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Modeling the Volatility of Shanghai Composite Index with GARCH Family Models

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Abstract

Since GARCH family models have been introduced to the world, people often use them to analyse volatility and they usually fit well. The aim of this paper is to find a better GARCH model to fit the Chinese Shanghai Composite Index data. This paper shows some properties of the common daily stock data, then it demonstrates the estimation results of the GARCH model and the asymmetric power ARCH (APARCH). At last, we compare the modeling results between GARCH model and APARCH model and conclude that APARCH model with AR(1) process has better performance.

Keywords: stock data, volatility, ARMA,GARCH, APARCH

CONTENT

1.Introduction

1.1 Background

Nowadays, with the development of economics all around the world, more and more people are concerned about the everyday change of the financial market. Most people prefer a higher rate of return than just putting money into banks to get less interest. To invest in stock market is a very common choice for those investors. It is well-known that higher benefit usually comes with higher risk in the investment environment. Then it is reasonable that the changes and trends issues of the stock market are concerned by investors much more.

People can get information about the situation of the stock market in many ways. Among all the symbols of the stock market situation, volatility is an essential label. "*For a definition, volatility is a measure for variation of the price of a financial instrument over time*" (Lin C, 1996). Therefore the future stock price uncertainty could be presented as volatility. Modeling and forecast volatility need new models other than the traditional linear regression models. The well-known traditional linear tools have their own limitations in the application since they always ignore the heteroskedasticity of the daily stock data.

1.2 Literature Review

The demand of allowing investors to handle volatility which is treated as the varying trends of the financial market urges people to create this new kind of model for heteroskedasticity. The high-frequency stock market daily data usually has some special properties such as fat-tailness, excess kurtosis and skewness. Modelling volatility of this kind data is a very tough task. As mentioned before, the most important property of the data, heteroskedasticity means that the error term variance of the data is not a constant. This property caused the limitation of traditional linear financial instruments. The first model of conditional heteroskedasticity, the autoregressive conditional heteroskedasticity (ARCH) was brought up by Engle (1982). Then the Generalized ARCH (GARCH) model which is proposed by Bollerslev (1986) and Taylor (1986) has replaced the ARCH model in most applications. After that, many GARCH family models came out, like Asymmetric Power ARCH (Ding, et al, 1993), Exponential GARCH (Nelson,1991), Glosten-Jagannathan-Runkle GARCH (GJRGARCH) model composed by Glosten, Jagannathan and Runkle (1993). Among the worldwide introduced GARCH family models, the overwhelmingly most popular GARCH model in applications has been the GARCH(1,1) model (Teräsvirta, 2006). GARCH family models are regarded as the best tools to analyse financial data considering the heteroskedasticity. Among them, GARCH (1,1) is widely used for this kind of data which has fat-tailness, excess kurtosis and skewness, because of those properties, we can also connect it to the Asymmetric Power ARCH model.

For the Shanghai composite index, Zhang, Cheng and Wang(2005) has tried to use GARCH(1,1) and EGARCH(1,1) to analyse it. The time period of the data they used for the paper is very short and they focused on the characteristic of EGARCH model in the empirical research.

1.3 Aim

The aim of this paper is to find the model which fits the high-frequency stock market data better between the mostly used GARCH(1,1) and APARCH(1,1) models. We also compare the results for all the models when they are with or without ARMA(1,0) process

2.Data

This paper uses the common stock index as the original dataset. In People's Republic Of China, two different stock exchanges are operated separately. One is Shanghai stock exchange, the other one is Shenzhen stock exchange. The Shanghai stock exchange is established much earlier and has larger amount of listed companies, listed stocks and more capitalization value than Shenzhen stock exchange. In summary, based on the technology and the geographical advantage of Shanghai, the Shanghai stock exchange becomes the most important and the main exchange in China. The above conditions are the reasons why the data from the Shanghai stock exchange will be used in our application.

The Shanghai Stock Exchange was established on December.19th.1990 and the time period of the dataset in this article starts from the very beginning and ends in December. 8th. 2011.

