Modeling and forecasting monthly electricity price of Sweden with periodic autoregressive models

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Abstract

Autoregressive model is widely used in economic data analysis. This thesis is aiming at finding the best model for modeling and forecasting monthly electricity price of Sweden among different autoregressive models. To fit data from Nord Pool Spot, models such as AR model (Hamilton 1994), periodic AR model (Franses and Paap 1994), periodic VAR model (Lütkepohl 2005) and SAR model (Brockwell and Davis 2002) are applied. Based on results of above models, residuals are tested whether they are white noises. Models whose residuals are white noises are used in forecasting price of one year. Comparing mean square errors of forecast results, the best model is periodic VAR model.

Key words: Autoregressive model, periodic autoregressive model, monthly electricity price of Sweden.

Contents

1. Introduction

Autoregressive model is widely used in economic data. It includes AR model (Hamilton 1994), periodic AR model (Franses and Paap 1994), periodic VAR model (Lütkepohl 2005) and SAR model (Brockwell and Davis 2002). In this thesis, different autoregressive models are applied to model monthly electricity price of Sweden. The aim is to find the best model among these four models.

In history, Denise and Jeremy (1989) have used autoregressive models including AR model, periodic AR model and periodic VAR model on seasonal U.K. consumption. Philip and Richard (1994) have used periodic AR model on several quarterly U.K. macroeconomic data. In this thesis, SAR model is added for comparison with the other models.

In this thesis, data section, based on plots of data and some summary statistics, the features of the data is summarized. In models section, for each model, the first thing is choice of the proper order p. It is decided based on Akaike information criterion, Bayesian information criterion or the plot of partial autocorrelation functions. With the order p chosen, the next step is to test if the model is applicable. Presence of periodicity is tested for periodic AR model. Bivariate Granger causality tests (Lütkepohl 2005) are used for periodic VAR model. Finally, to estimate the coefficients of the models, Yule-Walker estimation (Brockwell and Davis 2002) and maximum likelihood estimation are applied to AR model, periodic AR model and SAR model. For periodic VAR model, ordinary least squares are directly used. In results section, Box–Pierce test (Hamilton 1994) is used upon residuals of each model to find whether they are white noises. Only models whose residuals are white noises can be used for forecasting. In forecasting section, based on data from January in 2000 to February in 2010, monthly electricity prices of March 2010 to February 2011 are forecasted. Mean square errors are calculated as a statistic to find the best model. The best model is periodic VAR model. In conclusion section, the thesis is summarized and some possible improvements are put out.

The outline of the thesis is as follows. Section 2 is data section. Section 3 is models section which estimates the coefficients of models. Section 4 is results section and selects the models can be used for forecasting. Section 5 is forecast section to find the best model. Section 6 is conclusion section.

2. Data

This thesis uses price data from Nord Pool which is a multinational power market consisting of Norway, Sweden, and Denmark. The dataset from Nord Pool Spot has one variable, monthly price. It includes monthly price of Sweden Denmark and Norway from January in 2000 to February in 2011. When a country has different prices in several areas, the mean is calculated as the price of the country.

2.1 Run sequence plot

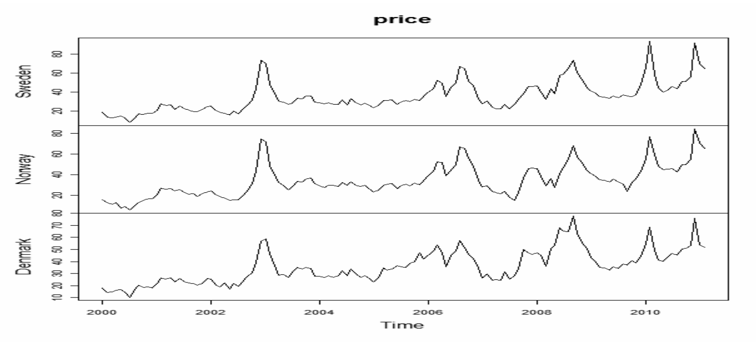

Figure1. Plot of monthly electricity price in three countries

Observing figure 1, the monthly electricity prices of three countries have similar trend. The three countries may have relationship with each other, so it is possible to consider vector autoregressive when modeling the data. To find the details of monthly electricity price in Sweden, figure 2 is plotted.

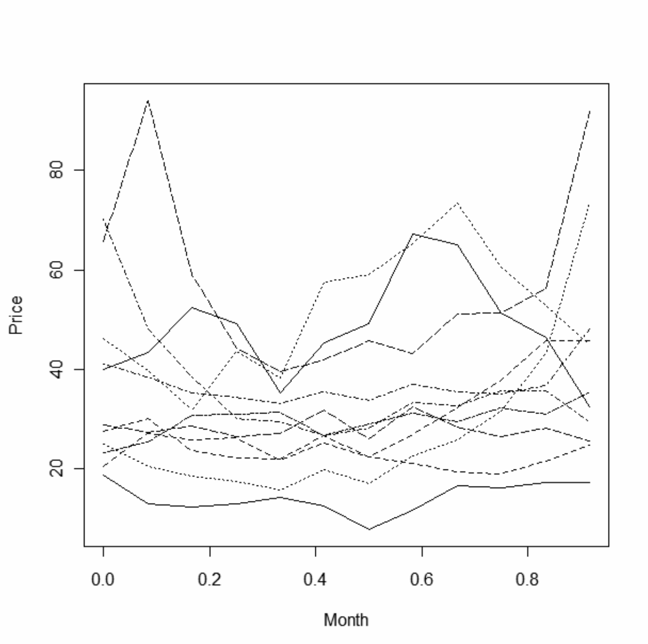

Figure2. Plot of monthly electricity price in Sweden

Note: Y-axis stands for the price of Sweden, X-axis stands for 12 months in a year. For the same month in the different year, price is increasing.

