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An Analysis of Cointegration Relation on Swedish National Account

Statistics D-level thesis, June 2011

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D-Level thesis in Statistics for M.S. Degree
June 2011

Abstract

The purpose of this essay is to apply the Johansen`s method to model a cointegration system using Swedish national account data and interpret the economic meaning according to the estimated long-run coefficient matrix. Our data includes five economic time series: Final Consumption, Investment, Export, Import and Gross Domestic Product, which are the main compounds of national account. In order to eliminate the impacts of large shocks by historical events, six dummy variables are raised. There exist two cointegration relations in the Swedish national account data. The estimated model is a partial model due to the restrictions on the loading weight in the long-run matrix. The result shows that Investment is weakly exogenous while Final Consumption and Gross Domestic Product are homogeneous.

Key Words: Cointegration, National account, Maximum likelihood estimation

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1. Introduction

Most of statistics model for time series are under the assumption of stationary, while in practice the economic time series data are usually non-stationary. In the study of the non-stationary vector time series, the concept of cointegration, finding certain combinations of non-stationary series which is stationary, was suggested first by Granger (1981, 1983), Granger and Weiss (1983), and studied further by Engle and Granger (1987).

Error correction model is a widely considered representation of cointegration system. Engle and Granger (1987) studied the relation between cointegration and error correction model, and suggested estimation for a regression on integrated regressors. After that, Johansen (1988) derived the full-information maximum likelihood estimators of the cointegration vectors for autoregressive process with independent Gaussian errors. Further extension was given by Johansen (1991) with some crucial likelihood based test for cointegration rank and restrictions on parameters.

Many studies have been carried out to analyze the Swedish economy, but few of them are targeting the national account. National account system is the most important accounting system in measuring the economic activity of a nation. It is very sensitive to the changes of the economic environment which makes it a good reflection of the status for economy. The ideal case for the national account data is to have stable growth for all series, namely in the long run there exist cointegration relations among these series. The purpose of this essay is to apply the Johansen`s method to model a cointegration system using Swedish national account data and interpret the economic meaning according to the estimated long-run coefficient matrix.

The structure of the thesis is the following: Section 2 introduces the Johansen`s produce of estimation and tests based on Gaussian likelihood. Section 3 describes the data of the Swedish national account. Section 4 illustrates the Johansen`s method for modeling a cointegration system; reports the results of tests for cointegration rank and restrictions on parameters along with interpretation of the estimated coefficients. Section 5 gives the conclusions.

2. Methodology

2.1 Johansen`s Cointegration Model

Multivariate time series are said to be cointegrated if they are individually integrated with order d_1 and certain linear combinations of them have a lower integrated order d_2, $d_1>d_2$. The mostly applied case is that a p-dimension I(1) vector time series X_t is cointegrated if there exist a non-zero vector ω such that $\omega' X_t$ is stationary.

According to Johansen (1988), the cointegration can be formulated in a vector error correction model (VECM) representation:

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \Pi X_{t-1} + \varepsilon_t \quad , \tag{2.1}$$

where Π is reduced rank restricted. Since Π is reduced rank, it can be decomposed as multiplying of two vectors, such that $\Pi = \alpha\beta'$ where α and β are $p \times r$ matrix, ($r < p$). The matrix Π is called long-run matrix, define the long-run effects to the VECM. $\beta' X_{t-1}$ represents r-cointegration relations inside the system. And α is the loading weight, illustrating the effect of cointegration relation on the VECM.

An extension of (2.1) including deterministic trend and dummy variables can be written as:

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \Pi X_{t-1} + \mu_0 + \mu_1 t + \Phi D_t + \varepsilon_t \, . \tag{2.2}$$

In order to partition the effect of deterministic trend and dummy variables into two components based on if the effect is inside the cointegration system or not. The decomposition can be done by:

$$
\begin{aligned}
\mu_0 &= \alpha\beta_0 + \gamma_0 \\
\mu_1 &= \alpha\beta_1 + \gamma_1 \\
\Phi &= \alpha\beta_D + \phi_D .
\end{aligned} \tag{2.3}
$$

Substitute (2.3) in (2.2), we get:

$$
\Delta X_t = \Gamma_1 \Delta X_{t-1} + \alpha\beta' X_{t-1} + \alpha\beta_0 + \alpha\beta_1 t + \gamma_0 + \gamma_1 t + \alpha\beta_D D_t + \phi_D D_t + \varepsilon_t \tag{2.4}
$$

$$
= \Gamma_1 \Delta X_{t-1} + \alpha \begin{bmatrix} \beta' & \beta_0 & \beta_1 & \beta_D \end{bmatrix} \begin{bmatrix} X_{t-1} \\ 1 \\ t \\ D_t \end{bmatrix} + \gamma_0 + \gamma_1 t + \phi_D D_t + \varepsilon_t \tag{2.5}
$$

where the parameters β_0, β_1 and β_D are called restricted parameters, which imply that the deterministic trend and dummy variables are restricted only in the cointegration relations.

2.2 Maximum Likelihood Estimation

The Maximum Likelihood Estimation of Gaussian case is first derived by Johansen (1988). To simplify the notations, rewrite (2.5) as:

$$
Z_{0t} = \alpha\tilde{\beta}' Z_{1t} + \Psi Z_{2t} + \varepsilon_t , \tag{2.6}
$$

where $\Psi = \begin{bmatrix} \Gamma_1 & \gamma_0 & \gamma_1 & \phi_D \end{bmatrix}$, $\tilde{\beta}' = \begin{bmatrix} \beta' & \beta_0 & \beta_1 & \beta_D \end{bmatrix}$, $Z_{1t} = \begin{bmatrix} X_{t-1} & 1 & t & D_t \end{bmatrix}$,

$Z'_{2t} = \begin{bmatrix} \Delta X_{t-1} & 1 & t & D_t \end{bmatrix}$, *and* $Z_{0t} = \Delta X_t$.