2.1 Index returns

Here is the definition of index return: let $\{y_t\}$ be the time series of the daily price of some financial asset. The serially return on the tth day is defined as

$$r_t = \log(y_t) - \log(y_{t-1})$$

Sometimes these time series of the return price are multiplied by 100 which make sure that they can be interpreted as percentage changes. The multiplication may reduce numerical errors since the original returns could be very small numbers.

2.2 Plots of data

Below is the plot of the daily data of the Shanghai Composite Index, including everyday high, low and closing price data and closing prices were used as the target to analyse. The Figure 1 shows this time-varying time series data has an obviously variability and apparently huge changes from 2006 to 2008.

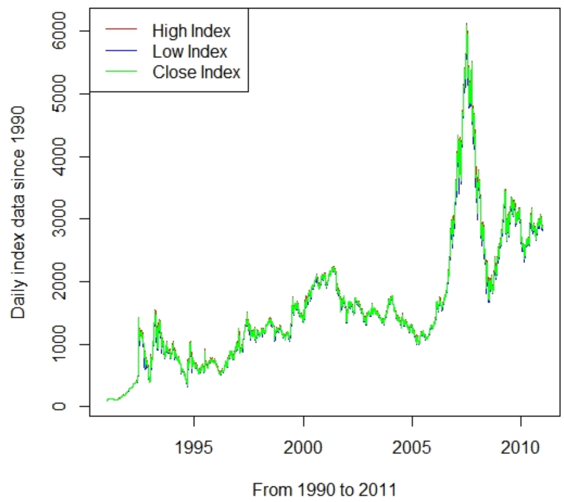

Figure 1: Plot of original daily data

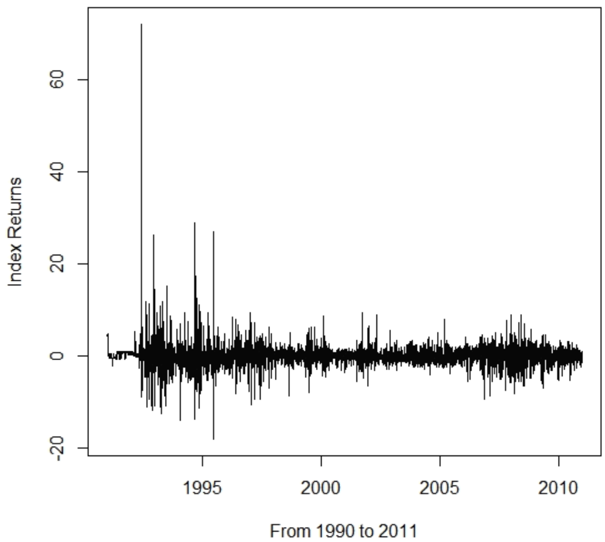

Figure 2: Plot of index return data

The Figure 2 is the plot of index return data. The volatilitys would be analysed by using the index return data instead of the original index data through the way that using the 100-times-log difference of each daily closing price which is mentioned in section 2.1.

2.3 Summary Statistics

The Table 1 demonstrates some summary statistics of the data.
There are 5137 observations in total in this dataset. After the log-difference process, we have 5136 observations. We can see that the mean and median of the data are both very near zero and the standard deviation is quite small. The kurtosis and skewness are excess which are exactly the characteristics of financial time series.

Table 1 : Summary Statistics of Shanghai Index Returns

	Values
Min.	-17.910
Max.	71.920
Median	0.061
Mean	0.067
Std. Dev.	2.523
Skewness	5.375
Kurtosis	141.661
Number of obs.	5136

Note: This table shows summary statistics of returns of Shanghai Composite Index daily closing price by using 100 times log-differences.

2.4 ARCH-LM test

The ARCH-LM test is a Lagrange multiplier (LM) test which is frequently used to test for the lag length of ARCH errors, in other words, the ARCH-LM test is about testing whether the series has ARCH effects at all (Engle, 1982).

For the ARCH-LM test, we run a regression here,

$$\varepsilon_t^2 = \alpha_0 + (\sum_{i=1}^{q} \alpha_i \varepsilon_{t-q}^2) + e_t$$

In this regression, $\alpha_1 = \alpha_2 = \ldots = \alpha_q = 0$ is the null hypothesis and the alternative hypothesis is $H_1 : \alpha_1 \neq 0$ or $\alpha_2 \neq 0$ or $\ldots \alpha_2 \neq 0$ which means there would be at least one of the estimated parameter α_i significant for sure. Besides, the test statistics follows χ^2 distribution with q degrees of freedom.