Observing figure 2, price of Sweden usually decreases from January to the middle of the year, then increases to the end of the year. The price for the same month in different year is increasing. For the similar shape of lines, the seasonality should be considered for modeling.

2.2 Summary statistics

Table1. Summary statistics of three countries

	Sweden	Denmark	Norway
Mean	35.27	35.87	33.97
Median	31.70	34.24	30.78
Maximum	94.00	77.67	84.32
Minimum	7.91	10.28	5.94

Note: The unit is sek/MWh, MWh is megawatt hour.

Observing table 1, prices of Sweden and Denmark are higher than Norway in mean and median. According to introduction of Nord Pool, electricity sector of Norway relies mostly on water power; that of Denmark relies mostly on fossil energy and new renewable energy; that of Sweden relies on water power and nuclear power. Comparing three countries, Norway has the lowest cost of electricity, so its price is lower than the other two countries.

3. Models

3.1 AR model

AR model (Hamilton 1994) is the most ordinary autoregressive model, it is expressed as,

$$y_t = \phi_1 y_{t-1} + ... + \phi_p y_{t-p} + \varepsilon_t, \varepsilon_t \sim WN(0,\sigma^2) \text{ which is notated as AR(p),} \quad (1)$$

where t $\in$ {p+1,p+2,...}, p $\in$ Z^+, ϕ_i is cofficient, y_t is price at month t.

To model data with AR(p), the first step is to choose the proper order p. Partial autocorrelation functions (Hamilton 1994), which are to check the dependence between price at month t and month (t-k) removing the effects of variables between them, are plotted in figure 3.

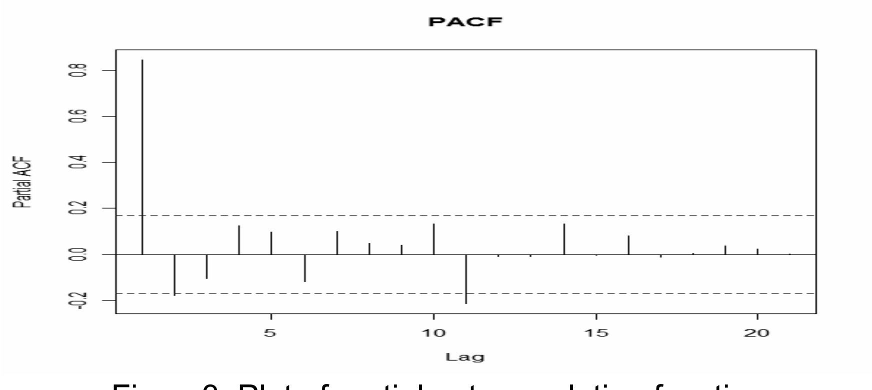

Figure3. Plot of partial autocorrelation function

Observing figure 3, partial autocorrelation between price at month t and (t-2) is not zero. When lag is bigger than 2, the partial autocorrelation can be regarded as zero except that lag is 11. When lag is 11, the pattern of partial autocorrelations has a spike in the figure 3. It shows the seasonality of the data. For AR(p), seasonality is not considered, it will be discussed in the next three models. Without considering the spike at lag 11, p is chosen as 2, it means that the price at month t just depends on the price at month (t-1) and (t-2).

When order p is 2, the model can be given as

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t, \varepsilon_t \sim WN(0, \sigma^2) \tag{2}$$

where $t \in \{3,4,...\}$, ϕ_1, ϕ_2 is cofficient, y_t is price at month t.

Apply Yule-Walker estimation (Brockwell and Davis 2002) to the monthly prices of Sweden from January in 2000 to February in 2011 to get the coefficients of equation (2). In AR(p), seasonality is not included into the model. To improve the model, periodic AR model is used.

3.2 PAR model

Periodic models are one kind of autoregressive models which allow the parameters varying in different seasons. In this way, it includes seasonality of data into modeling. For the monthly electricity price of Sweden, it shows some seasonality from the analysis of figure 3 in 3.1 and the seasonal period is chosen as 12 because it is monthly data.

3.2.1 Model form

The periodic model is given as,

$$y_t = \sum_{i=1}^{12} D_{i,t}(\phi_{1,i} y_{t-1} + ... + \phi_{p,i} y_{t-p}) + \varepsilon_t, D_{i,t} = \begin{cases} 0 & t \text{ is not in the month i} \\ 1 & t \text{ is in the month i} \end{cases}. \tag{3}$$

$\varepsilon_t \sim WN(0, \sigma^2)$, where $\phi_{1,i}, ..., \phi_{p,i}$ are coefficients at i-th month in a year, $i \in \{1,...,12\}$, and y_t is price at month t.