Then, estimate the following regression by OLS,

$$
\begin{aligned}
Z_{0t} &= \hat{B}_1' Z_{2t} + R_{0t} \\
Z_{1t} &= \hat{B}_2' Z_{2t} + R_{1t}
\end{aligned} \tag{2.7}
$$

and get the residual R_{0t} and R_{1t}. Thus, the concentrated model is:

$$
R_{0t} = \alpha\tilde{\beta}' R_{1t} + \varepsilon_t , \quad \varepsilon_t \sim N_p(0, \Omega) \tag{2.8}
$$

Assume $\tilde{\beta}'$ is known. α can be estimated as a function of $\tilde{\beta}$, $\alpha = \hat{\alpha}(\tilde{\beta})$,by post-multiplying (2.8) with $R'_{1t}\tilde{\beta}$ and dropping the error term. And then find the least squares estimator of α .

$$R_{0t}R_{1t}'\tilde{\beta} = \alpha\tilde{\beta}'R_{1t}R_{1t}'\tilde{\beta} \tag{2.9}$$

$$S_{01}\tilde{\beta} = \alpha\tilde{\beta}'S_{11}\tilde{\beta} \quad where \;\; S_{ij} = T^{-1}\sum_t R_{it}R_{jt}' \tag{2.10}$$

$$\hat{\alpha}(\tilde{\beta}) = S_{01}\tilde{\beta}(\tilde{\beta}'S_{11}\tilde{\beta})^{-1} \tag{2.11}$$

The error covariance matrix $\hat{\Omega}$ as a function of fixed $\tilde{\beta}$ and α is shown in (2.12),

$$\hat{\Omega} = S_{00} - S_{01}\tilde{\beta}\alpha' - \alpha\tilde{\beta}'S_{10} + \alpha\tilde{\beta}'S_{11}\tilde{\beta}\alpha' = S_{00} - S_{01}\tilde{\beta}(\tilde{\beta}'S_{11}\tilde{\beta})^{-1}\tilde{\beta}'S_{10} \; . \tag{2.12}$$

Under the multivariate normality assumption, maximizing the likelihood function of (2.8) is equivalent to minimizing the determinant of $\hat{\Omega}$:

$$\left|\hat{\Omega}\right| = \left|S_{00}\right| \frac{\left|\tilde{\beta}'(S_{11} - S_{10}S_{00}^{-1}S_{01})\tilde{\beta}\right|}{\left|\tilde{\beta}'S_{11}\tilde{\beta}\right|} . \tag{2.13}$$

Then (2.13) can be minimized by solving the eigenvalue problem as follows,

$$\left|\lambda S_{11} - S_{10}S_{00}^{-1}S_{01}\right| = 0 . \tag{2.14}$$

The minimum of $\left|\hat{\Omega}\right|$ is:

$$\left|\hat{\Omega}\right|_{\min} = \left|S_{00}\right| \prod_{i=1}^{p}(1-\lambda_i) . \tag{2.15}$$

Finally, the maximum likelihood estimation of $\tilde{\beta}_i'$ is obtained by normalizing the corresponding eigenvector of λ_i .The magnitude of λ is a measure of the "stationary" of the corresponding $\tilde{\beta}'X_{t-1}$, the larger the λ_i is, the "more" stationary the relation is. When $\lambda_i = 0$, the corresponding linear combination $\tilde{\beta}_i'X_{t-1}$ is non-stationary.

2.3 Trace Test

To determine the number of non-zero eigenvalue, namely the cointegration rank, trace test is introduced by Johansen (1991). The null hypothesis of trace test is:

H_0: Rank≤r, namely, there exist r cointegration relation at most.

The trace test uses the idea of likelihood ratio test. The test statistic τ_{p-r} is:

$$\tau_{p-r} = -2\ln Q(\frac{H_0}{H_1}) = T\ln\left\{\frac{|S_{00}|(1-\lambda_1)(1-\lambda_2)\cdots(1-\lambda_r)}{|S_{00}|(1-\lambda_1)(1-\lambda_2)\cdots(1-\lambda_r)\cdots(1-\lambda_p)}\right\}$$
$$= -T\ln((1-\lambda_{r+1})(1-\lambda_{r+2})\cdots(1-\lambda_p) \qquad (2.16)$$

If the estimated τ_{p-r} is large than an appropriate critical value C_{p-r}, then the null hypothesis is rejected, otherwise, the hypothesis is accepted. Notice that the distribution of τ_{p-r} is non-standard and should be determined by simulations. A lot of papers have been carried out concentrating on the distributions of critical value and Osterwald-Lenum, M. (1992) is one of the most famous work, giving useful tables for application.