The p-value of the test in this paper equals to 0.0108 which is a very small one. Therefore the null hypothesis that the series has no ARCH effects has been rejected. The result of the test allows us to fit GARCH family models to the data.

2.5 Autocorrelation function

Figure 3 shows the autocorrelation function results. If a financial time series is autocorrelated, it means that the future values should depend on current and past value which suggests that it is predictable. The results prove that the returns, return square, especially the absolute value of returns are frequently autocorrelated.

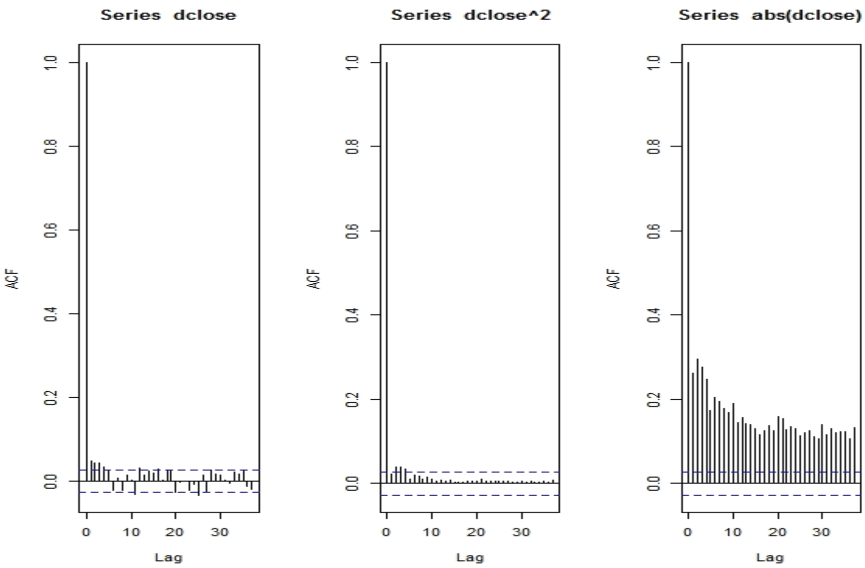

Figure 3: Autocorrelation Function of Returns, squared returns and absolute value of returns

3.Methodology

Engle proposed a new model in 1982 which is called AutoRegressive Conditional Heteroskedasticity (ARCH). This is the first model of conditional heteroskedasticity. Afterwards in 1986, Bollerslev brought up Generalized ARCH (GARCH) model. It replaced ARCH model in most applications eventually. In 1993, Ding, Engle and Granger, they carried out an asymmetric model called Asymmetric Power ARCH (APARCH). Models of Autoregressive Conditional Heteroskedasticity (ARCH) are the most popular way of parameterizing this dependence(Teräsvirta, 2006).

3.1 ARMA, ARCH, GARCH

The return series of a financial asset, $\{ r_t \}$, is often a serially sequence with zero mean and exhibits volatility clustering. This suggests that the conditional variance of $\{ r_t \}$ given past returns is not constant (Cryer and Chan, 2008). This is the reason why we cannot apply a simple linear regression to this kind of data since they consider the series as homoskedastic series indeed.

Here is the standard time series model:

$$r_t = E\,(r_t \mid \Omega_{t-1}) + \varepsilon_t$$

Following Bollerslev (1986), r_t here denote a real-valued discrete-time process with conditional mean and variance both varies with Ω_{t-1}, where Ω_{t-1} is the information set of all information through time *t*. r_t is also regarded as return in this paper.

Then we can take an AutoRegressive MovingAverage (ARMA) (p,q) model as the mean equation,

$$E\,(r_t \mid \Omega_{t-1}) = \mu_t(\theta)$$

$$\mu_t(\theta) = \varphi_0 + \varphi_1 r_{t-1} + \cdots + \varphi_p r_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$$

In financial applications, it is the common practice to apply ARMA model to the return series as mean equation. Since it may be too restrictive to assume that the observed process is a pure GARCH, adding up an ARMA part considerably extends the range of applications.