Since a dummy variable D is used in the equation (3), the periodic AR model is not possible to calculate in the same way as AR model. However, equation (3) can be transform into matrix form.

$$A_0 Y_t = A_1 Y_{t-1} + \omega_t, Y_t = (y_{12t-11}, \ldots, y_{12t}), order\ p \le 12, A_0 = \{a_{ij}\}_{12\times12}, A_1 = \{b_{ij}\}_{12\times12}. \quad (4)$$

$$\omega_t = diag(\varepsilon_1, \ldots, \varepsilon_p),\ b_{ij} = \begin{cases} 0 & j \ge 1, i > j \\ \phi_{12-j+i,i} & i \ge 2, j > i \end{cases},\ a_{ij} = \begin{cases} 0 & i \ge 2, j > i \\ 1 & i = j \\ -\phi_{(i-j),i} & j \ge 2, i > j \end{cases}.$$

For the equation (4), it is similar to equation (1), the estimation methods of AR(p) can also be used in periodic AR model. To modeling data with periodic AR model, the first step is to choose proper order p.

3.2.2 Choice on order p

Akaike information criterion and Bayesian information criterion are two widely used statistics to compare the fitness of different models. Usually, a model which has smaller AIC and BIC value fits better than other models. In periodic AR model, AIC and BIC are calculated in each month separately. To get a unique p, models which have smaller sum of AIC and BIC are better fitting.

Table 2. Value of AIC and BIC

	Order p					Order p			
Month	1	2	3	4		1	2	3	4
1	70.18	56.28	57.91	59.87	1	70.18	56.76	58.88	61.33
2	73.24	59.77	56.45	58.36	2	73.24	60.25	57.42	59.82
3	55.62	45.26	46.95	43.42	3	55.62	45.66	47.74	44.61
4	51.55	36.74	37.70	39.50	4	51.55	37.14	38.49	40.69
5	45.48	23.25	25.14	26.05	5	45.48	23.65	25.94	27.25
6	53.80	39.67	38.68	32.57	6	53.80	40.06	39.48	33.76
7	57.62	22.93	18.90	15.45	7	57.62	23.33	19.70	16.64
8	60.75	35.88	37.81	37.75	8	60.75	36.27	38.61	38.94
9	61.93	32.55	32.46	34.40	9	61.93	32.95	33.25	35.59
10	55.64	30.55	30.30	31.58	10	55.64	30.95	31.10	32.77
11	53.63	35.14	26.84	28.26	11	53.63	35.54	27.63	29.45
12	66.36	59.38	52.91	54.85	12	66.36	59.77	53.70	56.05
Sum	705.80	477.40	462.05	462.06	Sum	705.80	482.33	471.94	476.90

Note: This table combines the table of AIC and BIC. The left part is table of AIC and the right part is table of BIC. The AIC and BIC are calculated separately each month. Sums of AIC and BIC are statistics for choosing the proper order p.

Observing table 2, when order p is 1, sums of AIC and BIC are much bigger than other models. When order p is 2 3 or 4, sums of AIC and BIC are close. With the increasing of order p, the difficulty of modeling and forecasting increases much more. To achieve a compromise between accuracy and efficiency, order p is chosen as 2. The model can be given as,

$$y_t = \sum_{i=1}^{12} D_{i,t}(\phi_{1,i} y_{t-1} + \phi_{2,i} y_{t-2}) + \varepsilon_t, \varepsilon_t \sim WN(0, \sigma^2), D_{i,t} = \begin{cases} 0 & t \text{ is not in the month i} \\ 1 & t \text{ is in the month i} \end{cases}, \quad (5)$$

where $\phi_{1,i}, ..., \phi_{p,i}$ are coefficients at i-th month in a year, y_t is the price of month t.

For the equation (4), apply Yule-Walker estimation (Brockwell and Davis 2002) to the monthly prices of Sweden from January in 2000 to February in 2011 to get the coefficients. Based on the equation (4), the coefficients of equation (5) can be got. In periodic AR model, models are set based on the month electricity price data of Sweden, not including data of the other countries. From the analysis of figure 1 in 2.1, the price of Sweden may be influenced by the price of Denmark and Norway. To include data of the other countries, periodic vector autoregressive model is used.

3.3 Periodic VAR model

Periodic VAR model is one kind of periodic autoregressive model which uses vector series. Before modeling the data with vector series, which variables are included in the vector should be decided firstly. In other words, the countries whose prices have influence on Sweden must be found.

3.3.1 Bivariate Granger causality tests

Suppose the model is bivariate, it is given as

$$y_t = c_1 + a_0 y_{t-1} + ... + a_p y_{t-p} + b_1 x_{t-1} + ... + b_p x_{p-1} + u_t, u_t \sim WN(0, \sigma^2) \quad (6)$$

where y_t is monthly price of Sweden, and x_t is monthly price of other country. If y_t is not influence by x_t, the null hypothesis is: $H_0 : b_1 = ... = b_p = 0$. It means that (6) is transformed into univariate case: $y_t = c_1 + a_0 y_{t-1} + ... + a_p y_{t-p} + e_t, e_t \sim WN(0, \sigma^2)$.