2.4 Restrictions on *β*

Sometimes, we want to put some restriction on *β*, such as a zero row restriction, if accepted, means that the variable is not needed in the cointegration and can be omitted. Johansen (1991) gives the following method for testing the restrictions. Suppose we have s unrestricted coefficients or equivalently $m = p - s$ restrictions on parameters, then the null hypothesis can be written as:

$$\text{H}_0\text{:}\ \beta^c = H\beta \qquad \textit{where } \beta^c \textit{ is } p\times r, H \textit{ is } p\times s \textit{ and } \beta \textit{ is } s\times r.$$

The likelihood ratio test can be done by calculating the ratio between the value of the likelihood function in the restricted and unrestricted model,

$$\Lambda = \left(\frac{L_{\max}H_0}{L_{\max}H_1}\right) = \left(\frac{|S_{00}|\times(1-\lambda_1^c)(1-\lambda_2^c)\cdots(1-\lambda_r^c)}{|S_{00}|\times(1-\lambda_1)(1-\lambda_2)\cdots(1-\lambda_r)}\right)^{-\frac{T}{2}}. \qquad (2.18)$$

where λ_i^c is the eigenvalue from $\left|\lambda H'S_{11}H - H'S_{10}^{-1}S_{01}H\right| = 0$. The test statistic (2.19) asymptotically follows $\chi^2(rm)$ distribution.

$$-2\ln\Lambda = T\left\{\ln(1-\lambda_1^c) - \ln(1-\lambda_1) + \cdots + \ln(1-\lambda_r^c) - \ln(1-\lambda_r)\right\} \qquad (2.19)$$

2.5 Restrictions on α and partial model

We can also put some restriction on α. For example, the test of a zero row in α is the equivalent of testing whether a variable can be considered weakly exogenous. The likelihood ratio test for restrictions on α by Johansen (1991) is derived as follows:

$$\text{H}_0\text{:}\ \ \alpha^c = H\alpha = \begin{pmatrix}\alpha_1 & 0\end{pmatrix}' \qquad where\ \alpha^c\ is\ p\times r, H\ is\ p\times s\ and\ \alpha\ is\ s\times r$$

Then, under H_0 (2.8) will be rewritten as:

$$R_{0t} = \alpha^c \beta' R_{1t} + \varepsilon_t = H\alpha\beta' R_{1t} + \varepsilon_t = \begin{pmatrix}\alpha_1 & 0\end{pmatrix}' \beta' R_{1t}\ . \tag{2.20}$$

Decompose (2.20) into two "baby" model (2.21), and then do two auxiliary regression (2.22).

$$\begin{aligned}\Delta X_{1t} &= \alpha_1 \beta' R_{1t} + \varepsilon_{1t} \\ \Delta X_{2t} &= \varepsilon_{2t}\end{aligned} \tag{2.21}$$

$$\begin{aligned}R_{0t} &= \hat{B}_1 \Delta X_{2t} + R_{0,H\perp t} \\ R_{1t} &= \hat{B}2 \Delta X_{2t} + R_{1,H\perp t}\ .\end{aligned} \tag{2.22}$$

And the likelihood ratio test (2.23) asymptotically follows $\chi^2(rm)$ distribution.

$$-2\ln\Lambda = T\left\{\ln(1-\lambda_1^c) - \ln(1-\lambda_1) + \cdots + \ln(1-\lambda_r^c) - \ln(1-\lambda_r)\right\}, \tag{2.23}$$

where λ_i^c is based on model: $R_{0,H\perp t} = \alpha\beta' R_{1,H\perp t} + u_t$.

If a weakly exogenous hypothesis is accepted by the likelihood ratio test, then we can partition the time series as:

$$\Delta X_t = \begin{pmatrix}\Delta X_{1t} \\ \Delta X_{2t}\end{pmatrix} \quad where\ \Delta X_{2t}\ is\ weakly\ exogenous$$

And (2.2) can be rewritten as:

$$\Delta X_{1t} = a_1 \Delta X_{2t} + \Gamma_{1.1} \Delta X_{t-1} + \alpha\beta' X_{t-1} + \mu_{0.1} + \mu_{1.1} t + \Phi_1 D_t + \varepsilon_{1.2,t} \tag{2.24}$$

$$\Delta X_{2t} = \Gamma_{1.2} \Delta X_{t-1} + \mu_{0.2} + \mu_{1.2} t + \Phi_2 D_t + \varepsilon_{2,t} \beta \tag{2.25}$$

Because the weakly exogenous ΔX_{2t} does not contain any information about β, thus, we can only concentrate on the partial model (2.24) to get the fully efficient estimate of β. Juselius (2006) shows that a partial system has more stable parameters than the full system because the noise of weakly exogenous variables a not included.

3. Data

The original data is the Swedish national accounts data collected by Department of Economic History, Stockholm University. This thesis mainly uses five series: Final Consumption, Investment, Export, Import and Gross Domestic Product, range from 1830 to 2000. All the series are Nominal values measured in purchasers' prices, million SEK. Import and export are recorded on the c.i.f./f.o.b.-basis[1] and GDP are calculated by expenditure. Logarithm is applied to all variables.

3.1 Final Consumption

The Final Consumption measures the value of goods and services consumed by both private and government. The original data in Figure 3.1 left shows a time trend of increasing while the differenced data in Figure 3.1 right move in a tight range around a constant. Then it can be suggested that the Final Consumption may be a random walk with drift. It`s noticeable that a sharp wave appears in 1914- 1920, due to the outbreak of World War I. The war pushes up the consumption and then gives a

