$ must be satisfied throughout the iterations because otherwise $\{Q_t\}$ may become an explosive sequence.

4.2 Parameter Estimation

The equations (7) and (8) can be written as:

\[
\begin{pmatrix}
\text{RHS}_t \\
\text{RNK}_t
\end{pmatrix} = \begin{pmatrix}
\phi_0 \\
\varphi_0
\end{pmatrix} + \begin{pmatrix}
\phi_{t1} & \cdots & \phi_{2p} \\
\varphi_{t1} & \cdots & \varphi_{2p}
\end{pmatrix} \begin{pmatrix}
\text{RHS}_{t-1} \\
\vdots \\
\text{RNK}_{t-p}
\end{pmatrix} + \begin{pmatrix}
\varepsilon_{1,t} \\
\varepsilon_{2,t}
\end{pmatrix}
\] (17)

This is p-th order vector autoregressive model. We choose the value of p by using some model selection criteria, such as Akaike Information Criterion (AIC), Hanna-Quinn Information Criterion (HQ) and Schwarz Information Criterion (SC). All of these selection criteria are not the same when they are used in the univariate model.

\[
\text{AIC}(n) = \ln\{\det[\Sigma_{\tilde{u}}(n)]\} + \frac{2}{T}nK^2
\] (18)

\[
\text{HQ}(n) = \ln\{\det[\Sigma_{\tilde{u}}(n)]\} + \frac{2\ln\ln(T)}{T}nK^2
\] (19)

\[
\text{SC}(n) = \ln\{\det[\Sigma_{\tilde{u}}(n)]\} + \frac{\ln(T)}{T}nK^2
\] (20)

where $\Sigma_{\tilde{u}}(n) = T^{-1} \sum_{t=1}^{T} \tilde{u}_t \tilde{u}_t'$ and $n^*$ is the total number of the parameters in each equation and $n$ assigns the lag order.

In table 3, from fitted models with $p = 1, 2, \cdots, 20$, we choose the value of $p$ which has the minimal selection criteria; therefore, $p = 1$. 
The estimation result for VAR(1) is

\[
\begin{pmatrix}
\text{RHS}_t \\
\text{RNK}_t
\end{pmatrix} = \begin{pmatrix} 0.0247 \\ -0.0136 \end{pmatrix} + \begin{pmatrix} 0.0067 \\ 0.1012 \end{pmatrix} \begin{pmatrix} \text{RHS}_{t-1} \\
\text{RNK}_{t-1}
\end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\
\varepsilon_{2,t}
\end{pmatrix}
\]

(21)

Table 4 shows estimation results for the DCC-GARCH(1,1) model. The coefficients for all the parameters are positive. In particular, \( \alpha_{11} + \beta_{11} = 0.0797 + 0.9169 = 0.9966 < 1 \) and \( \alpha_{21} + \beta_{21} = 0.0936 + 0.8894 = 0.983 < 1 \), both of them are very close to 1, indicating that high persistence in the conditional variances. Furthermore, both of them are less than 1, this means that conditional variance is finite and the series are strictly stationary.
5. Conclusions

Figure 3 illustrates the dynamic condition correlation between the HS index and NK index. Between the two stock markets, it is clearly that they do exist a dynamic conditional correlation to change depend on the time change. Although sometimes the coefficient is large and sometimes it is small, the estimated conditional correlations have increased in recent years, implying higher linkages between the Japan market and the Hong Kong market. Furthermore, it seems that these correlations have a common positive trend. In other words, the linkages between these two markets tend to move together. The mean value of the dynamic conditional correlation coefficient is 0.52. It is a strong clear “linkage effects”. That implies that the two markets are not independent based on time. The reasons may be as follows:

1. In macroscopical sight, both Hong Kong and Japan are Asian financial centers and the economic transactions including trade, services, and capital movements are more and more intimate.

2. Microcosmic on look, many big public companies either listed in Hong Kong stock market or listed in Japan stock market, their contribution to the local stock index will be big and meanwhile they have a lot of businesses with other markets. For instance, Hong Kong Company will have a large number of businesses in Japan and vice versa.

3. The Asian stock markets are still not mature enough. There must be some interplay.

All the mentioned above will raise the clear “linkage effects” between Hong Kong and Japan stock markets.

Furthermore, they have influence on each other. We can call it a “volatility
"spillover effect" That means the volatility of HangSeng index will strengthen the volatility of the Tokyo Nikkei 225 index.

Figure 3  Dynamics Conditional correlation between HS and NK

Figure 4 shows the estimated volatilities, it is conditional standard deviations. The volatilities (conditional standard deviations) of the stock market is a Time-varying Volatility, for the first 1000 observations above, we can see that both of the Hongkong and Japan markets are queasy. The conditional standard deviations sometimes goes up and suddenly goes down. Because it is the year 1998-2002. The Asian financial crisis which happened in 1997 just went over. The economy of most Asian countries were serious attacked, especially the stock market. Even HongKong and Japan can not survive until several years later. Then the stock came to a relatively stable phase (obs1000-2500) and it is year 2002 - late 2007. During this period of time, the Asian economy kept a stability speed of development. So did the stock markets. In 2008, the American loan crisis erupts, the speed with which clouds of economic gloom and even despair have gathered over the global economy has been startling everywhere. The stock markets were hardest hit. So the results we get can explain the truth on some aspects.
Figure 4  Estimated Volatilities (Conditional Standard Deviations)
References