Apply ordinary least squares estimation to bivariate and univariate case separately, and calculate the residual sum of squares of each model. The statistic is

$$S0 = \frac{(RSS_0 - RSS_1)/p}{RSS_1/(n-2p-1)} \sim F(p, n-2p-1). \quad (7)$$

where RSS_0 is the RSS in univariate case, RSS_1 is the RSS in bivariate case.

Set the significant level as 0.05. When price of Norway is included in the bivariate vector, p value of S0 is always above 0.1 when order p<5, that does not reject null hypothesis. It means that price of Sweden is not influenced by Norway price. When price of Denmark is included in the bivariate vector, p value of S0 is always below 0.05 when order p<5, that rejects null hypothesis. It means that price of Sweden has some relationship with price of Denmark.

According to the bivariate Granger causality tests results and introduction of Nord Pool, Norway has exported much more electricity than it has imported. Moreover, Norway has the largest amount of electricity using and producing among the three countries. So the electricity market in Norway is not easy to be influenced by other countries. On the other side, Sweden imported just a small amount of electricity every year compared to its producing and using of electricity. So its market is not influence significantly by Norway and Denmark, either. For Denmark, it has the smallest amount of producing and using in electricity among three countries. So the small amount of importing from Sweden has big influence on Denmark. The monthly electricity price of Denmark is influenced by price of Sweden.

3.3.2 Modeling

Similar with equation (4), the model is given as,

$$A_0 Y_t = A_1 Y_{t-1} + \omega_t, Y_t = (y_{12t-11}, ..., y_{12t}), y_t = (y_{1,t}, y_{2,t})^T, p \le 12 \quad (8)$$

where $y_{1,t}$ is price of Sweden, $y_{2,t}$ is price of Denmark, A_0, A_1, ω_t is the same in the equation (4), ϕ_{ij} is coefficients matrice.

Observing equation (8), it is easy to understand but hard to calculate. Extracting the price of Sweden from the vector, equation (8) is transformed into,

$$y_{1,t} = \sum_{i=1}^{12} D_{i,t}(\phi_{1,i} y_{1,t-1} + ... + \phi_{p,i} y_{1,t-p} + \varphi_{1,i} y_{2,t-1} + ... + \varphi_{p,i} y_{2,t-p}) + \varepsilon_t, \varepsilon_t \sim WN(0, \sigma^2) \quad (9)$$

where $y_{1,t}$ is the same in (8), $D_{i,t} = \begin{cases} 0 & t \text{ is not in the month i} \\ 1 & t \text{ is in the month i} \end{cases}$, .

Observing equation, the first step is to choose the proper order p. In section 3.2, order p is chosen as 2 for periodic AR model. If order p is 2, the periodic VAR model will be better than the model in section 3.2 since vector model includes the effects of other countries. With the increasing of order p, the calculation of equation (9) becomes much more difficult. To achieve a compromise, order p is chosen as 1 which is close to value 2.

When order p is 1, equation (9) is transformed into,

$$y_{1,t} = \sum_{i=1}^{12} D_{i,t}(\phi_{1,i} y_{1,t-1} + \varphi_{1,i} y_{2,t-1}) + \varepsilon_t, \varepsilon_t \sim WN(0, \sigma^2). \quad (10)$$

To estimate the coefficients of (10), suppose $\beta_i = (\phi_{1,i}, \varphi_{1,i}), i = 1, ..., 12$ which stand of coefficients of i-th month in a year. Maximum likelihood estimation is widely used for estimating coefficients. To calculated the MLE of β_i, equation (10) is transformed into

$$y_{1,t} = \beta_i y_{t-1} + \varepsilon_t, y_t = (y_{1,t}, y_{2,t})^T, t = i + 12k, 0 < i \le 12, k \in Z. \quad (11)$$

For (11), likelihood function is $\frac{-n}{2}\ln(2\pi\sigma^2)-\frac{1}{2\sigma^2}\sum_{i=t_1}^{t_n}(y_i-\beta_i y_{t-1})^2$, which is $\ln(L(\beta_i,\sigma^2))$, where n is the number of i-th month in a year among the observations.

For fixed β_i, $\frac{\partial \ln(L(\beta_i,\sigma^2))}{\partial(\sigma^2)}=\frac{-n}{2\sigma^2}+\frac{1}{2(\sigma^2)^2}\sum_{i=t_1}^{t_n}(y_i-\beta_i y_{t-1})^2$. If $\frac{\partial \ln(L(\beta_i,\hat{\sigma}^2))}{\partial(\hat{\sigma}^2)}$ is zero, $\hat{\sigma}^2=\frac{1}{n}\sum_{i=t_1}^{t_n}(y_i-\beta_i y_{t-1})^2$. The profile likelihood of β is $\frac{n}{2}-\ln(\frac{2\pi}{n}\sum_{i=1}^{t_n}(y_i-\beta_i y_{t-1})^2)^{2n}$ which is $\ln(L(\beta_i))$. To maximize $\ln(L(\beta_i))$, it is equivalent to minimize $\sum_{i=1}^{t_n}(y_i-\beta_i y_{t-1})^2$, that is the same as OLS estimation.