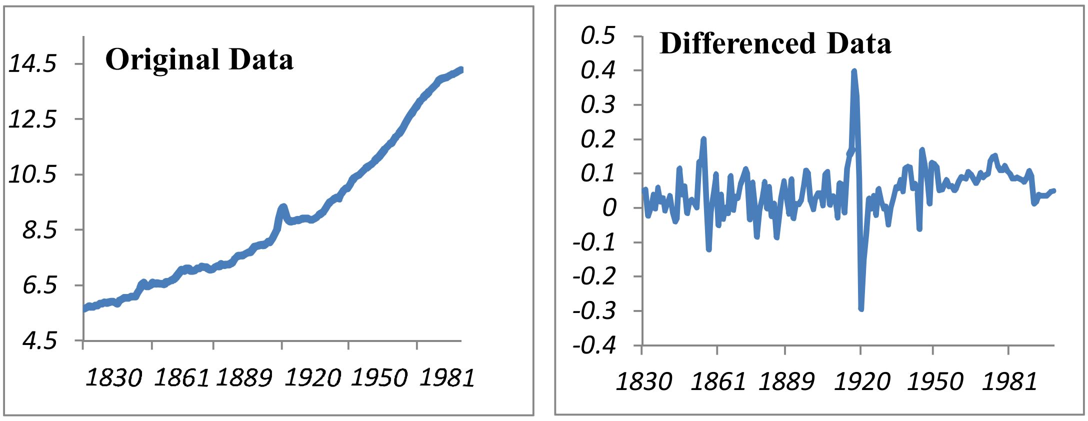


Figure 3.1 Logarithm Final Consumption

[1] c.i.f./f.o.b: Cost Insurance and Freight / Free On Board

strongly negative impact during the post-war time. The same shape of wave around 1944 can be explained by the World War II in similar way.

3.2 Investment

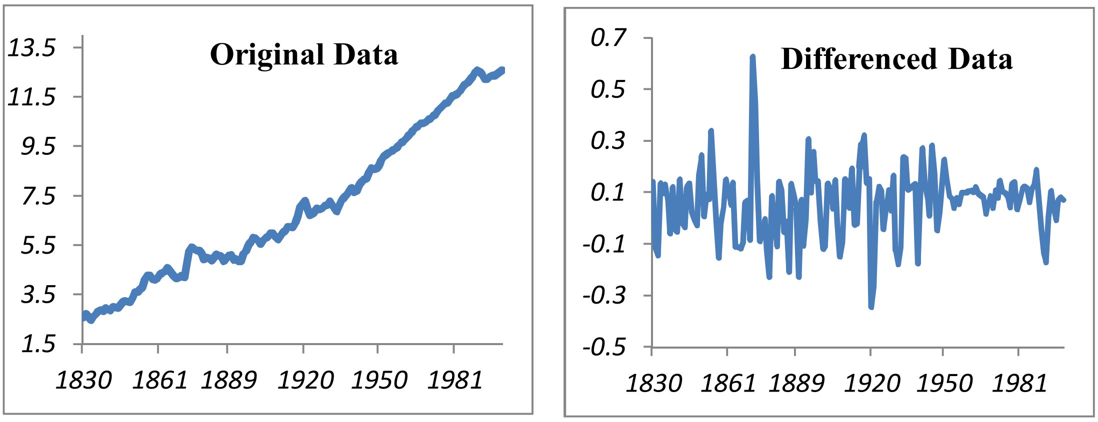


Figure 3.2 Logarithm Investment

The Investment shares the same shape from Final Consumption. A random walk with drift is also a proper suggestion for the series. In 1872, a great storm tide swapped the Baltic Sea, thus large amount of money went into the post-disaster reconstruction which gives a peak of Investment in Figure 3.2 right. The World War I also has a strong effect on Investment explain the bottom in 1920. Furthermore, the Great Depression in 1930 and the World War II on 1940s should be taken into consideration due to its non-ignorable influence.

3.3 Foreign Trade

The foreign trade contains two terms, Export and Import. As Figure 3.3 and 3.4 shows, Export and Import also follow a random walk with drift. It seems Import is more stable then Export. Around 1848, affected by the Industrial Revolution, Sweden turn itself form agriculture-based to modern industrial country. During 1914 to 1920 and 1945 to 1951, the economy of Sweden is largely influenced by the First and Second World War. The Great Depression in 1930s also has strongly impact on the foreign trade.