Then an ARCH model (Engle,1982) can be treated as the variance function,

$$Var(r_t \mid \Omega_{t-1}) = E(\varepsilon_t^2 \mid \Omega_{t-1}) = h_t(\theta)$$

The ARCH(q) process function form is like this:

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2$$

The ARCH process introduced by Engle(1982) clearly recognizes the difference between the unconditional and the conditional variance allowing the latter to change over time as a function of past errors (Bollerslev, 1986).

As we mentioned above, in most applications, the ARCH model has been replaced by the Generalized ARCH (GARCH) model that Bollerslev (1986) and Taylor (1986) proposed independently.

The GARCH (p,q) process is given as

$$r_t = E(r_t \mid \Omega_{t-1}) + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 h_{t-1} + \cdots + \beta_p h_{t-p}$$

Here, the conditional distribution of ε_t is supposed to be normally distributed with zero mean and the conditional variance equals to h_t, same as ARCH process.

We call the term $\sum_{i=1}^{p} \beta_i h_{t-i}$ GARCH term. For p=0, the process is simply the ARCH(q) process; for p=q=0, ε_t is only white noise.

Usually the simplest GARCH(1,1) model works very well:

$$r_t = E(r_t \mid \Omega_{t-1}) + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$$

where $\omega > 0, \alpha > 0, \beta > 0$

3.2 APARCH

In particular, Ding, et al (1993) investigated the autocorrelation structure of $|r_t|^\delta$, where r_t is the daily stock market returns, and δ is a positive number as the function power. They found $|r_t|$ had significant positive auto-correlations for long lags which we also concluded in section 2.5. Motivated by this empirical result they proposed a new general class of ARCH models, which they call the Asymmetric Power ARCH (APARCH).

The variance equation of APARCH(p,q) can be written as:

$$\varepsilon_t = z_t \sqrt{h_t}$$

$$z_t \sim N(0,1)$$

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^{\delta} + \sum_{j=1}^{q} \beta_j h_{t-j}$$

$$\omega > 0, \delta > 0, \alpha_i > 0$$

$$-1 < \gamma_i < 1, i = 1, \ldots, p,\ \ \beta_j > 0, j = 1, \ldots q$$

This model is quite interesting since it couples the flexibility of a varying exponent with the asymmetry coefficient (to take the "leverage effect" into account). Moreover, the APARCH includes seven other ARCH extensions as special cases, such as:

- ARCH when $\delta = 2, \gamma_i = 0(i = 1, \ldots p), \beta_j = 0(j = 1, \ldots q)$
- GARCH when $\delta = 2, \gamma_i = 0(i = 1, \ldots p)$
- Taylor and Schwert GARCH(TS-GARCH) when $\delta = 1, \gamma_i = 0$
- Glosten, Jagannathan, and Runkle(GJR-GARCH) when $\delta = 2$
- T-GARCH when $\delta = 1$
- N-GARCH when $\gamma_i = 0(i = 1, \ldots p), \beta_j = 0(j = 1, \ldots q)$
- Log-ARCH Model when $\delta \to 0$

3.3 AIC & BIC

The Akaike information criterion (AIC) is to measure the relative goodness of fitting statistical models. It was developed by Hirotsugu Akaike, under the name of "an information criterion" (AIC), and was first published by Akaike in 1974. In the general case, the traditional AIC defined as

$$AIC = 2k - 2\ln(L)$$

where k is the number of parameters in the statistical model, and L is the maximized value of the likelihood function for the estimated model.

In this paper, AIC is calculated like this,

$$AIC = (2\,|\ln(L)| + 2k) / N$$

N here stands for sample size.

Among a group of candidate models for the data, the preferred model is the one with the minimum AIC value. Based on that, we can also come to another conclusion that we need a maximum value of the log-likelihood function because that the larger log-likelihood we have, the smaller AIC value we will get.

Besides, the Bayesian information criterion (BIC) is also a criterion for model selection. It is based on the likelihood function and it is quite closed to Akaike information criterion (AIC).