The result of equation (11) is $\hat{\beta}_i=(Y_{t-1}Y_{t-1}^{\ T})^{-1}Y_{t-1}Y_{1,t}$, $Y_{t-1}=(Y_{1,t-1},Y_{2,t-1})$. As for elements in Y_{t-1}, $Y_{1,t}$ *is* $(...,y_{1,t-12},y_{1,t},y_{1,t+12},...)$ and $Y_{2,t}$ is $(...,y_{2,t-12},y_{2,t},y_{2,t+12},...)$. Other parameters are the same with that in equation (11). The process of calculation of coefficients is similar with Yule-Walker estimation (Brockwell and Davis 2002), which is the ordinary least squares estimation of sample autocorrelations. To include seasonality into model, one way is to use periodic model, the other way is to use seasonal models. Seasonal AR model is used for comparisons with other models.

3.4 SAR model

SAR model is one kind of seasonal ARIMA model of whose moving average term and integrated term is zero. For data in this thesis, period is 12 since it is monthly data. For SAR model notated as seasonal ARIMA(p,0,0)×(P,0,0), p is chosen as 2, to keep the same autoregressive term with the above AR and PAR models. P is chosen as 1 since there is a spike at lag 11 in figure 3. The SAR model is given as,

$$\phi(L)\Phi(L)x_t=c+\varepsilon_t,\ \varepsilon_t\sim WN(0,\sigma^2),\ \phi(L)=1-\phi_1L-\phi_2L^2,\ \Phi(L)=1-\Phi_1L^{12}, \quad (12)$$

where y_t is price of Sweden in month t.

The coefficients of equation (12) are got from maximum likelihood estimation. After calculation of coefficients of models, the results must be discussed whether they support the assumption of models.

4. Results

4.1 Coefficients of models

The coefficients of AR model and PAR model are got separately from applying Yule-Walker estimation to equation (2) and equation (5). Apply ordinary least squares to equation (10) and the coefficients of periodic VAR model are the results. Coefficients of SAR model are from applying maximum likelihood estimation to equation (12). All the coefficients are presented in table 2.

Table3. Coefficients of models

	AR		Periodic AR		Periodic VAR		SAR			
Coefficient	1	2	1	2	1	2	1	2	3	4
Jan.	1	-0.18	0.87	0.22	0.51	0.47	35.55	-1.01	0.17	-0.07
Feb.	1	-0.18	1.56	-0.63	0.95	0.04	35.55	-1.01	0.17	-0.07
Mar.	1	-0.18	0.58	-0.12	-0.11	1.01	35.55	-1.01	0.17	-0.07
Apr.	1	-0.18	0.90	-0.12	-0.42	1.37	35.55	-1.01	0.17	-0.07
May.	1	-0.18	0.67	0.04	0.53	0.35	35.55	-1.01	0.17	-0.07
Jun.	1	-0.18	0.31	0.78	-0.001	1.05	35.55	-1.01	0.17	-0.07
Jul.	1	-0.18	0.90	0.44	0.93	0.06	35.55	-1.01	0.17	-0.07
Aug.	1	-0.18	0.96	0.16	1.55	-0.38	35.55	-1.01	0.17	-0.07
Sep.	1	-0.18	0.68	0.42	0.62	0.42	35.55	-1.01	0.17	-0.07
Oct.	1	-0.18	1.10	-0.41	0.38	0.55	35.55	-1.01	0.17	-0.07
Nov.	1	-0.18	1.76	-0.72	0.69	0.31	35.55	-1.01	0.17	-0.07
Dec.	1	-0.18	3.36	-2.01	1.51	-0.36	35.55	-1.01	0.17	-0.07

Note: This is a table recording coefficients of models in section 3. The first column in AR columns and that in periodic AR columns is coefficient for price of Sweden at month (t-1), and the second in AR columns is coefficient for price of Sweden at month (t-2) in equation (2). For periodic VAR model, the first column is coefficient for price of Sweden at month (t-1) and the second is coefficient for price of Denmark at month (t-1) in equation (10). For SAR model, the first column is coefficient for the constant, the second is coefficient for price of Sweden at month (t-1) in ordinary AR term, the third is coefficient for price of Sweden at month (t-2) in ordinary AR term, and the fourth is coefficient for the price of Sweden at month (t-1) in seasonal AR term in equation (12).

Observing table 3, for AR model and SAR model, its coefficients are not changed with varying of months. For periodic AR and periodic VAR model, the coefficients are different in each month. That is why they are called periodic model.

4.2 Box-Pierce tests

In equation (1) which is AR(p), (3) which is PAR(p), (8) which is periodic VAR(p) and (12) which is ARIMA(p,0,0)×(P,0,0), there is the same term ε_t which is a white noise. All these four models are set based on this assumption. ε_t is the error term, which can be estimated by residuals. If residuals are white noises, ε_t can also be regarded as white noises.

Suppose $\hat{\varepsilon}_t = y_t - \hat{y}_t$ which are residuals, the null hypothesis is $H_0 : \hat{\varepsilon}_t \sim WN(0, \sigma^2)$, and the statistic is $Q(10) = T(\hat{\rho}_1 + ... + \hat{\rho}_{10})$, which converges in distribution to a random variable follows $\chi^2(10)$, where $\hat{\rho}_j$ is the sample autocorrelation between price of Sweden at month t and month (t-j).