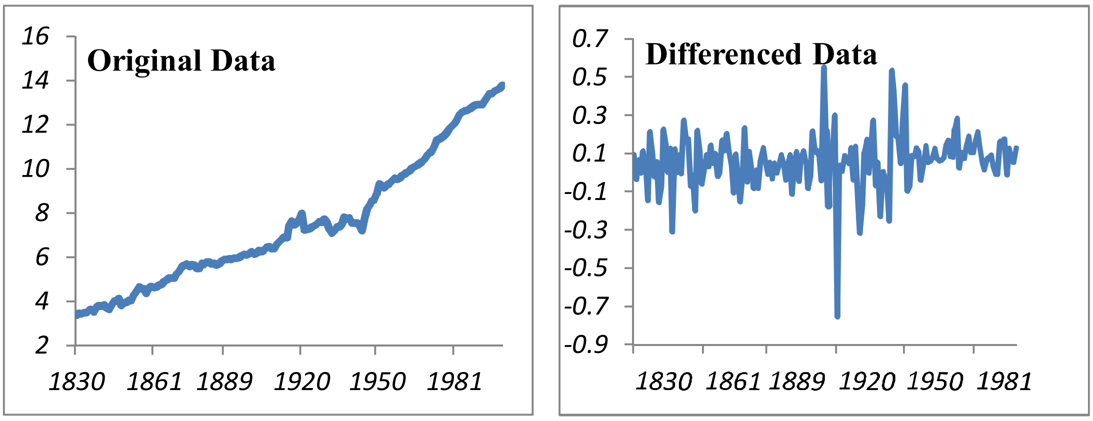


Figure 3.3 Logarithm Export

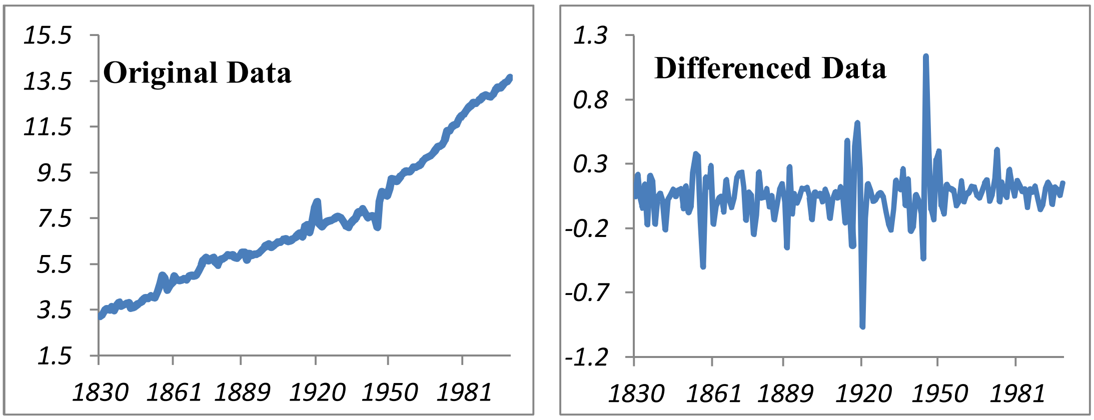


Figure 3.4 Logarithm Import

3.4 Gross Domestic Product

Gross Domestic Product measures the market value of all final goods and services, and is calculated by expenditure approach. It`s the most important indicator of a country's standard of living and represent the overall power of economy.

Figure 3.5 shows that the GDP of Sweden has firm growth during the past 170 years. The historical events also have great impact so that the data is very unstable if events occurred. During the war time, a shape ware is recorded because of the military needs and the change of outside environment. Around 1930s, the Great Depression also strongly influent the economy of Sweden, cause a low bottom in the differenced data.

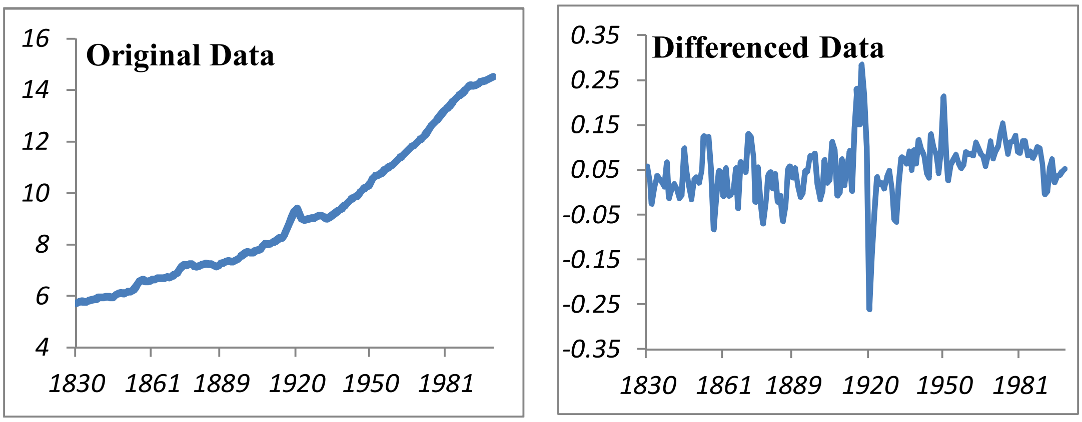


Figure 3.5 Gross Domestic Product

4. Results

4.1 Unit Root Test

The notation of variables is given in Appendx I. Although the shape of lines of each variabe suggest I(1) process is reasonable, it`s still necessary to test unit-root with Augmented Dickey–Fuller F test. The result in Table 4.1 strongly indicates that all the series are I(1) process.

Table 4.1 Augmented Dickey–Fuller test

Data	Series	Type of test	ADF P-Value
Before Difference	*FC*	With Constant, Without trend	0.99
	INV	With Constant, Without trend	0.99
	EX	With Constant, Without trend	0.99
	IM	With Constant, Without trend	0.99
	GDP	With Constant, Without trend	0.99
After Difference	*FC*	Without Constant and Trend	0.01
	INV	Without Constant and Trend	0.01
	EX	Without Constant and Trend	0.01
	IM	Without Constant and Trend	0.01
	GDP	Without Constant and Trend	0.01

4.2 Model Specification

The series show a sign with time trend, and the Augmented Dickey–Fuller test confirm that the series is a random walk with dirft, thus a restrict constant is added in the cointegration system. The historical events have great impact to the economic system of Sweden, so dummy variables are included to define the effects of different historical events. The full model can be expressed as (4.1).

$$\Delta Y_t = \Gamma \Delta Y_{t-1} + \alpha ECT_{t-1} + \Phi D_t + \varepsilon_t \tag{4.1}$$

where:

$$\begin{aligned}
&Y_t = (FC, INV, EX, IM, GDP)_t{}',\\
&ECT_{t-1} = \beta'(FC, INV, EX, IM, GDP, Cons)_{t-1}{}',\\
&D_t = (WWI, WWIob, ST, IR, GD)_t{}',\\
&\varepsilon_t \sim IN(0, \Omega).
\end{aligned}$$

4.2.1 Trace Test

Table 4.2 shows the result of trace test for cointegration rank of model (4.1). We have strong confidence to reject the hypothesis that rank ≤ 1, but it is hard to decide whether the cointegration rank should be taken as 2 or 3, because the test statistic of rank = 2 is quite close to the critical value at 1% level. Thus further consideration should be taken to choose the cointegration rank.