The formula for the BIC is:

$$-2 \cdot \ln p(x\,|\,k) \approx BIC = -2 \cdot \ln L + k \ln(n)$$

The assumption that the model errors or disturbances are independently and identically distributed according to a normal distribution is precisely suitable for the application case in this paper. Therefore the AIC & BIC values are two benchmarks for choosing a better fitting model here.

4.Application results

Both of the estimations are processed under the conditional distribution normality and processing with the 5000 observations. After the estimation, as mentioned before, the comparison between GARCH models is based on those criteria.

Table 2: Estimation results of parameters in GARCH models within standard error in brackets

	GARCH(1,1)	ARMA(1,0) +GARCH(1,1)	APARCH(1,1)	ARMA(1,0) +APARCH(1,1)
μ	0.023	0.023	0.044	0.047
	(0.021)	(0.021)	(0.007)	(0.026)
AR(1)	----------	0.005	----------	0.052
		(0.016)		(0.025)
ω	0.062	0.062	0.049	0.048
	(0.010)	(0.010)	(0.007)	(0.006)
α_1	0.259	0.259	0.214	0.215
	(0.019)	(0.019)	(0.013)	(0.013)
β_1	0.800	0.800	0.831	0.829
	(0.012)	(0.012)	(0.010)	(0.010)
γ_1	----------	----------	-0.067	-0.083
			(0.024)	(0.025)
δ	----------	----------	0.964	0.895
			(0.008)	(0.009)

Table 2 shows the estimation results of GARCH models. Since the standard GARCH(1,1) and APARCH(1,1) do not have an AutoRegressive process, then they do not have the parameter φ in that process. For another, APARCH process has two unique parameters γ and δ which GARCH process does not.

From Table 2, we can see that all the parameters come up with very small standard error which shows their significance.

Following the methodology, the first measuring of the comparison is log-likelihood which needs a larger value and then the AIC value and BIC value are opposite which require smaller value.

From Table 3, we can see that two APARCH models have both smaller AIC value and BIC value, meanwhile, a larger log-likelihood which all suggests that APARCH models fitted this data better than the well-known standard GARCH models. Between these two APARCH models, ARMA(1,0) plus APARCH(1,1) model performs even better which certifies that when we allow for an ARMA part, it will obtain a better result. This result shows that even though the GARCH models are capable of most the analysis of financial return time series, sometimes it still depends on the data itself, ARMA or other linear tools may remain useful in practical cases. But the ARMA(1,0)+GARCH(1,1) model does not show any better result in standard GARCH models which shows limitation and uncertainty of ARMA process.

Table 3: Comparison of GARCH family models

	GARCH(1,1)	ARMA(1,0) +GARCH(1,1)	APARCH(1,1)	ARMA(1,0) +APARCH(1,1)
Log-likelihood	-10256.69	-10255.27	-10213.76	*-10208.74*
AIC value	4.104	4.104	4.088	*4.086*
BIC value	4.109	4.111	4.096	*4.095*

5.Conclusion

Among all the estimations and comparisons based on some certain criteria, all the parameters are significant which shows that the Shanghai Composite Index data could be modeled by GARCH family models appropriately. The comparison result suggests that APARCH models could fit the data better. The best performance is from ARMA+APARCH model which suggests that it is efficient to combine the mean and variance function together.

However, adding ARMA process could not confirm that the goodness-of-fit would always show a better result for sure, such as the case of standard GARCH models, although it is considered to extend the range of applications.

All the results above shows that there may exist some special properties in Chinese stock index data. The time period of the data for this paper is quite long and there are very obvious up and down during the year 2006 and 2008, That could be a break in this data which is on the very advanced level modeling and forecasting and far beyond the range of this paper.

Another assumption in this paper is that the distribution of the error terms is supposed to be normally distributed. We could also consider it under the student-t distribution since t distribution has some properties such as fat-tailness which is exactly the same as the financial time series data. Estimations and comparisons by considering different distributions could be more beneficial for the applications.

The stock index is influenced by a lot of external factors such as inflation, tight or loose monetary policy and so on. It could hardly be predicted or modeled very accurately. Especially for China, the research of the composite index is much less than the other famous stock index around the world. Considering more different distributions, more different GARCH family models could be our further study.

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