Set significant level as 0.01, the statistic for AR model is 15.43, and p value for Q(10) is 0.12. So the test result does not reject the null hypothesis, and shows that the residuals are a white noise. The assumption of AR model that the error term can be regarded as a white noise is supported by this test result.

Applying the Box-Pierce test to periodic AR model, the statistic is 19.31, and p value is 0.04. For periodic VAR model, the statistic is 11.39, and p value is 0.33. For SAR model, the statistic is 16.35, and p value is 0.09. According to test results above, these four models all have residuals which are white noises. It means that the assumptions of models are supported by the results of tests. These four models can all be used in forecast section. To observe the details of the residuals, figure 4 is plotted.

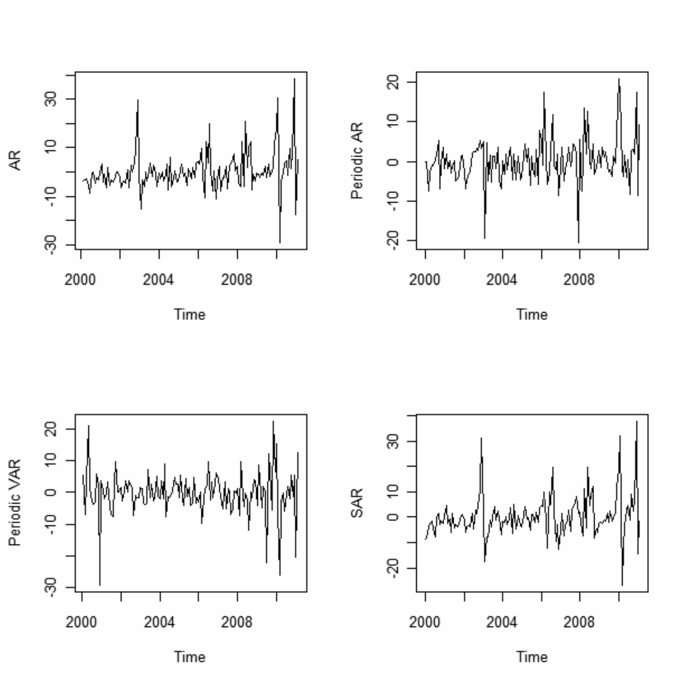

Figure4. Plot of residuals in four models

Observing figure 4, for AR model, residuals are bigger than others in year 2002, 2006 and 2010. For periodic AR model, residuals are bigger than others in year 2002, 2008 and 2010. For periodic VAR model, residuals are bigger than others in year 2001, 2009, 2010. For SAR model, residuals are bigger than others in year 2003, 2006, 2008，2010. The same thing among four plots in figure 4 is that residuals are bigger in the end and the beginning of a year. It may have some relationship with the season. When it is the end or the beginning of a year, the season is winter, and the using of electricity becomes much more than usual. If the weather changes suddenly, it makes the market more unpredictable.

5. Forecasting

In section 4, the four models all satisfy the assumption of the model. To choose the best model among these ones, forecast performance is considered. In this thesis, monthly electricity prices of Sweden from March 2010 to February 2011 are forecasted based on data from January in 2000 to February in 2010. The first step is t o set four new models on new data. Secondly, using new models and data, forecast the price of Sweden from March in 2010 to February in 2011 one by one. To compare the forecast performance, the model whose mean square error that is smaller is better than others. Mean square error is the expectation of the squares of error, which can be estimated as $\mathrm{MSE} = \frac{1}{12}\sum_{i=123}^{134}(y_t - \hat{y}_t)^2$, where y_t is the sample value of price at month t, and $\hat{y}_t$ is the forecast value of price at month t.

Table4. Mean square error of models

	AR	Periodic AR	Periodic VAR	SAR
MSE	585.48	430.03	178.52	896.71

Observing table 4, periodic VAR model has the smallest mean square error, which shows it has the best forecast performance. Mean square error just measure overall forecast performance. To observe the details of forecast performance, figure 5 is plotted.

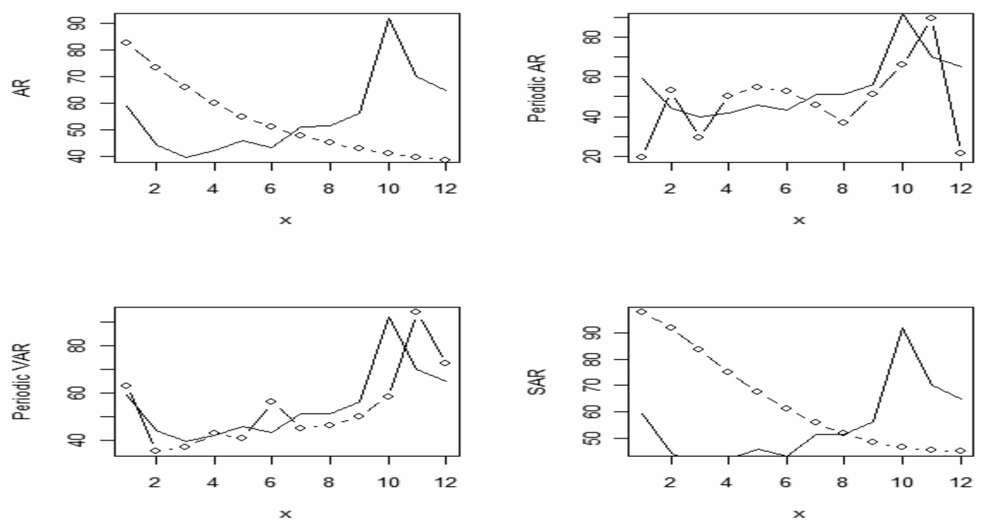

Figure5. Plot of sample and forecast values of price

Notes: In this figure, point lines stand for the forecast values, and solid lines stand for sample values. Y-axes have names of modes which are used for forecasting.