Table 4.2 Trace Test

Rank	Eigenvalues	Test statistic	Critical value 1%
r ≤ 4	0.01	1.56	12.97
r ≤ 3	0.06	11.62	24.6
r ≤ 2	0.16	40.39	41.07
r ≤ 1	0.26	91.57	60.16
r ≤ 0	0.40	178.17	84.45

4.2.2 Weight Matrix *α*

By considering that the t-values of the α-coefficients to the third cointegration are not very significant in Table 4.3, so rank = 2 is a preferable choice for the Swedish data. Also, the t-values of *INV* is not significant in the first two columns, which suggests that *INV* might be weakly exogenous such that we can replace the full model with a partial model.

Table 4.3 Weight Matrix α

	α_1	α_2	α_3	α_4	α_5
FC	**0.51**[6.36]	**-0.10**[-2.42]	**-0.45**[-3.94]	0.01[0.25]	0.00[0.01]
INV	0.35[1.78]	-0.08[-0.73]	-0.21[-0.59]	**-0.2**[-2.10]	0.08[0.87]
EX	**0.93**[4.40]	**-0.58**[-5.12]	0.51[1.69]	-0.14[-1.40]	-0.03[-0.34]
IM	**2.44**[9.30]	0.06[0.40]	-0.05[-0.14]	-0.16[-0.22]	-0.03[0.00]
GDP	**0.39**[5.37]	**-0.20**[-5.05]	-0.19[-1.84]	0.00[0.13]	0.02[0.00]

Note: T-values are given in the bracket, bolded if |t-value| > 2

4.2.3 Cointegrating Vectors *β*

According to the economic theory, Final Consumption is the main force of promoting the growth of Gross Domestic Product, so it`s reasonable to assume that they are homogeneous. The estimated cointegrating vectors *β* is given in Table 4.4 and it is easy to get that *FC* and *GDP* has a firm relation in the first two cointegration relations: $FC = -0.9 \times GDP$.

Table 4.4 Cointegrating Vectors *β*

	β_1	β_2	β_3	β_4	β_5	β_6
FC	1.00	1.00	1.00	1.00	1.00	1.00
INV	0.19	0.14	0.10	0.30	-0.01	-1.31
EX	0.43	0.40	-0.01	0.24	0.42	0.35
IM	-0.51	-0.43	-0.04	-0.12	-0.20	0.13
GDP	-1.11	-1.12	-1.03	-1.45	-1.21	0.26
Cons	0.42	0.40	0.16	1.18	0.56	-5.84

Note: Normalized with respect to the first row. *Cons* represent the constant in cointegration.

4.2.4 Test for Restrictions on β and α

Testing the hypothesis raised in the previous section can be done by put corresponding restrictions on the parameters. Matrix *HB* and *HA* will be used to restrict β and α when testing the weakly exogenous and homogeneous hypothesis.

$$H_1: FC = -0.9 \times GDP$$

$$H_2: INV \text{ is weakly exogenous.}$$

$$HB = \begin{bmatrix} 0 & 0 & 0 & -0.9 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad HA = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The results of likelihood ratio test for restrictions are given in Table 4.5. The p-values of tests suggest that both H_1 and H_2 are accepted. Instead of the full model (4.1), a partial model (4.2) is estimated due to the weakly exogenous of *INV*.

Table 4.5 Restriction Test

Hypothesis	LR statistic	P-value	Chi square df
H_1	0.04	0.98	2
H_2	3.42	0.18	2
H_1 and H_2	3.42	0.18	2

4.3 Estimation

The estimated partial model is given in (4.2) under the homogeneous and weakly exogenous conditions.

4.4 Fitted Values and Residuals Diagnostics

Fitted values and residuals plots are given in Figure 4.1. We can see that the fitted values in dotted line are almost coincided with the original data. And the R-square of the VECM are all larger than 0.6 also indicating that the model fitted well. The results of ARCH test for heteroscedasticity, Portmanteau test for serially correlated errors

$$
\begin{bmatrix} \Delta FC_t \\ \Delta EX_t \\ \Delta IM_t \\ \Delta GDP_t \end{bmatrix} = \left[\underbrace{\begin{matrix} \underset{[0.394]}{0.075} & \underset{[-0.143]}{-0.006} & \underset{[0.306]}{0.016} & \underset{[0.455]}{0.019} & \underset{[1.876]}{0.440} \\ \underset{[0.825]}{0.406} & \underset{[-1.203]}{-0.125} & \underset{[0.881]}{0.118} & \underset{[0.721]}{0.077} & \underset{[-1.409]}{-0.855} \\ \underset{[0.999]}{0.584} & \underset{[-0.359]}{-0.044} & \underset{[0.730]}{0.117} & \underset{[1.721]}{0.218} & \underset{[-1.392]}{-1.004} \\ \underset{[0.942]}{0.143} & \underset{[-0.591]}{-0.019} & \underset{[1.238]}{0.051} & \underset{[0.281]}{0.009} & \underset{[1.027]}{0.192} \end{matrix}}_{\Gamma} \right] \begin{bmatrix} \Delta FC_{t-1} \\ \Delta INV_{t-1} \\ \Delta EX_{t-1} \\ \Delta IM_{t-1} \\ \Delta GDP_{t-1} \end{bmatrix}
$$

$$
+ \left[\underbrace{\begin{matrix} \underset{[5.429]}{0.486} & \underset{[-2.277]}{-0.088} \\ \underset{[3.733]}{0.864} & \underset{[-4.954]}{-0.495} \\ \underset{[8.684]}{2.390} & \underset{[0.345]}{0.041} \\ \underset{[4.655]}{0.332} & \underset{[-5.273]}{-0.163} \end{matrix}}_{\alpha} \right] \left[\underbrace{\begin{matrix} 1 & 0.189 & 0.418 & -0.493 & -1.111 & 0.428 \\ 1 & 0.129 & 0.419 & -0.448 & -1.111 & 0.386 \end{matrix}}_{\beta'} \right] \begin{bmatrix} FC_{t-1} \\ INV_{t-1} \\ EX_{t-1} \\ IM_{t-1} \\ GDP_{t-1} \\ Cons \end{bmatrix} \qquad (4.2)
$$