Observing figure 5, forecast values of AR model and SAR model have different trends comparing with sample values. For AR model, it is because seasonality is not included. For SAR model, apply SAR integrated model to the new data, it is given as

$x_t = (1-L)^d (1-L^{12})^D y_t$, $\phi(L)\Phi(L)x_t = c + e_t, e_t \sim WN(0,\sigma^2)$, y_t is price at month t.
where $\phi(L) = 1-\phi_1 L - ... - \phi_p L^p, \Phi(L) = 1-\Phi_1 L^{12} - ... - \Phi_P (L^{12})^P$.

When p, d, P and D are in {0,1,2,3,4}, the smallest value of mean square error among different SAR integrated models is 322.3756. So the SAR model can forecast better than ordinary AR model. In figure 5, the SAR model does not forecast well because its autoregressive orders are not chosen properly.

In figure 5, periodic AR model forecast better than AR model and SAR model, but it has large error at the month 1 and 12. Considering that month 1 and 12 are in winter, it has been discussed in section 4.2 on figure 4 that the electricity market is much more unpredictable in winter. Periodic VAR model forecast best among all the models, whether comparing mean square errors or observing figure 5. It is the best model for modeling and forecasting monthly electricity price of Sweden.

6. Conclusion

In this thesis, it is aimed at finding the best model for modeling and forecasting monthly electricity price of Sweden among various autoregressive models. The data is from Nord Pool which is monthly electricity price including Sweden, Denmark and Norway. To model on the data, AR model, periodic AR model, periodic VAR model and SAR model are used. Based on results of models, the best model is chosen if the results support the assumption and the model has best forecast performance. Periodic VAR model is chosen as the best model for modeling and forecasting monthly electricity price of Sweden.

In the process of modeling, one step is to choose order p. In this essay, order p which is bigger than 12 is not considered. The difficulty of modeling and forecasting will increase with the increasing of order p. But for monthly data whose period is 12, bigger order p may make the model more fit to the real market.

In this thesis, monthly electricity price of Sweden is considered to be influenced by past prices of Sweden and Denmark. But in some months, the price may be influenced by some factors outside the electricity market of Nord Pool. For example, electricity produced from wind power in Sweden has been increasing fast. With the development of new renewable power, the electricity price of Sweden will changes because the cost of electricity production changes. For future research, these factors can be taken into modeling.

Appendices

1. Yule-Walker Estimation (Brockwell and Davis 2002)

Yule-Walker estimation is a widely used method to estimate the coefficients of autoregressive models. Suppose autoregressive model with order p is given as,

$$y_{t+1} = \phi_1 y_t + \ldots + \phi_p y_{t-p+1} + \varepsilon_{t+1}, \varepsilon_t \sim WN(0,\sigma^2).$$

The equation above is multiplied on both sides by y_t, and also taken expectance on both sides. It transform into,

$$\mathrm{E}(y_t\ y_{t+1}) = \phi_1 E(y_t y_t) + \ldots + \phi_p E(y_t y_{t-p+1}) + E(y_t \varepsilon_{t+1}), \varepsilon_t \sim WN(0,\sigma^2).$$

y_t has no relationship with ε_{t+1}, which means that $E(y_t \varepsilon_{t+1})$ equals zero. It is transformed into $\mathrm{E}(y_t\ y_{t+1}) = \phi_1 E(y_t y_t) + \ldots + \phi_p E(y_t y_{t-p+1})$, which can be transformed into $\hat{\gamma}_1 = \phi_1 \hat{\gamma}_0 + \phi_2 \hat{\gamma}_1 + \ldots + \phi_p \hat{\gamma}_{p-1}$, $\hat{\gamma}_i$ is i-th order sample covariance, by dividing both sides with (N-1), N is the amount of observations. By dividing both sides with $\hat{\gamma}_0$, finally the equation is transformed into $\hat{\rho}_1 = \phi_1 \hat{\rho}_0 + \ldots + \phi_p \hat{\rho}_{p-1}$, $\hat{\rho}_i$ is autocorrelation. Similarly, there is $\hat{\rho}_2 = \phi_1 \hat{\rho}_{-1} + \ldots + \phi_p \hat{\rho}_{p-2}$ and so on, noting that $\hat{\rho}_{-i} = \hat{\rho}_i$.

The results of the transformation for these equation form a matrix equation which is given as,

$$P\Phi = p, \text{ where } \Phi = (\phi_1, \ldots, \phi_p)^T, p = (\hat{\rho}_1, \ldots, \hat{\rho}_p)^T, P = (\rho_{ij})_{p \times p}, \rho_{ij} = \hat{\rho}_{|i-j|}.$$

The coefficients of AR(p) calculated by Yule-Walker estimation is $\hat{\Phi} = P^{-1} p$.

2. Part of codes

Some of these codes may help understand the content of thesis.

 # Data and packages

```
Price=read.table("dt.txt",header=T); library(tseries); library(MSBVAR);
library(pear)
Sweden=ts(Price$Sweden,start=c(2000,1),end=c(2011,2),frequency=12)
```

```
Denmark=ts(Price$Denmark,start=c(2000,1),end=c(2011,2),frequency=12)
price=ts(Price,start=c(2000,1),end=c(2011,2),frequency=12)
```

 # Plot of figure 2

```
Newsweden=matrix(rep(0,132),nrow=12)
Newsweden=as.data.frame(Newsweden)
for(i in 1:11){
A=12*i-11; B=12*i
Newsweden[,i]=Sweden[A:B]}
newsweden=ts(Newsweden,start=c(0,1),end=c(0,12),frequency=12)
ts.plot(newsweden,gpars=list(xlab="Month", ylab="Price", lty=c(1:12)))
```

 # Modeling of AR model

```
ts0=ar.yw(ts(Sweden)); res.0=ts0$resid
Box.test(res.0,lag=10,type=c("Box-Pierce"))
# All the Box-Pierce tests in this thesis use the same function and parameters.
```

 # Modeling of periodic AR model

```
plot(Sweden); pepacf(Sweden, plot=TRUE); peacf(Sweden)
ts1=pear(Sweden,2); res.1=residuals(ts1)
```

 # Modeling of periodic VAR model

```
granger.test(price,p) # p is in {1,2,…,5}.

b0=NULL; y1=NULL; y2=NULL; y=NULL; Y=NULL; RES=NULL;
RES0=NULL; N=length(Sweden)

for(i in 1:12){
n=floor((N-i-1)/12); y1=Sweden[i]; y2=Denmark[i]; y=Sweden[i+1]
for(j in 1:n){
y1=c(y1,Sweden[i+12*j]); y2=c(y2,Denmark[i+12*j]);
y=c(y,Sweden[i+1+12*j])}
Y=cbind(y1,y2); b=solve(t(Y)%*%Y)%*%t(Y)%*%y; RES=Y%*%b-y;
b0=cbind(b0,b); RES0=c(RES0,RES)}
b0# Coefficients of periodic VAR model
RES00=ts(RES0,start=c(2000,2),end=c(2011,2),frequency=12)# Residuals.
```

 # Modeling of SAR model

```
ts3=arima(Sweden, order = c(1,0,0), seasonal = list(order=c(1,0,0)))
```

 # Forecast using periodic AR mode

```
dt=Sweden[123:134]; nts0=ar(ts(Sweden[1:122]),1);
Dt0=predict(nts0,n.ahead=12); dt0=Dt0$pred; MSE0=sum((dt-dt0)*(dt-dt0))/12
```

 # Forecast using periodic AR model

```
new=ts(Sweden[1:122],start=c(2000,1),end=c(2010,2),frequency=12)
nts1=pear(new,2); Phi=nts1$phi; nPhi=NULL

for(i in 3:12){
nPhi1=Phi[i,]; nPhi=rbind(nPhi,nPhi1)}
nPhi=rbind(nPhi,Phi[1,]); nPhi=rbind(nPhi,Phi[2,]); Old1=Sweden[122:133];
Old2=Sweden[121:132]; Old=rbind(Old1,Old2)
Ans=nPhi%*%Old; dt1=diag(Ans); MSE1=sum((dt1-dt)*(dt1-dt))/12
```

 # Forecast using periodic VAR model

```
b1=NULL; y1=NULL; y2=NULL; y=NULL; Y=NULL; RES=NULL
RES1=NULL; N=length(Sweden); Old1=Sweden[122:133];
Old2=Denmark[122:133]; Old=cbind(Old1,Old2)

for(i in 1:12){
n=floor((N-i-1)/12); y1=Sweden[i]; y2=Denmark[i]; y=Sweden[i+1]
for(j in 1:(n-1)){
y1=c(y1,Sweden[i+12*j]); y2=c(y2,Denmark[i+12*j]);y=c(y,Sweden[i+1+12*j]}
Y=cbind(y1,y2); b=solve(t(Y)%*%Y)%*%t(Y)%*%y; RES=Y%*%b-y
b1=cbind(b1,b); RES1=c(RES1,RES)}
newb1=NULL
for(i in 2:12){
newb=b1[,i]; newb1=cbind(newb1,newb)}
newb1=cbind(newb1,b1[,1])
Ans=Old%*%newb1; dt2=diag(Ans); MSE2=sum((dt-dt2)*(dt-dt2))/12
```

 # Forecast using SAR model

```
nts3=arima(Sweden[1:122],order=c(2,0,0),seasonal=list(order=c(1,0,0),period=1
2)); Dt3=predict(nts3,12); dt3= Dt3$pred; MSE3=sum((dt-dt3)*(dt-dt3))/12
```

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