$$
+ \left[\underbrace{\begin{matrix} \underset{[3.115]}{0.102} \\ \underset{[2.437]}{0.207} \\ \underset{[4.274]}{0.432} \\ \underset{[6.592]}{0.173} \end{matrix}}_{\rho} \right] \Delta INV_t + \left[\underbrace{\begin{matrix} \underset{[8.454]}{0.341} & \underset{[2.214]}{0.088} & \underset{[-0.256]}{-0.011} & \underset{[-1.440]}{0.041} & \underset{[-0.105]}{-0.004} & \underset{[-0.186]}{-0.008} \\ \underset{[5.597]}{0.584} & \underset{[4.864]}{0.502} & \underset{[0.166]}{0.019} & \underset{[5.473]}{0.404} & \underset{[-3.421]}{-0.356} & \underset{[-3.036]}{-0.320} \\ \underset{[5.254]}{0.652} & \underset{[3.248]}{0.398} & \underset{[-0.644]}{-0.089} & \underset{[-1.373]}{-0.121} & \underset{[-0.801]}{-0.099} & \underset{[-1.183]}{-0.148} \\ \underset{[8.268]}{0.266} & \underset{[3.896]}{0.124} & \underset{[-0.376]}{-0.013} & \underset{[2.016]}{0.046} & \underset{[-0.996]}{-0.032} & \underset{[-1.770]}{-0.057} \end{matrix}}_{\Phi} \right] \begin{bmatrix} WWI \\ WWIob \\ ST \\ WWII \\ IR \\ GD \end{bmatrix} + \begin{bmatrix} \hat{\varepsilon}_{FC,t} \\ \hat{\varepsilon}_{EX,t} \\ \hat{\varepsilon}_{IM,t} \\ \hat{\varepsilon}_{GDP,t} \end{bmatrix}
$$

and Jarque-Bera tests for normality are given in Table 4.6. All the tests are accepted at 1% level, thus the model is well specified.

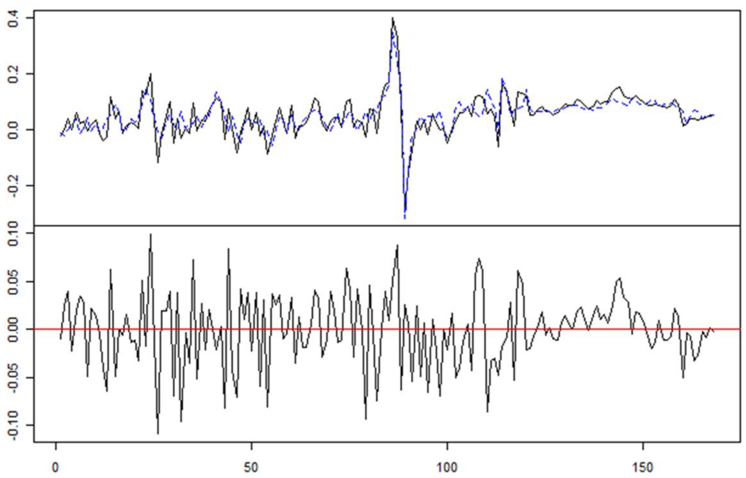
Table 4.6 Residual test

	ARCH	Normality	R-square	Portmanteau Test
FC	0.023	0.314	0.814	
EX	0.027	0.295	0.602	0.1035
IM	0.431	0.625	0.669	
GDP	0.808	0.335	0.862	

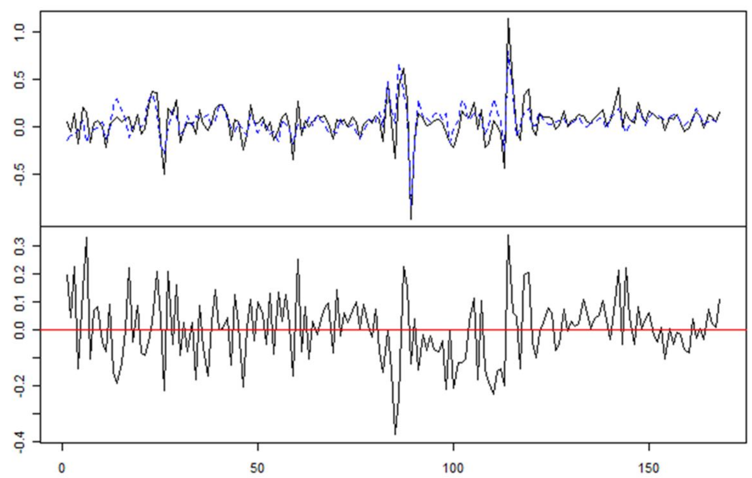
4.5 Interpretation

The estimated model (4.2) gives the coefficients with t-values in the bracket under the homogeneous and weakly exogenous conditions in section 4.2.

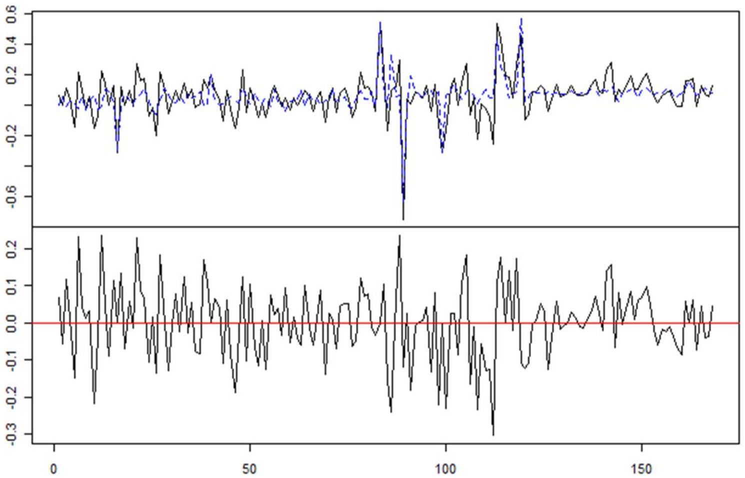
Matrix Γ shows the short-run dynamic relation of each variables. The most



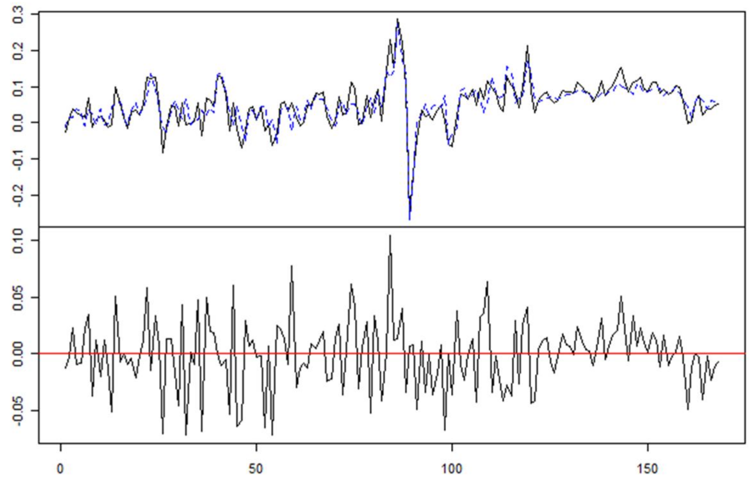
(a)



(c)



(b)



(d)

Figure 4.1 Fitted value in upper graph and residual in lower graph. (a) for FC, (b) for EX, (c) for IM and (d) for GDP.

significant relations are that change of *FC* positively depends on the change in *GDP* of last year, and change of *IM* will follows the same trend of last term.

The trace test suggests cointegration rank is 2, and the corresponding error correction terms are:

$$\begin{aligned} ECT_1 &= FC_{t-1} + 0.189INV_{t-1} + 0.418EX_{t-1} - 0.493IM_{t-1} - 1.111GDP_{t-1} + 0.428 \\ ECT_2 &= FC_{t-1} + 0.129INV_{t-1} + 0.419EX_{t-1} - 0.448IM_{t-1} - 1.111GDP_{t-1} + 0.386 \end{aligned} \tag{4.1}$$

Since the error correction terms is stationary time series, so the existence of cointegration relation offer support to the Swedish economic long-run stability. The highly significant loading matrix α indicates that the error correction terms play a vital role in the VECM model.

The coefficients in ρ describe the relations of the weakly exogenous *INV* and the rest series. The t-value of ρ are larger than 2.4 suggests *INV* has great influence on the economic system.

The Φ matrix shows the impact of different historical event on the series. The most influential event is the World War I, which affects all the series significantly, and the *EX* is the most sensitive indicator which is under the influence of all but *ST*.

5. Conclusions

This paper aims to find the cointegration relationship lying in the Swedish economy and interpret the economic meaning. First, we present that the series follow a random walk process with Augmented Dickey–Fuller test. And then, specify the full model with restricted constant to represent the deterministic time trend and a number of dummy variables to eliminate the unreasonable changes in time series caused by major historical events. After that, the hypothesis of weakly exogenous for *INV* and homogeneous for *FC* and *GDP* are accepted by doing likelihood ratio test for restrictions on parameters. In the end, we achieve the most important result: the Swedish national account has cointegration relations and the cointegration system have considerable effect on the economy. This result gives support to the long-run stability assumption of Swedish national account.

A partial model is estimated under the weakly exogenous and homogeneous assumption. The short run effect is not very significant, indicating that the national account data only have weak connection with the last period. The reason is that the economic system is not totally free, for example, the government will take measures to cool down the over-heated economy by raising tax or tightening credit. The error correction terms are highly significant, supporting that the whole economic system is stable in the long run. The coefficient ρ suggest a strong connection between Investment and the economic system in the same period with t-value larger than 2.4. And the matrix Φ give the influence of different historical events.

Although a desirable outcome is achieved by this paper, it still needs a lot of improvement. The future work can lead to the restriction on the short-run matrix due to its some insignificance.

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Appendix I

Table I Variables Notation

Variable	Notation	Comment
Final Consumption	*FC*	Both private and government.
Investment	*INV*	(-)
Export	*EX*	Recorded on the c.i.f./f.o.b.-basis.
Import	*IM*	Recorded on the c.i.f./f.o.b.-basis.
Gross Domestic Product	*GDP*	Measured by expenditure
Post World War I	*WWI*	0.5 for 1918, -1 for 1921, 0 for else.
Outbreak of World War I	*WWIob*	1 for 1915, 0 for else.
Storm Tide	*ST*	1 for 1872, 0 for else.
World War II	*WWII*	1 for 1945 and 1951, 0 for else.
The Industrial Revolution	*IR*	1 for 1848, 0 for else.
The Great Depression	*GD*	1 for 1931, 0 for else.